Examen de Matemáticas 1º de Bachillerato

Problema 1 Dados los números complejos $z_1 = 5 - 2i$ y $z_2 = -1 + 3i$. Se pide calcular:

a) $z_1 + z_2$ y $z_1 - z_2$

b) $z_1 \cdot z_2$

c) $\dfrac{z_1}{z_2}$

Solución:

a) $z_1 + z_2 = 4 + i$ y $z_1 - z_2 = 6 - 5i$

b) $z_1 \cdot z_2 = 1 + 17i$

c) $\dfrac{z_1}{z_2} = -\dfrac{11}{10} - \dfrac{17}{10}i$

Problema 2 Resolver la siguiente ecuación de segundo grado:

$$z^2 + 3z + 5 = 0$$

Solución:

$$z^2 + 3z + 5 = 0 \Longrightarrow z = \left\{ \begin{array}{l} -\dfrac{3}{2} + \dfrac{\sqrt{11}}{2}i \\ -\dfrac{3}{2} - \dfrac{\sqrt{11}}{2}i \end{array} \right.$$

Problema 3 Si $z = 3 - 7i$ calcular z^{10}.

Solución:

$$z = 3 - 7i = \sqrt{58}_{293^o 11' 55''} = \sqrt{58}(\cos 293^o 11' 55'' + i \sin 293^o 11' 55'')$$

$$z^{10} = (3 - 7i)^{10} = 58^5_{10 \cdot 293^o 11' 55''} = 58^5_{2931^o 59' 10''} = 58^5_{51^o 59' 10''} =$$

$$58^5(\cos 51^o 59' 10'' + i \sin 51^o 59' 10'')$$

Problema 4 Resolver la ecuación $z^3 - i = 7$.

Solución:

$$z^3 = 7 + i \Longrightarrow z = \sqrt[3]{7 + i}$$

$$7 + i = 5\sqrt{2}_{8^o 7' 48''} = 5\sqrt{2}(\cos 8^o 7' 48'' + i \sin 8^o 7' 48'')$$

$$z = \sqrt[3]{7 + i} = \left\{ \begin{array}{l} \sqrt[6]{50}_{2^o 42' 36''} = \sqrt[6]{50}(\cos 2^o 42' 36'' + i \sin 2^o 42' 36'') \\ \sqrt[6]{50}_{122^o 42' 36''} = \sqrt[6]{50}(\cos 122^o 42' 36'' + i \sin 122^o 42' 36'') \\ \sqrt[6]{50}_{242^o 42' 36''} = \sqrt[6]{50}(\cos 242^o 42' 36'' + i \sin 242^o 42' 36'') \end{array} \right.$$

Problema 5 Calcular las raices de $\sqrt[3]{-2+3i}$

Solución:

$$z = -2+3i = \sqrt{13}_{123^{\circ}41'24''} = \sqrt{13}(\cos 123^{\circ}41'24'' + i\sin 123^{\circ}41'24'')$$

$$\sqrt[3]{z} = \left\{ \begin{array}{l} \sqrt[6]{13}_{41^{\circ}13'48''} = \sqrt[6]{13}(\cos 41^{\circ}13'48'' + i\sin 41^{\circ}13'48'') \\ \sqrt[6]{13}_{161^{\circ}13'48''} = \sqrt[6]{13}(\cos 161^{\circ}13'48'' + i\sin 161^{\circ}13'48'') \\ \sqrt[6]{13}_{281^{\circ}13'48''} = \sqrt[6]{13}(\cos 281^{\circ}13'48'' + i\sin 281^{\circ}13'48'') \end{array} \right.$$