IDENTIDADES TRIGONOMÉTRICAS

EJERCICIOS RESUELTOS

1. Demuestra las siguientes identidades:

a) $\cos x \cdot (1 + \operatorname{tg} x) = \operatorname{sen} x \cdot (1 + \operatorname{ctg} x)$

b) $\dfrac{\operatorname{sen}(-x) \cdot \operatorname{ctg}(2\pi - x)}{\operatorname{tg}(\pi + x) \cdot \operatorname{sen}\left(\dfrac{\pi}{2} - x\right)} = \operatorname{ctg} x$

Solución:

a) $\cos x \cdot (1 + \operatorname{tg} x) = \operatorname{sen} x \cdot (1 + \operatorname{ctg} x)$

$$\cos x \cdot \left(1 + \frac{\operatorname{sen} x}{\cos x}\right) = \operatorname{sen} x \cdot \left(1 + \frac{\cos x}{\operatorname{sen} x}\right) \rightarrow \cos x \cdot \left(\frac{\cos x + \operatorname{sen} x}{\cos x}\right) = \operatorname{sen} x \cdot \left(\frac{\operatorname{sen} x + \cos x}{\operatorname{sen} x}\right) \rightarrow$$

$$\cancel{\cos x} \cdot \left(\frac{\cos x + \operatorname{sen} x}{\cancel{\cos x}}\right) = \cancel{\operatorname{sen} x} \cdot \left(\frac{\operatorname{sen} x + \cos x}{\cancel{\operatorname{sen} x}}\right) \rightarrow \cos x + \operatorname{sen} x = \cos x + \operatorname{sen} x$$

b) $\dfrac{\operatorname{sen}(-x) \cdot \operatorname{ctg}(2\pi - x)}{\operatorname{tg}(\pi + x) \cdot \operatorname{sen}\left(\dfrac{\pi}{2} - x\right)} = \operatorname{ctg} x$

Teniendo en cuenta:

a) sen (-x) = - sen x b) $\operatorname{ctg}(2\pi - x) = -\operatorname{ctg} x$ c) $\operatorname{tg}(\pi + x) = \operatorname{tg} x$ d) $\operatorname{sen}\left(\dfrac{\pi}{2} - x\right) = \cos x$

$$\frac{\operatorname{sen}(-x) \cdot \operatorname{ctg}(2\pi - x)}{\operatorname{tg}(\pi + x) \cdot \operatorname{sen}\left(\frac{\pi}{2} - x\right)} = \frac{(-\operatorname{sen} x) \cdot (-\operatorname{ctg} x)}{\operatorname{tg} x \cdot \cos x} = \frac{\cancel{\operatorname{sen} x} \cdot \dfrac{\cos x}{\cancel{\operatorname{sen} x}}}{\dfrac{\operatorname{sen} x}{\cancel{\cos x}} \cdot \cancel{\cos x}} = \frac{\cos x}{\operatorname{sen} x} = \operatorname{ctg} x$$

2. Demostrar las siguientes igualdades:

a) $\cos(2\pi - x) \cdot \operatorname{ctg} x + \operatorname{sen}(\pi - x) = \sec\left(\dfrac{\pi}{2} - x\right)$ **b) sec x – cos x = tg x · sen x**

Solución:

a) $\cos(2\pi - x) \cdot \operatorname{ctg} x + \operatorname{sen}(\pi - x) = \sec\left(\dfrac{\pi}{2} - x\right)$

$$\cos(2\pi - x) \cdot \operatorname{ctg} x + \operatorname{sen}(\pi - x) = \cos x \cdot \frac{\cos x}{\operatorname{sen} x} + \operatorname{sen} x = \frac{\cos^2 x}{\operatorname{sen} x} + \operatorname{sen} x = \frac{\cos^2 x + \operatorname{sen}^2 x}{\operatorname{sen} x} = \frac{1}{\operatorname{sen} x}$$

$$\sec\left(\frac{\pi}{2} - x\right) = \frac{1}{\cos\left(\frac{\pi}{2} - x\right)} = \frac{1}{\operatorname{sen} x}$$

b) sec x - cos x = tg x · sen x

$$\sec x - \cos x = \frac{1}{\cos x} - \cos x = \frac{1 - \cos^2 x}{\cos x} = \frac{\operatorname{sen}^2 x}{\cos x} = \operatorname{sen} x \cdot \frac{\operatorname{sen} x}{\cos x} = \operatorname{sen} x \cdot \operatorname{tg} x$$

3. Demuestra las siguientes identidades:

a) $\cos x + \cos y = (\operatorname{sen} x - \operatorname{sen} y)\operatorname{ctg}\left(\frac{x-y}{2}\right)$

b) $\operatorname{tg}\frac{x}{2} = \frac{\operatorname{sen} x}{1+\cos x}$

Solución:

a) $\cos x + \cos y = (\operatorname{sen} x - \operatorname{sen} y)\operatorname{ctg}\left(\frac{x-y}{2}\right)$

Aplicando las fórmulas de adición:

$$\cos x + \cos y = 2\cos\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$$

$$\operatorname{sen} x - \operatorname{sen} y = 2\cos\left(\frac{x+y}{2}\right)\operatorname{sen}\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = (\operatorname{sen} x - \operatorname{sen} y)\operatorname{ctg}\left(\frac{x-y}{2}\right) \longrightarrow 2\cos\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right) = 2\cos\left(\frac{x+y}{2}\right)\operatorname{sen}\left(\frac{x-y}{2}\right)\operatorname{ctg}\left(\frac{x-y}{2}\right) \longrightarrow$$

$$\cos\left(\frac{x-y}{2}\right) = \operatorname{sen}\left(\frac{x-y}{2}\right)\operatorname{ctg}\left(\frac{x-y}{2}\right) \longrightarrow \frac{\cos\left(\frac{x-y}{2}\right)}{\operatorname{sen}\left(\frac{x-y}{2}\right)} = \operatorname{ctg}\left(\frac{x-y}{2}\right) \longrightarrow \operatorname{ctg}\left(\frac{x-y}{2}\right) = \operatorname{ctg}\left(\frac{x-y}{2}\right)$$

b) $\operatorname{tg}\frac{x}{2} = \frac{\operatorname{sen} x}{1+\cos x}$

Aplicando la fórmula del ángulo mitad:

$$\operatorname{tg}\frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$\operatorname{tg}\frac{x}{2} = \frac{\sqrt{1-\cos x}}{\sqrt{1+\cos x}} = \frac{\sqrt{1-\cos x}}{\sqrt{1+\cos x}}\cdot\frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} = \frac{\sqrt{1-\cos^2 x}}{1+\cos x} = \frac{\operatorname{sen} x}{1+\cos x}$$

4. Demostrar la siguiente igualdad:

$$\frac{\operatorname{tg} x}{\operatorname{tg} 2x - \operatorname{tg} x} = \cos 2x$$

Solución:

$$\frac{\operatorname{tg} x}{\operatorname{tg} 2x - \operatorname{tg} x} = \frac{\operatorname{tg} x}{\frac{2\operatorname{tg} x}{1-\operatorname{tg}^2 x} - \operatorname{tg} x} = \frac{\operatorname{tg} x}{\frac{\operatorname{tg} x + \operatorname{tg}^3 x}{1-\operatorname{tg}^2 x}} = \frac{\operatorname{tg} x\left(1-\operatorname{tg}^2 x\right)}{\operatorname{tg} x\left(1+\operatorname{tg}^2 x\right)} = \frac{1-\frac{\operatorname{sen}^2 x}{\cos^2 x}}{1+\frac{\operatorname{sen}^2 x}{\cos^2 x}} = \frac{\cos^2 x - \operatorname{sen}^2 x}{\cos^2 x + \operatorname{sen}^2 x} = \cos 2x$$