

## LOGARITMOS

1. Calcula, sin usar calculadora los siguientes logaritmos:

$$\log_2 \sqrt[3]{2^2} = \quad \log_3 \left( \frac{\sqrt{3}}{3} \right) = \quad \log 0.0001 = \quad \ln e^{\frac{2}{3}} = \quad \log_{50} 1 = \quad \log \sqrt{10}$$

2. Calcula, utilizando el cambio de base y la calculadora  $\log_7 150$  y  $\log_2 25$ .

3. Halla el valor de x en cada caso:

$$\begin{array}{llll} a) 5^{x+1} = \frac{2}{10} & b) 3^{2x+2} = 243 & c) \log(x-2) = 2 & d) \log(5x) = 4 \\ e) \log_{\frac{1}{8}} x = \frac{1}{3} & f) \log_4 x = \frac{1}{2} & g) \log_x 343 = 3 & h) \log_x 16 = -2 \end{array}$$

4. Calcula x aplicando la definición de logaritmo:

$$\begin{array}{llll} a) 2^x = 16 & b) 2^x = 32 & c) 3^{1/x} = 9 & d) \log_2 64 = x \\ e) \log_3 81 = x & f) \log_{101} 10201 = x & g) \log_{16} 0,5 = x & h) \log_{10} 0,00001 = x \\ i) \log_x 125 = \frac{3}{2} & j) \log_x \frac{1}{3} = -\frac{1}{2} & k) \log_{125} \frac{1}{\sqrt{5}} = x & l) \log_{343} \sqrt{7} = x \end{array}$$

5. Calcula x aplicando la definición de logaritmo:

$$\begin{array}{llll} a) \log_{\frac{8}{3}} \frac{81}{16} = x & b) \log_{\frac{8}{3}} \frac{27}{125} = x & c) \log_8 \sqrt[4]{2} = x & d) x = \log_3(3\sqrt{3}) \\ e) x = \log_3 \left( \frac{\sqrt[4]{3}}{9} \right) & f) x = \log_{81}(3) & g) x = \log_{81} \left( \frac{\sqrt{3}}{3} \right) = & h) x = \log_{1/9} \left( \frac{\sqrt[4]{3}}{3} \right) = \\ i) x = \log_{\sqrt{3}/3} 81 & j) x = \log_{\sqrt{3}/3} \left( \frac{\sqrt[4]{3}}{3} \right) & k) \log_x \left( \frac{1}{2187} \right) = 7 & l) \log_{2/5} x = -1 \end{array}$$

6. Despeja x en los siguientes casos, ayudándote de la calculadora:

$$2^{x+3} = 15 \cdot 3^{2x-4} = 56 \quad \log_7 81 = x \quad x^{1,56} = 9,4$$

7. Si se sabe que  $\log A = 0,46$  y que  $\log B = 1,5$ , calcula razonadamente:

$$\log \left( \frac{100A}{B^2} \right) \quad \log \sqrt[5]{\frac{(A \cdot B)^3}{10}} \quad \log \frac{A^4}{\sqrt{B}}$$

8. Sabiendo que  $\log A = 1,28$  y  $\log B = 0,35$  calcula el valor de las siguientes expresiones:

$$(a) \log \left( \frac{0,01 \cdot A^3}{B} \right) = \quad (b) \log \sqrt[4]{\frac{B^3}{10A}} =$$

9. Sabiendo que  $\log 2 = 0.30103$  y que  $\log 3 = 0.47712$ , calcular:

a)  $\log 2000$       b)  $\log \sqrt{5}$       c)  $\log 25$       d)  $\log \sqrt[5]{8}$       e)  $\log \sqrt{160}$

f)  $\log 0,125$       g)  $\log 3\sqrt[3]{3}$       h)  $\log 40,5$       i)  $\log(0,64^3 \cdot \sqrt{0,32})$

j)  $\log \sqrt{2\sqrt{2\sqrt{2}}}$

10. Halla el resultado de las siguientes expresiones:

a)  $\log_5 125 - \log_3 243 + \log_4 256 =$

b)  $\log_3 1 + \log_2 64 + \log_3 9 + \log_7 49 =$

c)  $\log_2 4 + \log_3 81 - \log_6 216 + \log_4 64 =$

d)  $\log_3 \frac{1}{9} - \log_5 0,2 + \log_6 \frac{1}{36} - \log_2 0,5 =$

11. Demuestra que para cualquier base  $a$  se verifica:

$$\log_a 0,01 + 3 \log_a 100 - 4 \log_a 10 = 0$$

12. Desarrolla las siguientes expresiones, utilizando las propiedades de los logaritmos:

a)  $\log \frac{a^2 b}{c}$

b)  $\log(a^2 b^3 c)$

c)  $\log \frac{a^2 \sqrt[3]{b}}{\sqrt[4]{c^3}}$

d)  $\log \frac{m \sqrt[3]{n^4 \sqrt{m/n}}}{n}$

e)  $\log_2 \frac{1}{2^{3x}}$

f)  $\log_x \frac{\sqrt{x}}{\sqrt[3]{x^2}}$

13. Comprime las expresiones de manera que el logaritmo aparezca una sola vez:

a)  $\log x^4 - \log \sqrt{xy}$

b)  $\log x - 2 \log y$

c)  $3 \log x + \log(1-x)$

d)  $\frac{\log x}{2} + \frac{\log y}{4}$

e)  $-\log x - \log y$

f)  $\log x^{\log x}$

14. Elimina los logaritmos de las expresiones siguientes:

a)  $\log x + \log y = 1$

b)  $\log x - \log y = -1$

c)  $4 \log x - 3 \log y = 2$

d)  $\frac{2 \log x}{3} - 1 = \log y$

e)  $\log(\log x) = 1$

## LOGARITMOS - SOLUCIONES

$$1.- \log_2 \sqrt{2\sqrt[3]{2^2}} = \log_2 \sqrt[3]{2^3 \cdot 2^2} = \log_2 \sqrt[6]{2^5} = \log_2 2^{\frac{5}{6}} = \frac{5}{6}$$

$$\log_3 \left( \frac{\sqrt{3}}{3} \right) = \log_3 \sqrt{3} - \log_3 3 = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$\log 0,0001 = \log 10^{-4} = -4$$

$$\ln e^{\frac{2}{3}} = \frac{2}{3} ; \log_{50} 1 = 0 ; \log \sqrt{10} = \log 10^{\frac{1}{2}} = \frac{1}{2},$$

$$2.- \log_7 150 = \frac{\log 150}{\log 7} = 2,575$$

$$\log_2 25 = \frac{\log 25}{\log 2} = 4,644.$$

$$3.- (a) 5^{x+1} = \frac{2}{10} \Leftrightarrow 5^{x+1} = \frac{1}{5} \Leftrightarrow 5^{x+1} = 5^{-1} \Leftrightarrow x+1 = -1 \Rightarrow \boxed{x = -2}$$

$$(b) 3^{2x+2} = 243 \Leftrightarrow 3^{2x+2} = 3^5 \Leftrightarrow 2x+2 = 5 \Leftrightarrow 2x = 3 \Rightarrow \boxed{x = \frac{3}{2}}$$

$$(c) \log(x-2) = 2 \Leftrightarrow x-2 = 10^2 \Rightarrow \boxed{x = 102}$$

$$(d) \log(5x) = 4 \Leftrightarrow 5x = 10^4 \Rightarrow \boxed{x = \frac{10000}{5} = 2000}$$

$$(e) \log_{1/8} x = \frac{1}{3} \Leftrightarrow x = (\frac{1}{8})^{\frac{1}{3}} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$$

$$(f) \log_4 x = \frac{1}{2} \Leftrightarrow x = 4^{\frac{1}{2}} = \sqrt{4} = 2,$$

$$(g) \log_x 343 = 3 \Leftrightarrow x^3 = 343 = 7^3 \Leftrightarrow \boxed{x = 7}$$

$$(h) \log_x 16 = -2 \Leftrightarrow x^{-2} = 16 = 2^4 = 4^2 \Rightarrow \frac{1}{x^2} = 4^2 \Rightarrow 4x^2 = 1 \Rightarrow x^2 = \frac{1}{4}$$

$x = \frac{1}{2}$

$$4.- a) 2^x = 16 \Rightarrow \log_2 16 = \log_2 2^4 = 4, \quad b) 2^x = 32 \Rightarrow x = \log_2 32 = \log_2 2^5 = 5,$$

$$c) 3^{\frac{1}{x}} = 9 \Rightarrow \frac{1}{x} = \log_3 9 \Rightarrow \frac{1}{x} = \log_3 3^2 \Rightarrow \frac{1}{x} = 2 \Rightarrow \boxed{x = \frac{1}{2}}$$

$$d) x = \log_2 64 = \log_2 2^6 = 6, \quad e) x = \log_3 81 = \log_3 3^4 = 4,$$

$$f) x = \log_{10} 10201 = \log_{10} 101^2 = 2, \quad g) \log_{16} 0,5 = x \Rightarrow 16^x = \frac{1}{2} \Rightarrow$$

$$\Rightarrow 2^{-4x} = 2^1 \Rightarrow -4x = 1 \Rightarrow \boxed{x = -\frac{1}{4}}$$

$$h) x = \log 0,00001 = \log 10^{-5} = -5,$$

$$i) \log_x 125 = \frac{3}{2} \Rightarrow x^{\frac{3}{2}} = 5^3 \Rightarrow (x^2)^{\frac{3}{2}} = 5^3 \Rightarrow x^2 = 5 \Rightarrow \boxed{x = \sqrt{5}}$$

$$j) \log_x \frac{1}{3} = -\frac{1}{2} \Rightarrow x^{-\frac{1}{2}} = \frac{1}{3} \Rightarrow \frac{1}{x^{\frac{1}{2}}} = \frac{1}{3} \Rightarrow \sqrt{x} = 3 \Rightarrow \boxed{x = 9}$$

$$K) \log_{125} \frac{1}{\sqrt{5}} = x \Rightarrow 125^x = \frac{1}{\sqrt{5}} \Rightarrow 5^{3x} = 5^{-\frac{1}{2}} \Rightarrow 3x = -\frac{1}{2} \Rightarrow \\ \Rightarrow x = -\frac{1}{6}$$

$$l) \log_{343} \sqrt[3]{7} = x \Leftrightarrow 343^x = \sqrt[3]{7} \Rightarrow 7^{3x} = 7^{\frac{1}{2}} \Rightarrow 3x = \frac{1}{2} \Rightarrow x = \frac{1}{6}$$

$$5.- a) \log_{\frac{2}{3}} \frac{81}{16} = x \Rightarrow \left(\frac{2}{3}\right)^x = \frac{81}{16} = \left(\frac{3}{2}\right)^4 \Rightarrow \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^{-4} \Rightarrow x = -4$$

$$b) \log_{\frac{5}{3}} \frac{27}{125} = x \Rightarrow \log \left(\frac{5}{3}\right)^x = \left(\frac{3}{5}\right)^3 = \left(\frac{3}{5}\right)^{-3} \Rightarrow x = -3$$

$$c) \underline{3^x = 9 = 3^2} \Rightarrow \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2} \quad d) x = \log_2 64 = 6$$

$$d) x = \log_3 (3\sqrt{3}) = \log_3 3^{\frac{1+\frac{1}{2}}{2}} = \frac{3}{2}.$$

$$e) x = \log_3 \left( \frac{\sqrt{3}}{9} \right) = \log_3 \frac{3^{\frac{1}{2}}}{3^2} = \log_3 3^{\frac{1}{2}-2} = \frac{-7}{8}$$

$$f) x = \log_{81} 3 \Rightarrow 81^x = 3 \Rightarrow 3^{\frac{4x}{4}} = 3^1 \Rightarrow x = \frac{1}{4}$$

$$g) x = \log_{81} \left( \frac{\sqrt{3}}{3} \right) \Rightarrow (3^4)^x = \frac{3^{\frac{1}{2}}}{3} \Rightarrow 3^{\frac{4x}{4}} = 3^{\frac{1}{2}} \Rightarrow 4x = \frac{1}{2} \Rightarrow x = \frac{1}{8}$$

$$h) x = \log_{1/9} \left( \frac{\sqrt{3}}{3} \right) \Rightarrow \left( \frac{1}{9} \right)^x = 3^{\frac{1}{4}-1} \Rightarrow 3^{-2x} = 3^{\frac{-3}{4}} \Rightarrow -2x = \frac{-3}{4} \Rightarrow x = \frac{3}{8}$$

$$i) x = \log_{\sqrt{3}/3} 81 \Rightarrow \left( \frac{\sqrt{3}}{3} \right)^x = 81 \Leftrightarrow \left( 3^{\frac{1}{2}-1} \right)^x = 3^4 \Leftrightarrow 3^{\frac{-x}{2}} = 3^4 \Rightarrow x = -8$$

$$j) x = \log_{\sqrt{3}/3} \left( \frac{\sqrt{3}}{3} \right) \Rightarrow \left( \frac{\sqrt{3}}{3} \right)^x = \frac{\sqrt{3}}{3} \Rightarrow 3^{\frac{-x}{2}} = 3^{\frac{-3}{4}} \Rightarrow -\frac{x}{2} = \frac{-3}{4} \Rightarrow x = \frac{3}{2}$$

$$k) \log_x \left( \frac{1}{2187} \right) = 7 \Rightarrow x^7 = \frac{1}{2187} \Rightarrow x^7 = \left( \frac{1}{3} \right)^7 \Rightarrow x = \frac{1}{3}$$

$$l) \log_{2/5} x = -1 \Leftrightarrow \left( \frac{2}{5} \right)^{-1} = x \Leftrightarrow \frac{5}{2} = x$$

$$6.- (a) 2^{x+3} = 15 \xrightarrow[\log]{\text{Aplicamos}} \log 2^{x+3} = \log 15 \Rightarrow (x+3) \log 2 = \log 15$$

$$\Rightarrow x+3 = \frac{\log 15}{\log 2} \Rightarrow x = \frac{\log 15}{\log 2} - 3 = \underline{0'9069.}$$

$$(b) 3^{2x-4} = 56 \Rightarrow \log 3^{2x-4} = \log 56 \Rightarrow (2x-4) \log 3 = \log 56 \Rightarrow$$

$$2x-4 = \frac{\log 56}{\log 3} \Rightarrow x = \frac{1}{2} \left[ \frac{\log 56}{\log 3} + 4 \right] = \underline{3'832.}$$

$$(c) x = \log_7 81 = \frac{\ln 81}{\ln 7} = \underline{2'2581}$$

$$d) x^{156} = 9^4 \Rightarrow x = 9^4 = 4'205.$$

$$\text{o bien } 156 \cdot \log x = \log 9^4 \Rightarrow \log x = \frac{\log 9^4}{156} = \underline{x = 4'205.}$$

10º función inversa

$$7.- \log A = 0.46; \log B = 1.5$$

$$\text{a) } \log\left(\frac{100A}{B^2}\right) = \log 100A - \log B^2 = \log 100 + \log A - 2\log B = \\ = 2 + 0.46 - 2 \cdot 1.5 = \underline{-0.54}$$

$$\text{b) } \log \sqrt[5]{\frac{(A \cdot B)^3}{10}} = \frac{1}{5} [\log(A \cdot B)^3 - \log 10] = \frac{1}{5} [3\log A + 3\log B - \log 10] = \\ = \frac{1}{5} (3 \cdot 0.46 + 3 \cdot 1.5 - 1) = \underline{1.036.1}$$

$$\text{c) } \log \frac{A^4}{\sqrt{B}} = 4\log A - \frac{1}{2}\log B = 4 \cdot 0.46 - \frac{1}{2} \cdot 1.5 = \underline{1.09}$$

$$8.- \log A = 1.28; \log B = 0.35$$

$$\text{a) } \log\left(\frac{0.01 \cdot A^3}{B}\right) = \log 0.01 + \log A^3 - \log B = -2 + 3\log A - \log B \\ = -2 + 3 \cdot 1.28 - 0.35 = \underline{1.49}$$

$$\text{b) } \log \sqrt[4]{\frac{B^3}{10A}} = \frac{1}{4} [\log B^3 - \log(10A)] = \frac{1}{4} [3\log B - \log 10 - \log A] \\ = \frac{1}{4} [3 \cdot 0.35 - 1 - 1.28] = \underline{-0.3075}$$

$$9.- \log 2 = 0.30103; \log 3 = 0.47712$$

$$\text{a) } \log(2000) = \log(2 \cdot 1000) = \log 2 + \log 1000 = 0.30103 + 3 = \underline{3.30103}$$

$$\text{b) } \log \sqrt{5} = \frac{1}{2} \log 5 = \frac{1}{2} \log\left(\frac{10}{2}\right) = \frac{1}{2} [\log 10 - \log 2] = \\ = \frac{1}{2} (1 - 0.30103) = \underline{0.349485}$$

$$\text{c) } \log 25 = \log 5^2 = 2(\log 10 - \log 2) = \underline{1.39794}$$

$$\text{d) } \log \sqrt[5]{8} = \log \sqrt[5]{2^3} = \frac{3}{5} \log 2 = \frac{3}{5} \cdot 0.30103 = \underline{0.180618}$$

$$\text{e) } \log \sqrt{160} = \log 4\sqrt{10} = \log 4 + \log \sqrt{10} = 2\log 2 + \frac{1}{2} \log 10 = \\ = 2 \cdot 0.30103 + \frac{1}{2} \cdot 1 = \underline{1.10206}$$

$$\text{f) } \log 0.125 = \log \frac{125}{1000} = \log \frac{1}{8} = \log 2^{-3} = -3 \cdot \log 2 = \underline{-0.90309}$$

$$\text{g) } \log 3^{1.3} = \log \frac{33-3}{9} = \log \frac{30}{9} = \log \frac{10}{3} = \log 10 - \log 3 = 1 - 0.47712 \\ = \underline{0.52288}$$

$$\text{h) } \log 40.5 = \log \frac{405}{10} = \log \frac{81}{2} = \log 3^4 - \log 2 = 4 \cdot 0.47712 - 0.30103 \\ = \underline{1.60445}$$

$$\begin{aligned}
 i) \log(0.64^3 \cdot \sqrt{0.32}) &= \log 0.64^3 + \log \sqrt{0.32} = \\
 &= 3 \cdot \log \frac{64}{100} + \frac{1}{2} \log \frac{32}{100} = 3[\log 2^6 - \log 100] + \frac{1}{2} [\log 32 - \log 100] \\
 &= 3[6 \cdot \log 2 - 2] + \frac{1}{2} [5 \cdot \log 2 - 2] = 3[6 \cdot 0.30103 - 2] + \frac{1}{2} \\
 &\quad \cdot [5 \cdot 0.30103 - 2] = 3 \cdot (-0.19382) + 0.5 \cdot (-0.49485) = \underline{-0.828885} \\
 j) \log \sqrt{2\sqrt{2\sqrt{2}}} &= \log 2^{\frac{1}{2} + \frac{1}{4} + \frac{1}{8}} = \frac{7}{8} \log 2 = \frac{7}{8} \cdot 0.30103 = \underline{0.2634}
 \end{aligned}$$

10.-

- $\log_5 125 - \log_3 243 + \log_4 256 =$   
 $\log_5 5^3 - \log_3 3^5 + \log_4 4^4 = 3 - 5 + 4 = \underline{2}$
- $\log_3 1 + \log_2 64 + \log_3 9 + \log_7 49 =$   
 $0 + \log_2 2^6 + \log_3 3^2 + \log_7 7^2 = 0 + 6 + 2 + 2 = \underline{10}$
- $\log_2 \frac{4}{2^2} + \log_3 \frac{81}{3^4} - \log_6 \frac{216}{6^3} + \log_4 \frac{64}{4^3} = 2 + 4 - 3 + 3 = \underline{6}$
- $\log_3 \frac{1}{9} - \log_5 \frac{1}{0.2} + \log_6 \frac{1}{36} - \log_2 \frac{1}{0.5} =$   
 $\log_3 3^{-2} - \log_5 5^{-1} + \log_6 6^{-2} - \log_2 2^{-1} =$   
 $-2 - (-1) + (-2) - (-1) = -2 + 1 - 2 + 1 = \underline{-2}$

11.-

$$\begin{aligned}
 &\log_a 10^{-2} + 3 \cdot \log_a 10^2 - 4 \log_a 10 = \\
 &= -2 \log_a 10 + 6 \log_a 10 - 4 \log_a 10 = \underbrace{(-2+6-4)}_0 \cdot \log_a 10 = 0 \quad \text{e.q.d.}
 \end{aligned}$$

12.-

- $\log \frac{a^2 b}{c} = \log a^2 b - \log c = 2 \log a + \log b - \log c$
- $\log (a^2 b^3 c) = 2 \log a + 3 \log b + \log c$
- $\log \frac{a^2 \sqrt[3]{b}}{\sqrt[4]{c^3}} = 2 \log a + \frac{1}{3} \log b - \frac{3}{4} \log c$
- $\log \frac{m \cdot \sqrt[3]{n^4 \sqrt{mn/n}}}{n} = \log m + \frac{1}{3} \log(n^4 \sqrt{mn/n}) - \log n =$   
 $= \log m + \frac{1}{3}(4 \log n + \frac{1}{2} \log \frac{m}{n}) - \log n =$   
 $= \log m + \frac{4}{3} \log n + \frac{1}{6} \log m - \frac{1}{6} \log n - \log n = \frac{7}{6} \log m + \frac{1}{6} \log n$

$$e) \log_2 \frac{1}{2^{3x}} = \log_2 1 - \log_2 2^{3x} = 0 - 3x \cdot \frac{\log_2 2}{1} = -3x$$

$$f) \log_x \frac{\sqrt{x}}{\sqrt[3]{x^2}} = \log_x \sqrt{x} - \log_x \sqrt[3]{x^2} = \frac{1}{2} - \frac{2}{3} = \frac{-1}{6}$$

13:- a)  $\log x^4 - \log \sqrt{xy} = \log \frac{x^4}{\sqrt{xy}}$

b)  $\log x - 2\log y = \log \frac{x}{y^2}$

c)  $3\log x + \log(1-x) = \log x^3 + \log(1-x) = \log x^3(1-x)$ .

d)  $\frac{\log x}{2} + \frac{\log y}{4} = \log x^{1/2} + \log y^{1/4} = \log(\sqrt{x} \cdot \sqrt[4]{y})$ .

e)  $-\log x - \log y = \log x^{-1} + \log y^{-1} = \log\left(\frac{1}{xy}\right)$ .

f)  $\log x^{\log x} = \log x \cdot \log x = (\log x)^2$

14:- a)  $\log x + \log y = 1 \Rightarrow \log(x \cdot y) = \log 10 \Rightarrow \boxed{xy = 10}$

b)  $\log x - \log y = -1 \Rightarrow \log\left(\frac{x}{y}\right) = \log 10^{-1} \Rightarrow \boxed{\frac{x}{y} = \frac{1}{10}}$

c)  $4\log x - 3\log y = 2 \Rightarrow \log x^4 - \log y^3 = \log 10^2 \Rightarrow$   
 $\Rightarrow \log \frac{x^4}{y^3} = \log 100 \Rightarrow \boxed{\frac{x^4}{y^3} = 100}$

d)  $\frac{2\log x}{3} - 1 = \log y \Rightarrow \frac{2}{3}\log x - \log 10 = \log y$

$$\log \sqrt[3]{x^2} - \log 10 = \log y \Rightarrow \log \frac{\sqrt[3]{x^2}}{10} = \log y$$

$$\boxed{\frac{\sqrt[3]{x^2}}{10} = y}$$

e)  $\log(\log x) = 1 \Rightarrow \log(\log x) = \log 10 \Rightarrow \log x = 10 \Rightarrow$

$$\Rightarrow \boxed{x = 10^{10}}$$