Examen de Matemáticas 1º de Bachillerato

Enero 2013

Problema 1 Dados los números complejos $z_1 = 2 - i$ y $z_2 = -3 + 7i$. Se pide calcular:

a) $z_1 + z_2$ y $z_1 - z_2$

b) $z_1 \cdot z_2$

c) $\dfrac{z_1}{z_2}$

Solución:

a) $z_1 + z_2 = -1 + 6i$ y $z_1 - z_2 = 5 - 8i$

b) $z_1 \cdot z_2 = 1 + 17i$

c) $\dfrac{z_1}{z_2} = -\dfrac{13}{58} - \dfrac{11}{58}i$

Problema 2 Resolver la siguiente ecuación de segundo grado:

$$z^2 - 2z + 5 = 0$$

Solución:

$$z^2 - 2z + 5 = 0 \Longrightarrow z = \left\{ \begin{array}{l} 1 + 2i \\ 1 - 2i \end{array} \right.$$

Problema 3 Si $z = 4 - 5i$ calcular z^{10}.

Solución:

$$z = 4 - 5i = \sqrt{41}_{308^{\circ}39'35''} = \sqrt{41}(\cos 308^{\circ}39'35'' + i \sin 308^{\circ}39'35'')$$

$$z^{10} = (4 - 5i)^{10} = 41^5_{206^{\circ}35'53''} = 41^5(\cos 206^{\circ}35'53'' + i \sin 206^{\circ}35'53'')$$

Problema 4 Resolver la ecuación $z^3 - i = 5$.

Solución:

$$z^3 = 5 + i \Longrightarrow z = \sqrt[3]{5 + i}$$

$$5 + i = \sqrt{26}_{11^{\circ}18'36''} = \sqrt{26}(\cos 11^{\circ}18'36'' + i \sin 11^{\circ}18'36'')$$

$$z = \sqrt[3]{5 + i} = \left\{ \begin{array}{l} \sqrt[6]{26}_{3^{\circ}46'12''} = \sqrt[6]{26}(\cos 3^{\circ}46'12'' + i \sin 3^{\circ}46'12'') \\ \sqrt[6]{26}_{123^{\circ}46'12''} = \sqrt[6]{26}(\cos 123^{\circ}46'12'' + i \sin 123^{\circ}46'12'') \\ \sqrt[6]{26}_{243^{\circ}46'12''} = \sqrt[6]{26}(\cos 243^{\circ}46'12'' + i \sin 243^{\circ}46'12'') \end{array} \right.$$

Problema 5 Calcular las raices de $\sqrt[3]{-3+i}$

Solución:

$$z = -3 + i = \sqrt{10}_{161^{\circ}33'54''} = \sqrt{10}(\cos 161^{\circ}33'54'' + i \sin 161^{\circ}33'54'')$$

$$\sqrt[3]{z} = \begin{cases} \sqrt[6]{10}_{53^{\circ}51'18''} = \sqrt[6]{10}(\cos 53^{\circ}51'18'' + i \sin 53^{\circ}51'18'') \\ \sqrt[6]{10}_{173^{\circ}51'18''} = \sqrt[6]{10}(\cos 173^{\circ}51'18'' + i \sin 173^{\circ}51'18'') \\ \sqrt[6]{10}_{293^{\circ}51'18''} = \sqrt[6]{10}(\cos 293^{\circ}51'18'' + i \sin 293^{\circ}51'18'') \end{cases}$$