

OPERACIONES CON FRACCIONES ALGEBRAICAS

Realiza las siguientes operaciones con fracciones simplificando al máximo:

$$1) \frac{3-x}{x} + \frac{2x}{x-1} - \frac{x-1}{x}$$

$$2) \frac{3}{x+1} - \frac{2}{x-1}$$

$$3) \frac{x-1}{x+1} - \frac{x+1}{x-1} + \frac{4}{x^2-1}$$

$$4) \frac{x+3}{x-4} - \frac{x-3}{x+4} - \frac{14x}{x^2-16}$$

$$5) \frac{x}{x-1} + \frac{x+1}{x^2-4x+3}$$

$$6) \frac{x-6-x^3}{x^2+4x+4} - \frac{5}{x+2} + x$$

$$7) \frac{2}{1-x} + \frac{4}{1+x} - \frac{4}{1-x^2}$$

$$8) \frac{6}{1+x} + \frac{4}{x-1} - \frac{12x}{x^2-1}$$

$$9) \frac{2x^2-2}{x^2+2x+1} \cdot \frac{x^2-1}{x^2+x-2}$$

$$10) \frac{8x^2}{x^2-3x+2} \cdot \frac{x^2-4x+4}{4x^4-16x^2}$$

$$11) \frac{2x^2}{4x-8} : \frac{x^3-x^2}{x^2-3x+2}$$

$$12) \frac{x^2-5x+6}{x^2-x-2} : \frac{x^2+2x}{x^2+x}$$

$$13) \frac{x^2+x-2}{x+3} \cdot \frac{x^2+6x+9}{x^2+2x-3}$$

$$14) \frac{x^2-9}{x^2+9} : \frac{x^2-6x+9}{x^4-81}$$

$$15) \left(x - \frac{1}{x}\right) \cdot \left(x + \frac{1}{x}\right) \cdot \frac{x}{x^2-1}$$

$$16) \left(\frac{x^2+4}{x} + 4\right) : \frac{x+2}{2x}$$

$$17) \frac{x^2-4}{x^3} \cdot \left(\frac{x}{x+2}\right)^2$$

$$18) \left(\frac{1}{2x} - \frac{2}{x^2}\right) : \left(\frac{x}{4} - \frac{4}{x}\right)$$

$$19) \frac{x^2-4x}{x^2-16} : \left(1 - \frac{4}{x+4}\right)$$

$$20) \left(\frac{1}{2} - \frac{1}{x}\right) : \left(\frac{1}{4} - \frac{1}{x^2}\right)$$

Soluciones:

$$1) \frac{3-x}{x} + \frac{2x}{x-1} - \frac{x-1}{x} = \frac{(3-x)(x-1)}{x(x-1)} + \frac{2x^2}{x(x-1)} - \frac{(x-1)^2}{x(x-1)} = \frac{3x-3-\cancel{x^2}+x+2\cancel{x^2}-\cancel{x^2}+2x-1}{x(x-1)} = \frac{6x-4}{x^2-x}$$

$$2) \frac{3}{x+1} - \frac{2}{x-1} = \frac{3(x-1)-2(x+1)}{(x+1)(x-1)} = \frac{3x-3-2x-2}{(x+1)(x-1)} = \frac{x-5}{x^2-1}$$

$$3) \frac{x-1}{x+1} - \frac{x+1}{x-1} + \frac{4}{x^2-1} \\ = \frac{(x-1)^2 - (x+1)^2 - 4}{(x+1)(x-1)} = \frac{\cancel{x^2} - 2x + \cancel{1} - \cancel{x^2} - 2x - \cancel{1} - 4}{(x+1)(x-1)} = \frac{-4x-4}{(x+1)(x-1)} = \frac{-4(\cancel{x+1})}{(\cancel{x+1})(x-1)} = \frac{-4}{x-1}$$

$$4) \frac{x+3}{x-4} - \frac{x-3}{x+4} - \frac{14x}{x^2-16} = \frac{(x+3)(x+4) - (x-3)(x-4) - 14x}{(x-4)(x+4)} = \frac{\cancel{x^2} + 3x + 4x + \cancel{12} - \cancel{x^2} + 3x + 4x - \cancel{12} - 14x}{(x-4)(x+4)} = 0$$

$$5) \frac{x}{x-1} + \frac{x+1}{x^2-4x+3} = \frac{x(x-3)+x+1}{(x-1)(x-3)} = \frac{x^2-3x+x+1}{(x-1)(x-3)} = \frac{x^2-2x+1}{(x-1)(x-3)} = \frac{(x-1)^2}{(x-1)(x-3)} = \frac{x-1}{x-3}$$

$$6) \frac{x-6-x^3}{x^2+4x+4} - \frac{5}{x+2} + x = \frac{x-6-x^3-5(x+2)+x(x+2)^2}{(x+2)^2} = \frac{\cancel{x} - 6 - \cancel{x^3} - 5\cancel{x} - 10 + \cancel{x^3} + 4x^2 + 4\cancel{x}}{(x+2)^2} = \frac{4x^2-16}{(x+2)^2} = \\ = \frac{4(x-2)(\cancel{x+2})}{(x+2)(\cancel{x+2})} = \frac{4x-8}{x+2}$$

$$7) \frac{2}{1-x} + \frac{4}{1+x} - \frac{4}{1-x^2} = \frac{2(1+x)+4(1-x)-4}{(1-x)(1+x)} = \frac{2+2x+\cancel{4}-4x-\cancel{4}}{(1-x)(1+x)} = \frac{2-2x}{(1-x)(1+x)} = \frac{2(\cancel{1-x})}{(\cancel{1-x})(1+x)} = \frac{2}{1+x}$$

$$8) \frac{6}{1+x} + \frac{4}{x-1} - \frac{12x}{x^2-1} = \frac{6(x-1)+4(1+x)-12x}{(1+x)(x-1)} = \frac{6x-6+4+4x-12x}{(1+x)(x-1)} = \frac{-2-2x}{(1+x)(x-1)} = \frac{-2(1+x)}{(1+x)(x-1)} = \frac{-2}{x-1}$$

$$9) \frac{2x^2-2}{x^2+2x+1} \cdot \frac{x^2-1}{x^2+x-2} = \frac{(2x^2-2)(x^2-1)}{(x^2+2x+1)(x^2+x-2)} = \frac{2(x-1)^2(\cancel{x+1})^2}{(\cancel{x+1})^2(x+2)(\cancel{x-1})} = \frac{2x-2}{x+2}$$

$$10) \frac{8x^2}{x^2-3x+2} \cdot \frac{x^2-4x+4}{4x^4-16x^2} = \frac{8x^2(x^2-4x+4)}{(x^2-3x+2)(4x^4-16x^2)} = \frac{8\cancel{x^2}(\cancel{x-2})^2}{(\cancel{x-2})(x-1)4\cancel{x^2}(\cancel{x-2})(x+2)} = \frac{2}{x^2+x-2}$$

$$11) \frac{2x^2}{4x-8} : \frac{x^3-x^2}{x^2-3x+2} = \frac{2x^2(x^2-3x+2)}{(4x-8)(x^3-x^2)} = \frac{2\cancel{x^2}(\cancel{x-2})(\cancel{x-1})}{4(\cancel{x-2}) \cdot \cancel{x^2}(\cancel{x-1})} = \frac{1}{2}$$

$$12) \frac{x^2-5x+6}{x^2-x-2} : \frac{x^2+2x}{x^2+x} = \frac{(x^2-5x+6)(x^2+x)}{(x^2-x-2)(x^2+2x)} = \frac{(x-3)(\cancel{x-2})(\cancel{x+1})\cancel{x}}{(\cancel{x-2})(\cancel{x+1})(x+2)\cancel{x}} = \frac{x-3}{x+2}$$

$$13) \frac{x^2+x-2}{x+3} \cdot \frac{x^2+6x+9}{x^2+2x-3} = \frac{(x^2+x-2)(x^2+6x+9)}{(x+3)(x^2+2x-3)} = \frac{(x+2)(x-1)(x+3)^2}{(x+3)(x+3)(x-1)} = \frac{(x+2)(\cancel{x-1})(\cancel{x+3})^2}{(\cancel{x+3})(\cancel{x+3})(\cancel{x-1})} = x+2$$

$$14) \frac{x^2-9}{x^2+9} : \frac{x^2-6x+9}{x^4-81} = \frac{(x^2-9)(x^4-81)}{(x^2+9)(x^2-6x+9)} = \frac{(\cancel{x^2+9})(\cancel{x-3})^2(x+3)^2}{(\cancel{x^2+9})(\cancel{x-3})^2} = (x+3)^2$$

$$15) \left(x - \frac{1}{x}\right) \cdot \left(x + \frac{1}{x}\right) \cdot \frac{x}{x^2-1} = \frac{\cancel{x^2-1}}{\cancel{x}} \cdot \frac{x^2+1}{x} \cdot \frac{\cancel{x}}{\cancel{x^2-1}} = \frac{x^2+1}{x}$$

$$16) \left(\frac{x^2+4}{x} + 4\right) : \frac{x+2}{2x} = \frac{x^2+4+4x}{x} : \frac{x+2}{2x} = \frac{2x(x^2+4+4x)}{x(x+2)} = \frac{2\cancel{x}(x+2)^2}{\cancel{x}(x+2)} = 2x+4$$

$$17) \frac{x^2-4}{x^3} \cdot \left(\frac{x}{x+2}\right)^2 = \frac{x^2-4}{x^3} \cdot \frac{x^2}{(x+2)^2} = \frac{(\cancel{x+2})(x-2)\cancel{x^2}}{x^{\cancel{3}}(x+2)^2} = \frac{x-2}{x^2+2x}$$

$$18) \left(\frac{1}{2x} - \frac{2}{x^2}\right) : \left(\frac{x}{4} - \frac{4}{x}\right) = \frac{x-4}{2x^2} : \frac{x^2-16}{4x} = \frac{4\cancel{x}(\cancel{x-4})}{2x^{\cancel{2}}(\cancel{x-4})(x+4)} = \frac{2}{x^2+4x}$$

$$19) \frac{x^2-4x}{x^2-16} : \left(1 - \frac{4}{x+4}\right) = \frac{x^2-4x}{x^2-16} : \frac{x+4-4}{x+4} = \frac{(x^2-4x)(x+4)}{x(x^2-16)} = \frac{x(x-4)(x+4)}{x(x^2-16)} = 1$$

$$20) \left(\frac{1}{2} - \frac{1}{x}\right) : \left(\frac{1}{4} - \frac{1}{x^2}\right) = \frac{x-2}{2x} : \frac{x^2-4}{4x^2} = \frac{4x(x-2)}{2x(x^2-4)} = \frac{4\cancel{x}(\cancel{x-2})}{2\cancel{x}(\cancel{x-2})(x+2)} = \frac{2}{x+2}$$