Medida de ángulos y Razones Trigonométricas

1. Determina, sin usar la calculadora, el signo del seno, coseno y tangente de los siguientes ángulos:

a) –200º b) 3487º c) 6π/5 rad

2. Construye con regla y compás, sin usar un transportador de ángulos, dos ángulos menores de 360º que tengan:

a) $\cos\alpha = -\frac{1}{4}$ b) $sen\alpha = \frac{1}{\sqrt{2}}$ c) $\operatorname{tg}\alpha = 1'5$ d) un coseno triple del seno

3. Deduce la expresión simplificada de las restantes razones trigonométricas en función de la cotangente.

4. Escribe, sin utilizar calculadora, los valores **exactos** del seno, el coseno y la tangente de los siguientes ángulos:

a) 1935º b) -1470º c) 32π/6 rad

5. Utilizando únicamente que cosec 296º = -1´1, escribe redondeando los resultados con dos cifras decimales, el seno y el coseno de los siguientes ángulos:

a) 64º b) 244º c) 26º d) 116º

6. Halla, sin utilizar calculadora, los valores **exactos** de las restantes razones trigonométricas de:

a) $\cos ec\alpha = 2$ siendo $\frac{\pi}{2} < \alpha < \pi$ b) $\cot g\alpha = \frac{1}{3}$ siendo $\pi < \alpha < \frac{3\pi}{2}$

7. Demuestra las siguientes identidades:

a) $\cos^4\alpha - sen^4\alpha = 2\cos^2\alpha - 1$ b) $(\cos ec\alpha + \cot g\alpha)(\cos ec\alpha - \cot g\alpha) = 1$

8. Simplifica lo más posible las siguientes expresiones: a) $\frac{\sec^2 a - \cos^2 a}{tg^2 a}$ b) $\frac{\cos eca}{1 + ctg^2 a}$

9. Sin hacer uso de la calculadora, escribe las expresiones generales de todos los ángulos que:

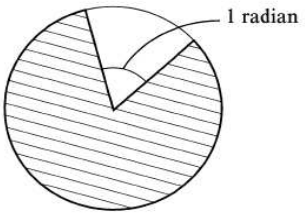
a) tengan un coseno igual al de 117º
b) tengan una tangente igual a la de 41º
c) tengan 0´5 como valor de seno
d) tengan $\sqrt{3}$ como valor de tangente

10. Escribe, aproximadas al grado más cercano, los ángulos (0º < x < 360º) que cumplan:

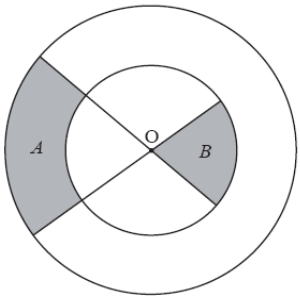
a) sen x = 0´5983 b) tg x = 0'47960 c) ctg x = – 1´19572
d) cos x = – 0´7583 e) sen x = - 0´1045 f) cos x = 0´1908

11. Cortamos un disco circular en veinte sectores cuyas áreas están en progresión aritmética. El ángulo de mayor sector es doble del ángulo del sector menor. Halle el ángulo del menor sector.

12. El diagrama muestra un círculo de 5 cm de radio. Halle el perímetro y el área de la región sombreada.



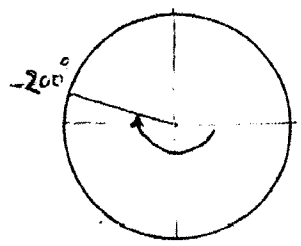
13. El diagrama muestra dos círculos concéntricos, el radio del interior es 1 cm. Halla el radio del círculo exterior sabiendo que el área de la región A es doble que la de la región B.



14. En los escaparates de las relojerías suelen mostrar las agujas de los relojes en posición simétrica marcando un poco antes de las diez y diez. Precisa hora, minuto y segundo en que sucede.

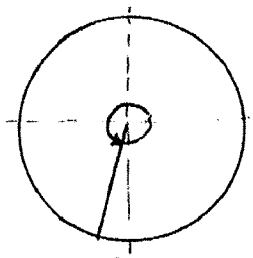
1)

(a)



SENO	+
COSENO	−
TANG	−

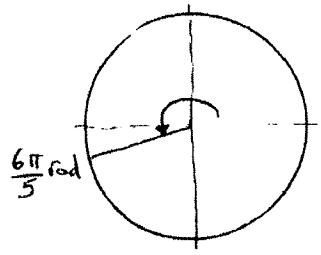
(b)



$$\begin{array}{r|l} 3487 & 360 \\ \hline 247 & 9 \end{array}$$

SENO	−
COSENO	−
TANG	+

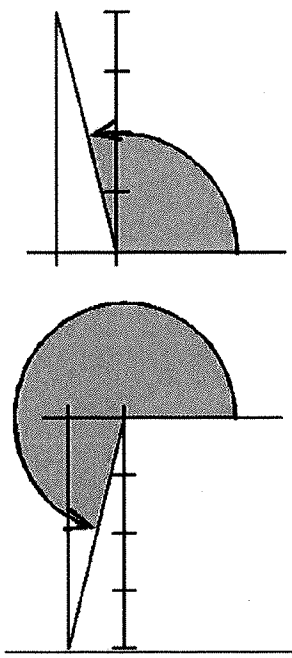
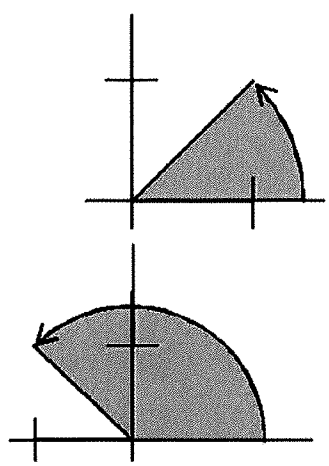
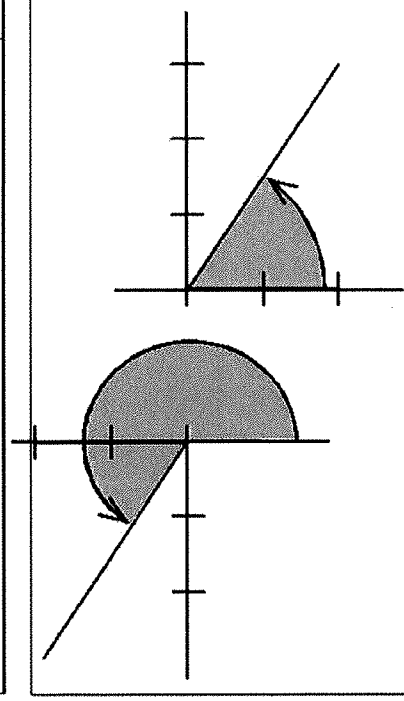
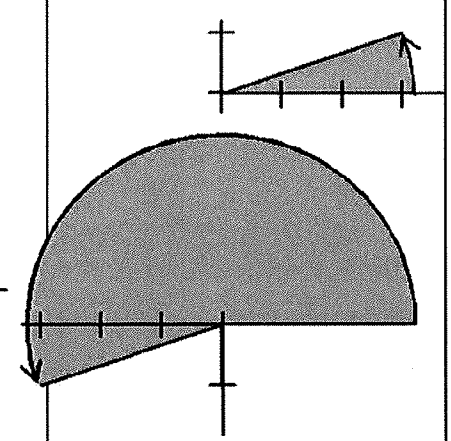
(c)



$$\frac{6\pi}{5}\text{ rad} \equiv \frac{6\times 180^\circ}{5} = 216^\circ$$

SENO	−
COSENO	−
TANG	+

2)

$\cos\alpha = -\frac{1}{4}$	$sen\alpha = \frac{\sqrt{2}}{2}$	$tg\alpha = 1'5$	$\cos\alpha = 3sen\alpha$
			

3)

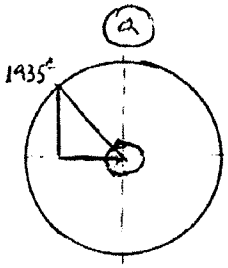
$$sen^2a + cos^2a = 1 \Rightarrow \frac{sen^2a}{sen^2a} + \frac{cos^2a}{sen^2a} = \frac{1}{sen^2a} \Rightarrow 1 + ctg^2a = cosec^2a \Rightarrow$$

$$\Rightarrow \boxed{cosec\,a = \pm\sqrt{1+ctg^2a}} \Rightarrow \boxed{sen\,a = \frac{\pm 1}{\sqrt{1+ctg^2a}}} \Rightarrow cos\,a = sen\,a \cdot ctg\,a = \boxed{\frac{\pm ctg\,a}{\sqrt{1+ctg^2a}}} \Rightarrow$$

$$\Rightarrow \boxed{sec\,a = \frac{\pm\sqrt{1+ctg^2a}}{ctg\,a}} \qquad \boxed{tg\,a = \frac{1}{ctg\,a}}$$

4

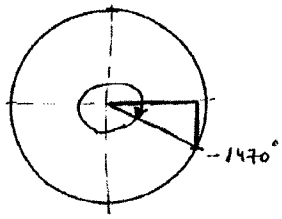
ⓐ



$$\begin{array}{r|l} 1935 & 360 \\ \underline{135} & 5 \end{array}$$

$$\boxed{\begin{aligned} \operatorname{sen} 1935^\circ &= \operatorname{sen} 135^\circ = \sqrt{2}/2 \\ \cos 1935^\circ &= \cos 135^\circ = -\sqrt{2}/2 \\ \operatorname{tg} 1935^\circ &= -1 \end{aligned}}$$

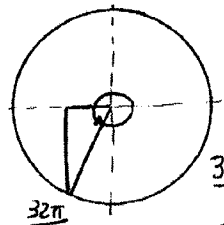
ⓑ



$$\begin{array}{r|l} 1470 & 360 \\ \underline{030} & 4 \end{array}$$

$$\boxed{\begin{aligned} \operatorname{sen}(-1470^\circ) &= \operatorname{sen}(-30) = -\operatorname{sen} 30 = -\tfrac{1}{2} \\ \cos(-1470^\circ) &= \cos(-30) = \cos 30 = \sqrt{3}/2 \\ \operatorname{tg}(-1470^\circ) &= -1/\sqrt{3} \end{aligned}}$$

ⓒ

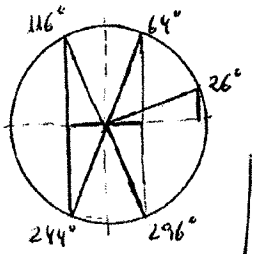


$$\frac{32\pi}{6} = \frac{32 \cdot 180}{6} = 960^\circ$$

$$\begin{array}{r|l} 960 & 360 \\ \underline{240} & 2 \end{array}$$

$$\boxed{\begin{aligned} \operatorname{sen} \frac{32\pi}{6} &= \operatorname{sen} 240^\circ = -\sqrt{3}/2 \\ \cos \frac{32\pi}{6} &= \cos 240^\circ = -1/2 \\ \operatorname{tg} \frac{32\pi}{6} &= \sqrt{3} \end{aligned}}$$

5



$$\operatorname{cosec} 296^\circ = -1'1 \Rightarrow \operatorname{sen} 296^\circ = -\frac{1}{1'1} = -0'91$$

$$\cos 296^\circ = -\sqrt{1-(-0'91)^2} = +0'42$$

ⓐ	ⓑ	ⓒ	ⓓ
$\operatorname{sen} 64^\circ = 0'91$	$\operatorname{sen} 244^\circ = -0'91$	$\operatorname{sen} 26^\circ = 0'42$	$\operatorname{sen} 116^\circ = 0'91$
$\cos 64^\circ = 0'42$	$\cos 244^\circ = -0'42$	$\cos 26^\circ = 0'91$	$\cos 116^\circ = -0'42$

6

ⓐ $\left.\begin{array}{l} \operatorname{cosec} a = 2 \\ (\pi/2 < a < \pi) \end{array}\right| \Rightarrow \operatorname{sen} a = \boxed{\frac{1}{2}} \Rightarrow \cos a = \pm\sqrt{1-\frac{1}{4}} = \boxed{-\frac{\sqrt{3}}{2}} \Rightarrow \sec a = \boxed{-\frac{2}{\sqrt{3}}}$

$$\operatorname{tg} a = \frac{1/2}{-\sqrt{3}/2} = \boxed{-\frac{1}{\sqrt{3}}} \Rightarrow \boxed{\operatorname{ctg} a = -\sqrt{3}}$$

ⓑ $\left.\begin{array}{l} \operatorname{ctg} a = \frac{1}{3} \\ \pi < a < \frac{3\pi}{2} \end{array}\right| \Rightarrow \frac{\cos a}{\operatorname{sen} a} = \frac{1}{3} \Rightarrow \operatorname{sen} a = 3\cos a \Rightarrow (3\cos a)^2 + \cos^2 a = 1 \Rightarrow$

$$\Rightarrow 10\cos^2 a = 1 \Rightarrow \cos a = \boxed{-\frac{1}{\sqrt{10}}} \Rightarrow \operatorname{sen} a = \boxed{-\frac{3}{\sqrt{10}}} \Rightarrow \boxed{\operatorname{tg} a = 3}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\boxed{\sec a = -\sqrt{10}} \qquad \boxed{\operatorname{cosec} a = -\frac{\sqrt{10}}{3}}$$

7

a) $\cos^4\alpha - \operatorname{sen}^4\alpha = 2\cos^2\alpha - 1$

$$\cos^4\alpha - \operatorname{sen}^4\alpha = (\cos^2\alpha + \operatorname{sen}^2\alpha)(\cos^2\alpha - \operatorname{sen}^2\alpha) = 1 \cdot [\cos^2\alpha - (1-\cos^2\alpha)] = 2\cos^2\alpha - 1 \checkmark$$

b) $(\operatorname{cosec}\alpha + \operatorname{ctg}\alpha)(\operatorname{cosec}\alpha - \operatorname{ctg}\alpha) = 1$

$$(\operatorname{cosec}\alpha + \operatorname{ctg}\alpha)(\operatorname{cosec}\alpha - \operatorname{ctg}\alpha) = \operatorname{cosec}^2\alpha - \operatorname{ctg}^2\alpha = \frac{1}{\operatorname{sen}^2\alpha} - \frac{\cos^2\alpha}{\operatorname{sen}^2\alpha} = \frac{1-\cos^2\alpha}{\operatorname{sen}^2\alpha} = \frac{\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha} = 1 \checkmark$$

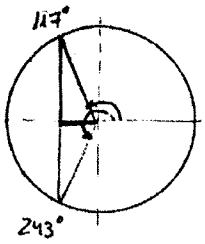
8

a) $$\frac{\sec^2\alpha - \cos^2\alpha}{\operatorname{tg}^2\alpha} = \frac{\frac{1}{\cos^2\alpha} - \cos^2\alpha}{\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}} = \frac{1-\cos^4\alpha}{\operatorname{sen}^2\alpha} = \frac{(1+\cos^2\alpha)(1-\cos^2\alpha)}{\operatorname{sen}^2\alpha} = \frac{(1+\cos^2\alpha)\cdot\cancel{\operatorname{sen}^2\alpha}}{\cancel{\operatorname{sen}^2\alpha}} = \boxed{1+\cos^2\alpha}$$

b) $$\frac{\operatorname{cosec}\alpha}{1+\operatorname{ctg}^2\alpha} = \frac{\frac{1}{\operatorname{sen}\alpha}}{1+\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}} = \frac{\operatorname{sen}\alpha}{\frac{1}{\operatorname{sen}^2\alpha}+\cos^2\alpha} = \frac{\operatorname{sen}\alpha}{1} = \boxed{\operatorname{sen}\alpha}$$

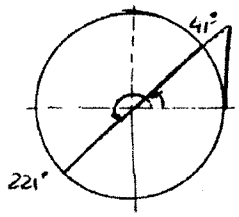
9]

ⓐ



$\alpha = 117° + m\,360°$
$\alpha = 243 + m \cdot 360°$
$m \in \mathbb{Z}$

ⓑ

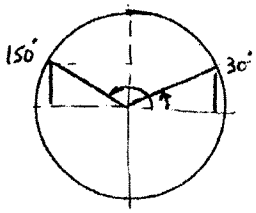


$\alpha = 41° + m \cdot 360°$
$\alpha = 221° + m \cdot 360°$
$m \in \mathbb{Z}$

O también:

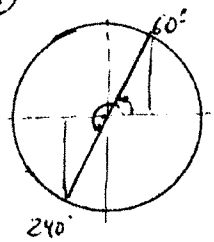
$\alpha = 41° + m \cdot 180°$
$m \in \mathbb{Z}$

ⓒ



$\alpha = 30° + m \cdot 360°$
$\alpha = 150° + m \cdot 360°$
$m \in \mathbb{Z}$

ⓓ



$\alpha = 60° + m \cdot 360°$
$\alpha = 240° + m \cdot 360°$
$m \in \mathbb{Z}$

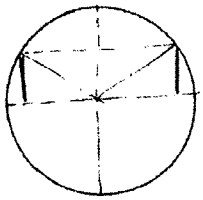
O también:

$\alpha = 60° + m \cdot 180°$
$m \in \mathbb{Z}$

10]

ⓐ $\operatorname{sen} x = 0'5983 \Rightarrow$

$x = 37 + m \cdot 360°$
$x = 143 + m \cdot 360°$
$m \in \mathbb{Z}$

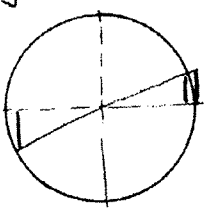


ⓑ $\operatorname{tg} x = 0'47960 \Rightarrow$

$x = 26° + m \cdot 360°$
$x = 206° + m \cdot 360°$
$m \in \mathbb{Z}$

O también:

$x = 26° + m \cdot 180°$
$m \in \mathbb{Z}$



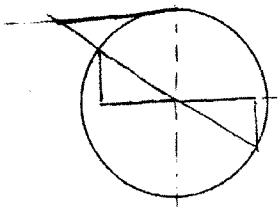
ⓒ $\operatorname{ctg} x = -1'19572 \Rightarrow x = -39'91$

$\Downarrow$

$x = 320° + m \cdot 360°$
$x = 140° + m \cdot 360°$
$m \in \mathbb{Z}$

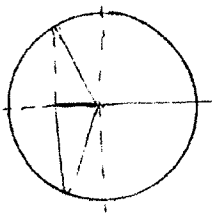
O también:

$x = 140° + m \cdot 180°$
$m \in \mathbb{Z}$



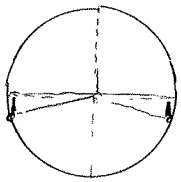
ⓓ $\cos x = -0'7583 \Rightarrow$

$x = 139° + m \cdot 360°$
$x = 221° + m \cdot 360°$
$m \in \mathbb{Z}$



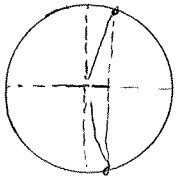
(e) $\operatorname{sen} x = -0'1045 \Rightarrow x = -6^\circ \equiv \boxed{354^\circ + m\cdot 360^\circ}$ $\quad m \in \mathbb{Z}$

$x = \boxed{186^\circ + m\cdot 360^\circ}$



(f) $\cos = 0'1908 \Rightarrow x = \boxed{79^\circ + m\cdot 360^\circ}$ $\quad m \in \mathbb{Z}$

$x = \boxed{281^\circ + m\cdot 360^\circ}$



11| Como el área de un sector $\left(\frac{R^2\theta}{2}\right)$ es proporcional al ángulo, que el mayor ángulo sea doble del menor significa que el mayor área será doble de la menor.

$A_1, A_2, \ldots, A_{20}$

$S_{20} = \pi R^2 \Rightarrow \frac{A_1 + A_{20}}{2}\cdot 20 = \pi R^2 \Rightarrow \frac{A_1 + 2A_1}{2}\cdot 20 = \pi R^2$; $30A_1 = \pi R^2$; $A_1 = \frac{\pi R^2}{30}$

$A_{20} = 2A_1$

$A_1 = \frac{R^2\theta_1}{2} \Rightarrow \frac{\cancel{R^2}\theta_1}{2} = \frac{\pi \cancel{R^2}}{30} \Rightarrow \theta_1 = \boxed{\frac{\pi}{15}\text{ rad}}$

<u>También</u>

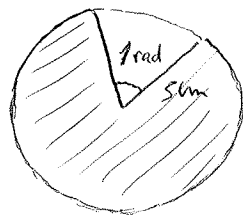
Al ser áreas proporcionales a ángulos, que las primeras estén en progresión aritmética implicaría que lo están los ángulos: $\theta_1, \theta_2, \ldots, \theta_{20}$

$S_{20} = 2\pi \Rightarrow \frac{\theta_1 + \theta_{20}}{2}\cdot 20 = 2\pi$; $\frac{\theta_1 + 2\theta_1}{2}\cdot 20 = 2\pi$; $30\theta_1 = 2\pi \Rightarrow \theta_1 = \frac{\pi}{15}$ ✓

12| 2π rad —— $2\pi\cdot 5$

$2\pi - 1$ rad —— Arco

$\text{Arco} = \frac{\cancel{2\pi}\cdot 5\cdot(2\pi-1)}{\cancel{2\pi}} = 10\pi - 5 \simeq 26'4$ cm



$\text{Perímetro} = \text{Arco} + 2\cdot R = 10\pi - 5 + 2\cdot 5 =$

$= \boxed{10\pi + 5} \simeq 36'4$ cm

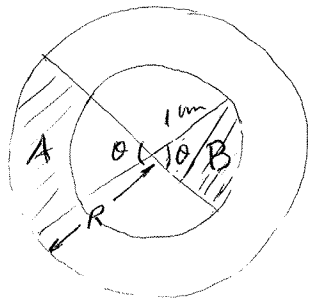
2π rad —— $\pi\cdot 5^2$

$2\pi - 1$ rad —— Sector

$\text{Sector} = \frac{\cancel{\pi}\cdot 25\cdot(2\pi-1)}{2\cancel{\pi}} = \boxed{\frac{50\pi - 25}{2}} \simeq 66'04$ cm^2

13| $\text{Area } B = \frac{\pi\cdot 1^2\cdot\theta}{2} = \frac{\pi\theta}{2}$

$\text{Area } A = \frac{\pi\cdot R^2\theta}{2} - \frac{\pi\cdot 1^2\theta}{2} = \frac{\pi(R^2-1)\theta}{2}$



$\text{Area } A = 2\cdot\text{Area } B \Rightarrow \frac{\cancel{\pi}(R^2-1)\cancel{\theta}}{\cancel{2}} = 2\frac{\cancel{\pi}\cancel{\theta}}{\cancel{2}}$

$R^2 - 1 = 2$

$R^2 = 3$; $R = \boxed{\sqrt{3}\text{ cm}}$

14

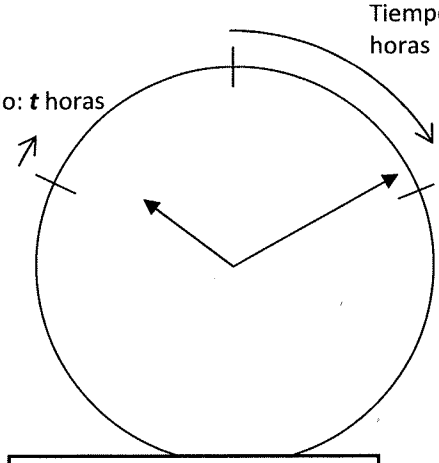
Las velocidades angulares de las agujas son:

Aguja Horaria = $30\,{}^{\circ}/_{h}$

Aguja Minutera = $360\,{}^{\circ}/_{h}$

Aguja Horaria
Tiempo transcurrido: ***t*** horas
Angulo girado: ***a***

Aguja Minutera
Tiempo transcurrido: ***t*** horas



A partir de las diez en punto, y en un mismo tiempo ***t***:

$$\left.\begin{array}{r} 30 = \dfrac{a}{t} \rightarrow a = 30t \\ 360 = \dfrac{60-a}{t} \end{array}\right\} \Rightarrow 360t = 60 - 30t$$

$$390t = 60 \Rightarrow t = \frac{60}{390} = \frac{2}{13}h \approx 9\min 14seg$$

Son casi las 10h 9min 14seg