EL NÚMERO REAL. LOGARITMOS

1. Calcula, sin usar calculadora los siguientes logaritmos:

$\log_2 \sqrt{2\sqrt[3]{2^2}} =$ $\qquad \log_3\left(\frac{\sqrt{3}}{3}\right) =$ $\qquad \log 0.0001 =$ $\qquad Ln\, e^{2/3} =$ $\qquad \log_{50} 1 =$ $\qquad \log\sqrt{10}$

2. Calcula, utilizando el cambio de base y la calculadora $\log_7 150$ y $\log_2 25$.

3. Halla el valor de x en cada caso:

$a)\ 5^{x+1} = \frac{2}{10}$ $\qquad b)\ 3^{2x+2} = 243$ $\qquad c)\ \log(x-2) = 2$ $\qquad d)\ \log(5x) = 4$

$e)\ \log_{1/8} x = \frac{1}{3}$ $\qquad f)\ \log_4 x = \frac{1}{2}$ $\qquad g)\ \log_x 343 = 3$ $\qquad h)\ \log_x 16 = -2$

4. Calcula x aplicando la definición de logaritmo:

$a)\ 2^x = 16$ $\qquad b)\ 2^x = 32$ $\qquad c)\ 3^{1/x} = 9$ $\qquad d)\ \log_2 64 = x$

$e)\ \log_3 81 = x$ $\qquad f)\ \log_{101} 10201 = x$ $\qquad g)\ \log_{16} 0,5 = x$ $\qquad h)\ \log_{10} 0,00001 = x$

$i)\ \log_x 125 = \frac{3}{2}$ $\qquad j)\ \log_x \frac{1}{3} = -\frac{1}{2}$ $\qquad k)\ \log_{125} \frac{1}{\sqrt{5}} = x$ $\qquad l)\ \log_{343} \sqrt{7} = x$

5. Calcula x aplicando la definición de logaritmo:

$a)\ \log_{\frac{2}{3}} \frac{81}{16} = x$ $\qquad b)\ \log_{\frac{5}{3}} \frac{27}{125} = x$ $\qquad c)\ \log_8 \sqrt[4]{2} = x$ $\qquad d)\ x = \log_3(3\sqrt{3})$

$e)\ x = \log_3\left(\frac{\sqrt[4]{3}}{9}\right)$ $\qquad f)\ x = \log_{81}(3)$ $\qquad g)\ x = \log_{81}\left(\frac{\sqrt{3}}{3}\right) =$ $\qquad h)\ x = \log_{1/9}\left(\frac{\sqrt[4]{3}}{3}\right) =$

$i)\ x = \log_{\sqrt{3}/3} 81$ $\qquad j)\ x = \log_{\sqrt{3}/3}\left(\frac{\sqrt[4]{3}}{3}\right)$ $\qquad k)\ \log_x\left(\frac{1}{2187}\right) = 7$ $\qquad l)\ \log_{2/5} x = -1$

6. Despeja x en los siguientes casos, ayudándote de la calculadora:

$2^{x+3} = 15$ $\qquad 3^{2x-4} = 56$ $\qquad \log_7 81 = x$ $\qquad x^{1,56} = 9,4$

7. Si se sabe que log A= 0,46 y que log B = 1,5, calcula razonadamente:

$\log\left(\frac{100A}{B^2}\right)$ $\qquad \log\sqrt[5]{\frac{(A\cdot B)^3}{10}}$ $\qquad \log\frac{A^4}{\sqrt{B}}$

8. Sabiendo que *log A=1,28* y *log B=0,35* calcula el valor de las siguientes expresiones:

$(a)\ \log\left(\frac{0,01\cdot A^3}{B}\right) =$ $\qquad (b)\ \log\sqrt[4]{\frac{B^3}{10A}} =$

9. Sabiendo que log 2= 0.30103 y que log 3 = 0.47712, calcular:

a) $\log 2000$ *b*) $\log\sqrt{5}$ *c*) $\log 25$ *d*) $\log\sqrt[5]{8}$ *e*) $\log\sqrt{160}$

f) $\log 0{,}125$ *g*) $\log 3.\widehat{\widehat{3}}$ *h*) $\log 40{,}5$ *i*) $\log\left(0{,}64^3\cdot\sqrt{0{,}32}\right)$

j) $\log\sqrt{2\sqrt{2\sqrt{2}}}$

10. Halla el resultado de las siguientes expresiones:

a) $\log_5 125 - \log_3 243 + \log_4 256 =$

b) $\log_3 1 + \log_2 64 + \log_3 9 + \log_7 49 =$

c) $\log_2 4 + \log_3 81 - \log_6 216 + \log_4 64 =$

d) $\log_3 \frac{1}{9} - \log_5 0,2 + \log_6 \frac{1}{36} - \log_2 0,5 =$

11. Demuestra que para cualquier base ***a*** se verifica:

$$\log_a 0{,}01 + 3\log_a 100 - 4\log_a 10 = 0$$

12. Desarrolla las siguientes expresiones, utilizando las propiedades de los logaritmos:

a) $\log\frac{a^2b}{c}$ *b*) $\log(a^2b^3c)$ *c*) $\log\frac{a^2\sqrt[3]{b}}{\sqrt[4]{c^3}}$

d) $\log\frac{m\sqrt[3]{n^4\sqrt{m/n}}}{n}$ *e*) $\log_2\frac{1}{2^{3x}}$ *f*) $\log_x\frac{\sqrt{x}}{\sqrt[3]{x^2}}$

13. Comprime las expresiones de manera que el logaritmo aparezca una sola

a) $\log x^4 - \log\sqrt{xy}$ *b*) $\log x - 2\log y$ *c*) $3\log x + \log(1-x)$

d) $\frac{\log x}{2} + \frac{\log y}{4}$ *e*) $-\log x - \log y$ *f*) $\log x^{\log x}$

vez:

14. Elimina los logaritmos de las expresiones siguientes:

a) $\log x + \log y = 1$ *b*) $\log x - \log y = -1$ *c*) $4\log x - 3\log y = 2$

d) $\frac{2\log x}{3} - 1 = \log y$ *e*) $\log(\log x) = 1$

LOGARITMOS - SOLUCIONES

1.- $\log_2 \sqrt{2\sqrt[3]{2^2}} = \log_2 \sqrt{\sqrt[3]{2^3\cdot 2^2}} = \log_2 \sqrt[6]{2^5} = \log_2 2^{5/6} = \frac{5}{6}$

$\log_3\left(\frac{\sqrt{3}}{3}\right) = \log_3\sqrt{3} - \log_3 3 = \frac{1}{2} - 1 = -1/2$

$\log 0{,}0001 = \log 10^{-4} = -4$

$\ln e^{2/3} = \frac{2}{3}$; $\log_{50} 1 = 0$; $\log\sqrt{10} = \log 10^{1/2} = \frac{1}{2}$

2.- $\log_7 150 = \frac{\log 150}{\log 7} = 2'575$

$\log_2 25 = \frac{\log 25}{\log 2} = 4'644$.

3.- (a) $5^{x+1} = \frac{2}{10} \iff 5^{x+1} = \frac{1}{5} \iff 5^{x+1} = 5^{-1} \iff x+1=-1 \Rightarrow \boxed{x=-2}$

(b) $3^{2x+2} = 243 \iff 3^{2x+2} = 3^5 \iff 2x+2=5 \iff 2x=3 \iff \boxed{x=\frac{3}{2}}$

(c) $\log(x-2) = 2 \iff x-2 = 10^2 \Rightarrow \boxed{x=102}$

(d) $\log(5x) = 4 \iff 5x = 10^4 \Rightarrow \boxed{x = \frac{10000}{5} = 2000}$

(e) $\log_{1/8} x = \frac{1}{3} \iff x = \left(\frac{1}{8}\right)^{1/3} = \sqrt[3]{1/8} = 1/2$

(f) $\log_4 x = \frac{1}{2} \iff x = 4^{1/2} = \sqrt{4} = 2$

g) $\log_x 343 = 3 \iff x^3 = 343 = 7^3 \iff \boxed{x=7}$

h) $\log_x 16 = -2 \iff x^{-2} = 16 = 2^4 = 4^2 \Rightarrow \frac{1}{x^2} = 4^2 \Rightarrow 4^2x^2 = 1 \Rightarrow x^2 = \frac{1}{4^2}$

$\boxed{x = 1/4}$

4.- a) $2^x = 16 \Rightarrow x = \log_2 16 = \log_2 2^4 = 4$ b) $2^x = 32 \Rightarrow x = \log_2 32 = \log_2 2^5 = 5$

c) $3^{1/x} = 9 \Rightarrow \frac{1}{x} = \log_3 9 \Rightarrow x^{-1} = \log_3 3^2 \Rightarrow x^{-1} = 2 \Rightarrow \boxed{x = 1/2}$

d) $x = \log_2 64 = \log_2 2^6 = 6$ e) $x = \log_3 81 = \log_3 3^4 = 4$.

f) $x = \log_{101} 10201 = \log_{101} 101^2 = 2$ g) $\log_{16} 0'5 = x \Rightarrow 16^x = \frac{1}{2} \Rightarrow$

$\Rightarrow 2^{-4x} = 2^{-1} \Rightarrow -4x = -1$ $\boxed{x = 1/4}$ h) $x = \log 0'00001 = \log 10^{-5} = -5$

i) $\log_x 125 = 3/2 \Rightarrow x^{3/2} = 5^3 \Rightarrow (x^2)^3 = 5^3 \Rightarrow x^2 = 5 \Rightarrow \boxed{x = \sqrt{5}}$

j) $\log_x \frac{1}{3} = -\frac{1}{2} \Rightarrow x^{-1/2} = \frac{1}{3} \iff \frac{1}{x^{1/2}} = \frac{1}{3} \Rightarrow \sqrt{x} = 3 \Rightarrow \boxed{x = 9}$

k) $\log_{125}\frac{1}{\sqrt{5}}=x \Rightarrow 125^x=\frac{1}{\sqrt{5}} \Rightarrow 5^{3x}=5^{-1/2} \Rightarrow 3x=-\frac{1}{2} \Rightarrow$

$\Rightarrow \boxed{x=-1/6}$

l) $\log_{343}\sqrt{7}=x \Leftrightarrow 343^x=\sqrt{7} \Rightarrow 7^{3x}=7^{1/2} \Rightarrow 3x=\frac{1}{2} \Rightarrow \boxed{x=\frac{1}{6}}$

5.- a) $\log_{2/3}\frac{81}{16}=x \Rightarrow \left(\frac{2}{3}\right)^x=\frac{81}{16}=\left(\frac{3}{2}\right)^4 \Rightarrow \left(\frac{2}{3}\right)^{x}=\left(\frac{2}{3}\right)^{-4} \Rightarrow \boxed{x=-4}$

b) $\log_{5/3}\frac{27}{125}=x \Rightarrow \cancel{\log}\ \left(\frac{5}{3}\right)^x=\left(\frac{3}{5}\right)^3=\left(\frac{5}{3}\right)^{-3} \Rightarrow \boxed{x=-3}$

c) $3^{1/x}=9=3^2 \Rightarrow \frac{1}{x}=2 \Rightarrow \boxed{x=1/2}$ d) $x=\log_2 64=6$

d) $x=\log_3(3\sqrt{3})=\log_3 3^{1+\frac{1}{2}}=3/2$.

e) $x=\log_3\left(\frac{\sqrt[4]{3}}{9}\right)=\log_3\frac{3^{1/4}}{3^2}=\log_3 3^{\frac{1}{4}-2}=\frac{-7}{8}$ //

f) $x=\log_{81}3 \Rightarrow 81^x=3 \Rightarrow 3^{4x}=3^1 \Rightarrow \boxed{x=\frac{1}{4}}$

g) $x=\log_{81}\left(\frac{\sqrt{3}}{3}\right) \Rightarrow (3^4)^x=\frac{3^{1/2}}{3} \Rightarrow 3^{4x}=3^{1/2} \Rightarrow 4x=\frac{1}{2} \Rightarrow \boxed{x=\frac{1}{8}}$

h) $x=\log_{1/9}\left(\frac{\sqrt[4]{3}}{3}\right) \Rightarrow \left(\frac{1}{9}\right)^x=3^{1/4-1} \Rightarrow 3^{-2x}=3^{-3/4} \Rightarrow -2x=\frac{-3}{4} \Rightarrow \boxed{x=\frac{3}{8}}$

i) $x=\log_{\sqrt{3}/3}81 \Rightarrow \left(\frac{\sqrt{3}}{3}\right)^x=81 \Leftrightarrow \left(3^{\frac{1}{2}-1}\right)^x=3^4 \Leftrightarrow 3^{-x/2}=3^4 \Rightarrow \boxed{x=-8}$

j) $x=\log_{\sqrt{3}/3}\left(\frac{\sqrt[4]{3}}{3}\right) \Rightarrow \left(\frac{\sqrt{3}}{3}\right)^x=\frac{\sqrt[4]{3}}{3} \Rightarrow 3^{-\frac{x}{2}}=3^{-3/4} \Rightarrow -\frac{x}{2}=\frac{-3}{4} \Rightarrow \boxed{x=\frac{3}{2}}$

k) $\log_x\left(\frac{1}{2187}\right)=7 \Rightarrow x^7=\frac{1}{2187} \Rightarrow x^7=\left(\frac{1}{3}\right)^7 \rightarrow x=\frac{1}{3}$

l) $\log_{2/5}x=-1 \Leftrightarrow \left(\frac{2}{5}\right)^{-1}=x \Leftrightarrow \boxed{\frac{5}{2}=x}$

6.- (a) $2^{x+3}=15 \xrightarrow[\log]{\text{Aplicamos}} \log 2^{x+3}=\log 15 \Rightarrow (x+3)\log 2=\log 15$

$\Rightarrow x+3=\frac{\log 15}{\log 2} \Rightarrow x=\frac{\log 15}{\log 2}-3=0'9069.$

(b) $3^{2x-4}=56 \Rightarrow \log 3^{2x-4}=\log 56 \Rightarrow (2x-4)\log 3=\log 56 \Rightarrow$

$2x-4=\frac{\log 56}{\log 3} \Rightarrow x=\frac{1}{2}\left[\frac{\log 56}{\log 3}+4\right]=3'832.$

(c) $x=\log_7 81=\frac{\ln 81}{\ln 7}=2'2581$

d) $x^{1'56}=9'4 \Rightarrow x=9'4^{(1/1'56)}=4'205.$

o bien $1'56\cdot\log x=\log 9'4 \Rightarrow \log x=\frac{\log 9'4}{1'56} \Rightarrow x=4'205.$

10^x función inversa

7.- $\log A = 0'46$; $\log B = 1'5$

a) $\log\left(\frac{100A}{B^2}\right) = \log 100A - \log B^2 = \log 100 + \log A - 2\log B =$
$= 2 + 0'46 - 2\cdot 1'5 = \underline{-0'54}$

b) $\log \sqrt[5]{\frac{(A\cdot B)^3}{10}} = \frac{1}{5}\left[\log (A\cdot B)^3 - \log 10\right] = \frac{1}{5}\left[3\log A + 3\log B - \log 10\right] =$
$= \frac{1}{5}\left(3\cdot 0'56 + 3\cdot 1'5 - 1\right) = \underline{1'036.}$

c) $\log \frac{A^4}{\sqrt{B}} = 4\log A - \frac{1}{2}\log B = 4\cdot 0'46 - \frac{1}{2}\cdot 1'5 = \underline{1'09}$

8.- $\log A = 1'28$; $\log B = 0'35$

(a) $\log\left(\frac{0'01\cdot A^3}{B}\right) = \log 0'01 + \log A^3 - \log B = -2 + 3\cdot\log A - \log B$
$= -2 + 3\cdot 1'28 - 0'35 = \underline{1'49}$

(b) $\log \sqrt[4]{\frac{B^3}{10A}} = \frac{1}{4}\left[\log B^3 - \log(10A)\right] = \frac{1}{4}\left[3\log B - \log 10 - \log A\right]$
$= \frac{1}{4}\left[3\cdot 0'35 - 1 - 1'28\right] = \underline{-0'3075}$

9.- $\log 2 = 0'30103$; $\log 3 = 0'47712$.

a) $\log(2000) = \log(2\cdot 1000) = \log 2 + \log 1000 = 0'30103 + 3 = \underline{3'30103}$

b) $\log\sqrt{5} = \frac{1}{2}\log 5 = \frac{1}{2}\log\left(\frac{10}{2}\right) = \frac{1}{2}\left[\log 10 - \log 2\right] =$
$= \frac{1}{2}\left(1 - 0'30103\right) = \underline{0'349485}$

c) $\log 25 = \log 5^2 = 2\left(\log(10) - \log 2\right) = \underline{1'39794}$

d) $\log\sqrt[5]{8} = \log\sqrt[5]{2^3} = \frac{3}{5}\log 2 = \frac{3}{5}\cdot 0'30103 = \underline{0'180618}$

e) $\log\sqrt{160} = \log 4\sqrt{10} = \log 4 + \log\sqrt{10} = 2\log 2 + \frac{1}{2}\log 10 =$
$= 2\cdot 0'30103 + \frac{1}{2}\cdot 1 = \underline{1'10206}$

f) $\log 0'125 = \log\frac{125}{1000} = \log\frac{1}{8} = \log 2^{-3} = -3\cdot\log 2 = \underline{-0'90309}$

g) $\log 3'\overset{\frown}{3} = \log\frac{33-3}{9} = \log\frac{30}{9} = \log\frac{10}{3} = \log 10 - \log 3 = 1 - 0'47712$
$= \underline{0'52288}$

h) $\log 40'5 = \log\frac{405}{10} = \log\frac{81}{2} = \log 3^4 - \log 2 = 4\cdot 0'47712 - 0'30103$
$= \underline{1'60745}$

i) $\log\left(0'64^3\cdot\sqrt{0'32}\right) = \log 0'64^3 + \log\sqrt{0'32} =$

$= 3\cdot\log\frac{64}{100} + \frac{1}{2}\log\frac{32}{100} = 3\left[\log 2^6 - \log 100\right] + \frac{1}{2}\left[\log 32 - \log 100\right]$

$= 3\cdot\left[6\cdot\log 2 - 2\right] + \frac{1}{2}\cdot\left[5\cdot\log 2 - 2\right] = 3\cdot\left[6\cdot 0'30103 - 2\right] + \frac{1}{2}\cdot$

$\cdot\left[5\cdot 0'30103 - 2\right] = 3\cdot(-0'19382) + 0'5\cdot(-0'49485) = \underline{-0'828885}$

j) $\log\sqrt{2\sqrt{2\sqrt{2}}} = \log 2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}} = \frac{7}{8}\log 2 = \frac{7}{8}\cdot 0'30103 = \underline{0'2634}$

10.- a) $\log_5 125 - \log_3 243 + \log_4 256 =$

$\log_5 5^3 - \log_3 3^5 + \log_4 4^4 = 3 - 5 + 4 = \underline{2}$

b) $\log_3 1 + \log_2 64 + \log_3 9 + \log_7 49 =$

$0 + \log_2 2^6 + \log_3 3^2 + \log_7 7^2 = 0 + 6 + 2 + 2 = \underline{10}$

c) $\log_2 \underset{2^2}{4} + \log_3 \underset{3^4}{81} - \log_6 \underset{6^3}{216} + \log_4 \underset{4^3}{64} = 2 + 4 - 3 + 3 = \underline{6}$

d) $\log_3\frac{1}{9} - \log_5 0'2 + \log_6\frac{1}{36} - \log_2 0'5 =$

$\log_3 3^{-2} - \log_5 5^{-1} + \log_6 6^{-2} - \log_2 2^{-1} =$

$-2 - (-1) + (-2) - (-1) = -2 + 1 - 2 + 1 = \underline{-2}$

$0'2 = \frac{2}{10} = \frac{1}{5} = 5^{-1}$

$0'5 = \frac{1}{2} = 2^{-1}$

11.- $\log_a 10^{-2} + 3\cdot\log_a 10^2 - 4\log_a 10 =$

$= -2\log_a 10 + 6\log_a 10 - 4\log_a 10 = \underbrace{(-2+6-4)}_{0}\cdot\log_a 10 = 0$

c.q.d.

12.- a) $\log\frac{a^2b}{c} = \log a^2b - \log c = 2\log a + \log b - \log c.$

b) $\log(a^2b^3c) = 2\log a + 3\log b + \log c$

c) $\log\frac{a^2\sqrt[3]{b}}{\sqrt[4]{c^3}} = 2\log a + \frac{1}{3}\log b - \frac{3}{4}\log c$

d) $\log\frac{m\cdot\sqrt[3]{n^4\sqrt{m/n}}}{n} = \log m + \frac{1}{3}\log\left(n^4\sqrt{m/n}\right) - \log n =$

$= \log m + \frac{1}{3}\left(4\log n + \frac{1}{2}\log\frac{m}{n}\right) - \log n =$

$= \log m + \frac{4}{3}\log n + \frac{1}{6}\log m - \frac{1}{6}\log n - \log n = \frac{7}{6}\log m + \frac{1}{6}\log n$

e) $\log_2 \frac{1}{2^{3x}} = \log_2 1 - \log_2 2^{3x} = 0 - 3x \cdot \underbrace{\log_2 2}_{1} = -3x$

f) $\log_x \frac{\sqrt{x}}{\sqrt[3]{x^2}} = \log_x \sqrt{x} - \log_x \sqrt[3]{x^2} = \frac{1}{2} - \frac{2}{3} = \frac{-1}{6}$.

13.- a) $\log x^4 - \log \sqrt{xy} = \log \frac{x^4}{\sqrt{xy}}$

b) $\log x - 2\log y = \log \frac{x}{y^2}$

c) $3 \log x + \log(1-x) = \log x^3 + \log(1-x) = \log x^3(1-x)$.

d) $\frac{\log x}{2} + \frac{\log y}{4} = \log x^{1/2} + \log y^{1/4} = \log(\sqrt{x} \cdot \sqrt[4]{y})$.

e) $-\log x - \log y = \log x^{-1} + \log y^{-1} = \log(\frac{1}{xy})$.

f) $\log x^{\log x} = \log x \cdot \log x = (\log x)^2$

14.- a) $\log x + \log y = 1 \Rightarrow \log(x \cdot y) = \log 10 \Rightarrow \boxed{xy = 10}$

b) $\log x - \log y = -1 \Rightarrow \log(\frac{x}{y}) = \log 10^{-1} \Rightarrow \boxed{\frac{x}{y} = \frac{1}{10}}$

c) $4 \log x - 3 \log y = 2 \Rightarrow \log x^4 - \log y^3 = \log 10^2 \Rightarrow$

$\Rightarrow \log \frac{x^4}{y^3} = \log 100 \Rightarrow \boxed{\frac{x^4}{y^3} = 100}$

d) $\frac{2 \log x}{3} - 1 = \log y \Rightarrow \frac{2}{3} \log x - \log 10 = \log y$

$\log \sqrt[3]{x^2} - \log 10 = \log y \Rightarrow \log \frac{\sqrt[3]{x^2}}{10} = \log y$

$\boxed{\frac{\sqrt[3]{x^2}}{10} = y}$

e) $\log(\log x) = 1 \Rightarrow \log(\log x) = \log 10 \Rightarrow \log x = 10 \Rightarrow$

$\Rightarrow \boxed{x = 10^{10}}$