

Ejercicio 1.

Calcula el valor de la expresión:

$$\frac{\log_a \left(\sqrt[3]{a^2 \cdot \sqrt{a}} \right)^5 - \log_{\sqrt{b}} \left(\frac{b}{\sqrt{b^{-3}}} \right)^{-2}}{\log_b \frac{1}{\sqrt{b^3}} \cdot \log_{\frac{1}{a}} \left(\sqrt{a} \cdot \sqrt[3]{\frac{1}{a}} \right)} = \frac{\log_a \left(\sqrt[3]{\sqrt{a^4 \cdot a}} \right)^5 - \log_{\sqrt{b}} \left(b^{1+\frac{3}{2}} \right)^{-2}}{\log_b b^{-\frac{3}{2}} \cdot \log_{\frac{1}{a}} \left(a^{\frac{1}{2}} \cdot a^{-\frac{1}{3}} \right)} = \frac{5 \cdot \log_a a^{\frac{5}{6}} + 2 \cdot \log_{\sqrt{b}} b^{\frac{5}{2}}}{-\frac{3}{2} \cdot \log_{\frac{1}{a}} a^{\frac{1}{6}}} =$$

$$= \frac{5 \cdot \frac{5}{6} + 2 \cdot \log_{\sqrt{b}} (\sqrt{b})^5}{-\frac{3}{2} \cdot \log_{\frac{1}{a}} \left(\frac{1}{a} \right)^{\frac{1}{6}}} = \frac{\frac{25}{6} + 2 \cdot 5}{-\frac{3}{2} \cdot \left(-\frac{1}{6} \right)} = \frac{\frac{85}{6}}{\frac{1}{4}} = \frac{170}{3}$$

Ejercicio 2.

Si $\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2+x}}$, ¿cuál es el valor de x ? Comprueba las soluciones.

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2+x}} \Rightarrow \sqrt{2} = 1 + \frac{1}{\frac{4+2x+1}{2+x}} \Rightarrow \sqrt{2} = 1 + \frac{2+x}{5+2x} \Rightarrow \sqrt{2} = \frac{7+3x}{5+2x} \Rightarrow 2 = \frac{(7+3x)^2}{(5+2x)^2} \Rightarrow$$

$$\Rightarrow 2 = \frac{49+42x+9x^2}{25+20x+4x^2} \Rightarrow 50+40x+8x^2 = 49+42x+9x^2 \Rightarrow x^2+2x-1=0 \Rightarrow \begin{cases} x = -1 + \sqrt{2} \\ x = -1 - \sqrt{2} \end{cases}$$

$$\text{si } x = -1 + \sqrt{2} \Rightarrow 1 + \frac{1}{2 + \frac{1}{2+(-1+\sqrt{2})}} = 1 + \frac{1}{2 + \frac{1}{1+\sqrt{2}}} = 1 + \frac{1}{\frac{3+2\sqrt{2}}{1+\sqrt{2}}} = 1 + \frac{1+\sqrt{2}}{3+2\sqrt{2}} =$$

$$= \frac{4+3\sqrt{2}}{3+2\sqrt{2}} = \frac{(4+3\sqrt{2})(3-2\sqrt{2})}{(3+2\sqrt{2})(3-2\sqrt{2})} = \frac{12+9\sqrt{2}-8\sqrt{2}-12}{9-8} = \sqrt{2}$$

$$\text{si } x = -1 - \sqrt{2} \Rightarrow 1 + \frac{1}{2 + \frac{1}{2+(-1-\sqrt{2})}} = 1 + \frac{1}{2 + \frac{1}{1-\sqrt{2}}} = 1 + \frac{1}{\frac{3-2\sqrt{2}}{1-\sqrt{2}}} = 1 + \frac{1-\sqrt{2}}{3-2\sqrt{2}} =$$

$$= \frac{4-3\sqrt{2}}{3-2\sqrt{2}} = \frac{(4-3\sqrt{2})(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})} = \frac{12-9\sqrt{2}+8\sqrt{2}-12}{9-8} = -\sqrt{2} \neq \sqrt{2}$$

Ejercicio 3.

a) Escribe en forma de intervalo el conjunto de números que verifica la desigualdad: $\left| \frac{3+2x}{4} \right| \geq 2^{-2}$

$$\left| \frac{3+2x}{4} \right| \geq 2^{-2} \Rightarrow \left| \frac{3+2x}{4} \right| \geq \frac{1}{4} \Rightarrow \begin{cases} \frac{3+2x}{4} \geq \frac{1}{4} \Rightarrow 3+2x \geq 1 \Rightarrow x \geq -1, x \in [-1, +\infty) \\ \frac{3+2x}{4} \leq -\frac{1}{4} \Rightarrow 3+2x \leq -1 \Rightarrow x \leq -2, x \in (-\infty, -2] \end{cases}$$

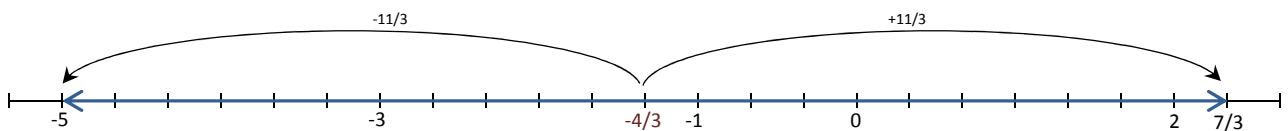
Los números $x \in (-\infty, -2] \cup [-1, +\infty)$ verifican la condición $\left| \frac{3+2x}{4} \right| \geq 2^{-2}$

b) Escribe, usando valor absoluto, la condición que deben cumplir los puntos del intervalo $\left(-5, \frac{7}{3}\right)$

$\left(-5, \frac{7}{3}\right) \Rightarrow$ distancia entre los extremos: $\frac{7}{3} - (-5) = \frac{22}{3}$; la mitad $\frac{11}{3} =$ radio del entorno

El centro del entorno será el punto medio entre -5 y $\frac{7}{3}$: $\frac{-5 + \frac{7}{3}}{2} = \frac{-4}{3}$

$$d\left(x, -\frac{4}{3}\right) < \frac{11}{3} \Rightarrow \left|x - \left(-\frac{4}{3}\right)\right| < \frac{11}{3} \Rightarrow \left|x + \frac{4}{3}\right| < \frac{11}{3}$$



Ejercicio 4.

Calcula, razonadamente, los siguientes límites de sucesiones:

$$\begin{aligned} a) \lim_{n \rightarrow \infty} \left(\frac{1+2+2^2+2^3+\dots+2^n}{1+3+3^2+3^3+\dots+3^n} \right) &= \lim_{n \rightarrow \infty} \left(\frac{\frac{2^{n+1}-1}{2-1}}{\frac{3^{n+1}-1}{3-1}} \right) = \lim_{n \rightarrow \infty} \left(\frac{2^{n+1}-1}{\frac{3^{n+1}-1}{2}} \right) = \lim_{n \rightarrow \infty} \left(\frac{2 \cdot (2^{n+1}-1)}{3^{n+1}-1} \right) = \\ &= \left(\frac{\infty}{\infty}, \text{dividimos por } 3^{n+1} \right) = \lim_{n \rightarrow \infty} \left(\frac{\frac{2 \cdot (2^{n+1}-1)}{3^{n+1}}}{\frac{3^{n+1}-1}{3^{n+1}}} \right) = \lim_{n \rightarrow \infty} \left(\frac{2 \cdot \left(\left(\frac{2}{3}\right)^{n+1} - \frac{1}{3^{n+1}} \right)}{1 - \frac{1}{3^{n+1}}} \right) = \frac{2 \cdot (0-0)}{1-0} = 0 \end{aligned}$$

$$\begin{aligned}
 b) \lim_{n \rightarrow \infty} \left(\frac{5n+2}{5n} \right)^{2n} &= (\text{indeterminación } 1^\infty) = \lim_{n \rightarrow \infty} \left(\frac{5n}{5n} + \frac{2}{5n} \right)^{2n} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{5n} \right)^{2n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{\frac{5n}{2}} \right)^{2n} = \\
 &= \lim_{n \rightarrow \infty} \left(1 + \frac{1}{\frac{5n}{2}} \right)^{\frac{5n}{2} \cdot \frac{2}{5n} \cdot 2n} = e^{\lim_{n \rightarrow \infty} \frac{4n}{5n}} = e^{\lim_{n \rightarrow \infty} \frac{4}{5}} = e^{\frac{4}{5}}
 \end{aligned}$$

Ejercicio 5.

Resuelve la inecuación $2x \cdot (3x^2 + 3x + 1) \leq (2x + 1)^2 - 3x^2$

$$6x^3 + 6x^2 + 2x \leq 4x^2 + 4x + 1 - 3x^2 \Rightarrow 6x^3 + 5x^2 - 2x - 1 \leq 0 \Rightarrow (x+1)(6x^2 - x - 1) \leq 0$$

$$6x^2 - x - 1 = 0 \Rightarrow x = \frac{1}{2}, x = -\frac{1}{3} \Rightarrow 6x^2 - x - 1 = 6\left(x - \frac{1}{2}\right)\left(x + \frac{1}{3}\right)$$

Entonces la inecuación queda $6(x+1)\left(x - \frac{1}{2}\right)\left(x + \frac{1}{3}\right) \leq 0$

Ahora analizamos el signo del producto:

| | $(-\infty, -1)$ | -1 | $\left(-1, -\frac{1}{3}\right)$ | $-\frac{1}{3}$ | $\left(-\frac{1}{3}, \frac{1}{2}\right)$ | $\frac{1}{2}$ | $\left(\frac{1}{2}, +\infty\right)$ |
|---|-----------------|------|---------------------------------|----------------|--|---------------|-------------------------------------|
| $x+1$ | - | 0 | + | + | + | + | + |
| $x + \frac{1}{3}$ | - | - | - | 0 | + | + | + |
| $x - \frac{1}{2}$ | - | - | - | - | - | 0 | + |
| $(x+1)\left(x + \frac{1}{3}\right)\left(x - \frac{1}{2}\right)$ | - | 0 | + | 0 | - | 0 | + |

Solución: $x \in (-\infty, -1] \cup \left[-\frac{1}{3}, \frac{1}{2}\right]$

Ejercicio 6.

Resuelve las siguientes ecuaciones:

a) $4^{x+1} + 3 = 7 \cdot 2^x$

$$\begin{aligned} (2^2)^{x+1} - 7 \cdot 2^x + 3 = 0 &\Rightarrow 2^{2x+2} - 7 \cdot 2^x + 3 = 0 \Rightarrow 4 \cdot 2^{2x} - 7 \cdot 2^x + 3 = 0 \Rightarrow (\text{cambio } 2^x = z) \Rightarrow \\ \Rightarrow 4 \cdot z^2 - 7 \cdot z + 3 = 0 &\Rightarrow \begin{cases} x=1 \Rightarrow 2^x = 1 \Rightarrow \boxed{x=0} \\ x = \frac{3}{4} \Rightarrow 2^x = \frac{3}{4} \Rightarrow \log 2^x = \log \frac{3}{4} \Rightarrow x \log 2 = \log 3 - \log 4 \Rightarrow \\ \Rightarrow \boxed{x = \frac{\log 3 - \log 4}{\log 2}} \end{cases} \end{aligned}$$

b) $\log(2x+5) - \frac{1}{2} \log(6x+1) - \log 3 = 0$

$$\log(2x+5) = \frac{1}{2} \log(6x+1) + \log 3 \Rightarrow \log(2x+5) = \log(6x+1)^{\frac{1}{2}} + \log 3 \Rightarrow$$

$$\Rightarrow \log(2x+5) = \log(\sqrt{6x+1}) \cdot 3 \Rightarrow 2x+5 = 3 \cdot \sqrt{6x+1} \Rightarrow (2x+5)^2 = 9(6x+1) \Rightarrow$$

$$\Rightarrow 4x^2 + 20x + 25 = 54x + 9 \Rightarrow 4x^2 - 34x + 16 = 0 \Rightarrow 2x^2 - 17x + 8 = 0 \Rightarrow$$

$$\Rightarrow \begin{cases} x = 8 \\ x = \frac{1}{2} \end{cases} \text{ ambas soluciones son válidas.}$$