

T5: ECUACIONES

1.- Resolver las ecuaciones: (3 ptos)

a) $3x^2 - 27 = 0$

b) $-(x^2 - 3) = 5x + 3$

c) $(x - 3) \cdot (x + 2) = 0$

d) $x^2 - 2x + 3 = 0$

2.- Resolver las ecuaciones: (2 ptos)

a) $\frac{x-1}{6} - \frac{x-2}{2} - \frac{x-3}{3} = 2$

b) $-\frac{x-5}{6} = \frac{x-1}{9} - \frac{x-3}{4}$

3.- Resolver las ecuaciones: (2 ptos)

a) $x^4 + 3x^2 - 4 = 0$

b) $\frac{x^2 - 1}{x^2 + 1} = \frac{x^2 - 1}{3}$

4.- Resolver las ecuaciones:

a) $2x^4 + 7x^3 - 4x^2 = 0$

b) $x^3 + x^2 + x = 0$

(2 puntos)

5.- Sin resolver las ecuaciones, indicar razonadamente el número de soluciones que tienen:

a) $x^2 - 5x + 7 = 0$

b) $x^2 - 4x + 4 = 0$

(1 punto)

$$\textcircled{1} \text{ a) } 3x^2 - 27 = 0 \Rightarrow 3x^2 = 27 \Rightarrow x^2 = \frac{27}{3} \Rightarrow x^2 = 9 \Rightarrow \boxed{x = \pm\sqrt{9} = \pm 3}$$

$$\text{b) } -(x^2 - 3) = 5x + 3 \Rightarrow -x^2 + 3 = 5x + 3 \Rightarrow 0 = x^2 + 5x + \cancel{3} - \cancel{3} \Rightarrow$$

$$\Rightarrow x^2 + 5x = 0 \Rightarrow x(x + 5) = 0 \begin{cases} \boxed{x = 0} \\ x + 5 = 0 \Rightarrow \boxed{x = -5} \end{cases}$$

$$\text{c) } (x - 3) \cdot (x + 2) = 0 \Rightarrow \begin{cases} x - 3 = 0 \Rightarrow \boxed{x = 3} \\ x + 2 = 0 \Rightarrow \boxed{x = -2} \end{cases}$$

$$\text{d) } x^2 - 2x + 3 = 0 \begin{cases} a = 1 \\ b = -2 \\ c = 3 \end{cases}$$

$$\text{Como } \Delta = b^2 - 4ac = (-2)^2 - 4 \cdot 1 \cdot 3 = 4 - 12 = -8 < 0 \Rightarrow \boxed{\nexists \text{ soluc. real}}$$

$$\textcircled{2} \text{ a) } \frac{x-1}{6} - \frac{x-2}{2} - \frac{x-3}{3} = 2 \quad \text{mcm}(6, 2, 3) = 6$$

Multiplico por 6 a los 2 miembros:

$$1 \cdot (x-1) - 3 \cdot (x-2) - 2 \cdot (x-3) = 6 \cdot 2$$

$$x - 1 - 3x + 6 - 2x + 6 = 12$$

$$-4x + 11 = 12$$

$$11 - 12 = 4x$$

$$-1 = 4x \Rightarrow \boxed{x = \frac{-1}{4}}$$

$$\text{b) } -\frac{x-5}{6} = \frac{x-1}{9} - \frac{x-3}{4} \quad \text{mcm}(6, 9, 4) = 36$$

Multiplico por 36 a los miembros:

$$-6 \cdot (x-5) = 4 \cdot (x-1) - 9 \cdot (x-3)$$

$$-6x + 30 = 4x - 4 - 9x + 27$$

$$-6x + 30 = -5x + 23$$

$$30 - 23 = -5x + 6x \Rightarrow \boxed{7 = x}$$

$$\textcircled{3} \text{ a) } X^4 + 3X^2 - 4 = 0 \quad \begin{cases} m = X^2 \\ m^2 = X^4 \end{cases}$$

$$m^2 + 3m - 4 = 0 \Rightarrow \begin{cases} a=1 \\ b=3 \\ c=-4 \end{cases}$$

$$m = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1} = \frac{-3 \pm \sqrt{9+16}}{2} = \frac{-3 \pm \sqrt{25}}{2} = \frac{-3 \pm 5}{2} \begin{cases} \frac{-3+5}{2} = \frac{2}{2} = 1 \\ \frac{-3-5}{2} = \frac{-8}{2} = -4 \end{cases}$$

$$\text{Si } m = 1 \Rightarrow X^2 = 1 \Rightarrow \boxed{X = \pm \sqrt{1} = \pm 1}$$

$$\text{Si } m = -4 \Rightarrow X^2 = -4 \Rightarrow X = \pm \sqrt{-4} = \nexists \text{ soluc. real}$$

$$\text{b) } \frac{X^2-1}{X^2+1} = \frac{X^2-1}{3} \Rightarrow 3 \cdot (X^2-1) = (X^2+1)(X^2-1) \Rightarrow 3X^2-3 = X^4-1 \Rightarrow$$

$$\Rightarrow 0 = X^4 - 3X^2 - 1 + 3 \Rightarrow X^4 - 3X^2 + 2 = 0 \Rightarrow \begin{cases} m = X^2 \\ m^2 = X^4 \end{cases}$$

$$m^2 - 3m + 2 = 0 \quad \begin{cases} a=1 \\ b=-3 \\ c=2 \end{cases}$$

$$m = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm \sqrt{1}}{2} = \frac{3 \pm 1}{2} \begin{cases} \frac{3+1}{2} = \frac{4}{2} = 2 \\ \frac{3-1}{2} = \frac{2}{2} = 1 \end{cases}$$

$$\text{Si } m = 2 \Rightarrow X^2 = 2 \Rightarrow \boxed{X = \pm \sqrt{2}}$$

$$\text{Si } m = 1 \Rightarrow X^2 = 1 \Rightarrow \boxed{X = \pm \sqrt{1} = \pm 1}$$

$$\textcircled{4} \text{ a) } 2X^4 + 7X^3 - 4X^2 = 0 \Rightarrow X^2 \cdot (2X^2 + 7X - 4) = 0 \quad \begin{cases} X^2 = 0 \Rightarrow \boxed{X = 0 \text{ DOBLE}} \\ 2X^2 + 7X - 4 = 0 \end{cases}$$

$$2X^2 + 7X - 4 = 0 \quad \begin{cases} a=2 \\ b=7 \\ c=-4 \end{cases} \Rightarrow X = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 2 \cdot (-4)}}{2 \cdot 2} = \frac{-7 \pm \sqrt{49+32}}{4} = \frac{-7 \pm \sqrt{81}}{4} =$$

$$= \frac{-7 \pm 9}{4} \begin{cases} \frac{-7+9}{4} = \frac{2}{4} = \boxed{\frac{1}{2}} \\ \frac{-7-9}{4} = \frac{-16}{4} = \boxed{-4} \end{cases}$$

$$\text{b) } X^3 + X^2 + X = 0 \Rightarrow X(X^2 + X + 1) = 0 \Rightarrow \begin{cases} \boxed{X = 0} \\ X^2 + X + 1 = 0 \Rightarrow \Delta = 1^2 - 4 \cdot 1 \cdot 1 = 1 - 4 = -3 < 0 \\ \Rightarrow \nexists \text{ soluc. real.} \end{cases}$$

$$\textcircled{5} \text{ a) } X^2 - 5X + 7 = 0 \Rightarrow \Delta = b^2 - 4ac = (-5)^2 - 4 \cdot 1 \cdot 7 = 25 - 28 = -3 < 0 \Rightarrow \boxed{\nexists \text{ soluci3n real}}$$

$$\text{b) } X^2 - 4X + 4 = 0 \Rightarrow \Delta = b^2 - 4ac = (-4)^2 - 4 \cdot 1 \cdot 4 = 16 - 16 = 0 \Rightarrow \boxed{\text{Dos soluciones iguales}}$$