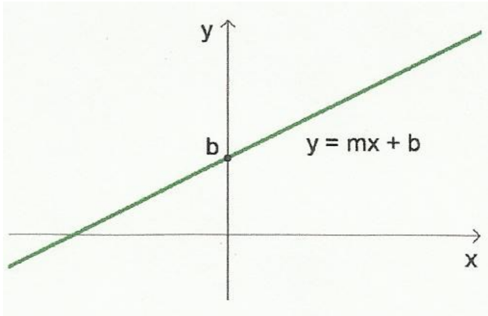
UNIT 9: ELEMENTARY FUNCTIONS:

Linear Functions: A function that can be graphically represented in the Cartesian Plane as a straight line is called a linear function.



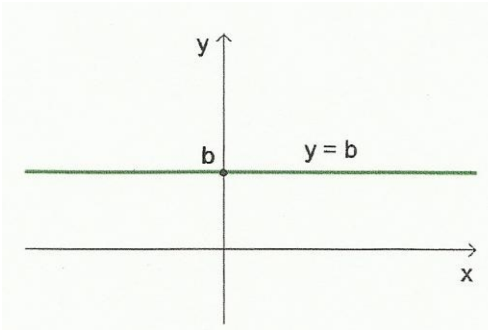
The equation of a linear function is

$$y=mx+n$$

m is the **slope or gradient** and **n** is the **y-intercept**.

Examples: y=-2x+5 , y=x-3

Remember that if m>0, the line is an increasing function, and if m<0, the line is a decreasing function.



If m=0, the equation of the function is

$$y=n$$

This type of linear functions are called **constant functions**.

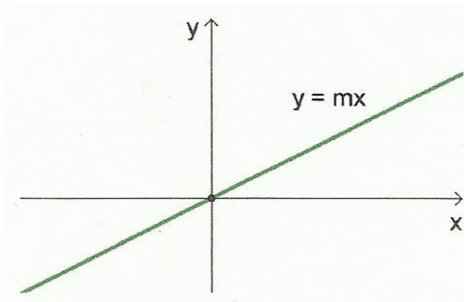
Their graphs are horizontal lines.

Examples: y=-2, y=4, y=0.

If n=0, the equation of the function is

$$y=mx$$

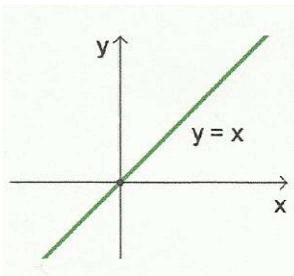
This type of linear functions are called **directly proportional function**.



The variable y is directly proportional to the variable x. The constant ratio, $m=\frac{y}{x}$, is called constant of proportionality. Their graphs is a straight line that passes through the origin.

Examples: y=2x, $y=\frac{2}{3}x$.

If m=1, the directly proportional function is y=x, and it is called **identity function**. This line is the angle bisector of the first and third quadrant.



Parabolas and Quadratic Functions:

A polynomial of second degree define a function whose algebraic expression is

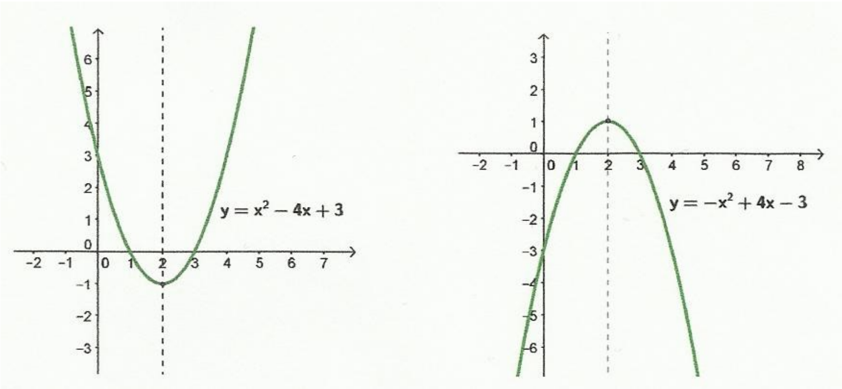
$$y=ax^2+bx+c \quad , \quad a\neq 0$$

It is called a **quadratic function** and its graph is called a **parabola**.

Examples: $y=x^2$, $y=x^2-1$, $y=-2x^2+3x-5$

The domain of a quadratic function is $\mathbb{R}$.

The parabola will open **upward** or **downward**.



A parabola will either an absolute minimum or an absolute maximum. This point is called the **vertex** of the parabola.

There is a line of symmetry which will divide the graphs into two halves. This line is called the **axis of symmetry** of the parabola.

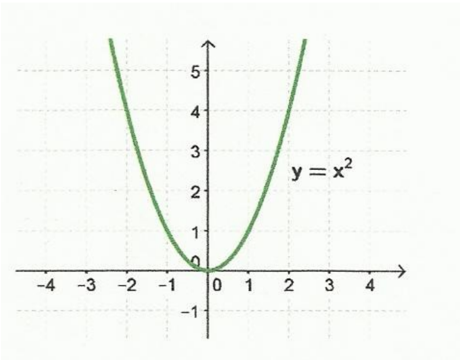
The parabola $y=x^2$:

The basic parabola is $y=x^2$.

The function is symmetrical about the x-axis.

Its vertex is the point (0,0), and it is also the absolute minimum.

The graph has two branches (one of them is decreasing and the other is increasing).



Graphs of the remaining quadratic functions are similar to the graph of $y=x^2$.

Functions of type $y=ax^2$:

Examples: Graph the functions:

$y=2x^2$ $y=3x^2$ $y=-x^2$

In these functions, the vertex of these parabolas is always (0,0) and their axis of symmetry is the y-axis. The greater $|a|$, the slimmer the parabola will be.

Functions of type $y=x^2+c$:

Examples: Graph the functions:

$y=x^2+2$ $y=x^2-3$ $y=x^2+1$

In this case, we translate *c* units up or down (it depends on the sign of *c*), but the shape is exactly the same.

Functions of type $y=(x\pm c)^2$:

Examples: Graph the functions:

$y=(x-3)^2$ $y=(x+2)^2$ $y=(x-1)^2$

If we have the function $y=(x-c)^2$ the function goes *c* units to the right, and with the function $y=(x+c)^2$ the function goes *c* units to the left.

Functions of type $y=ax^2+bx+c$:

A parabola $y=ax^2+bx+c$ can be represented calculating first the coordinates of its vertex: $V(x_V, y_V)$:

$x_V=\frac{-b}{a}$. The axis of symmetry of the parabola is the vertical line $x=x_V$.

Then, plot some points whose abscissa is close to the vertex of the parabola (on both sides of it).

Examples: Represent the graphs of the functions:

a) $y=x^2+2x-3$ b) $y=x^2-4x+3$

c) $y=-x^2+2x$

d) $y=x^2+x-2$

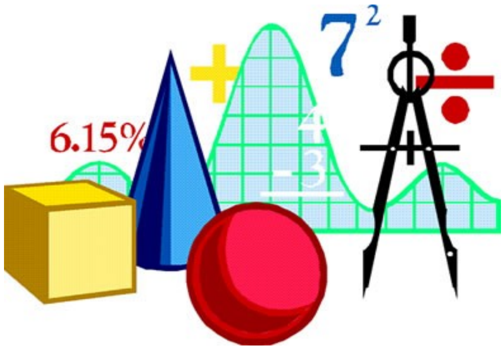
e) $y=-x^2+1$

f) $y=-x^2+3x+2$

The conclusions we can draw from these graphs are:

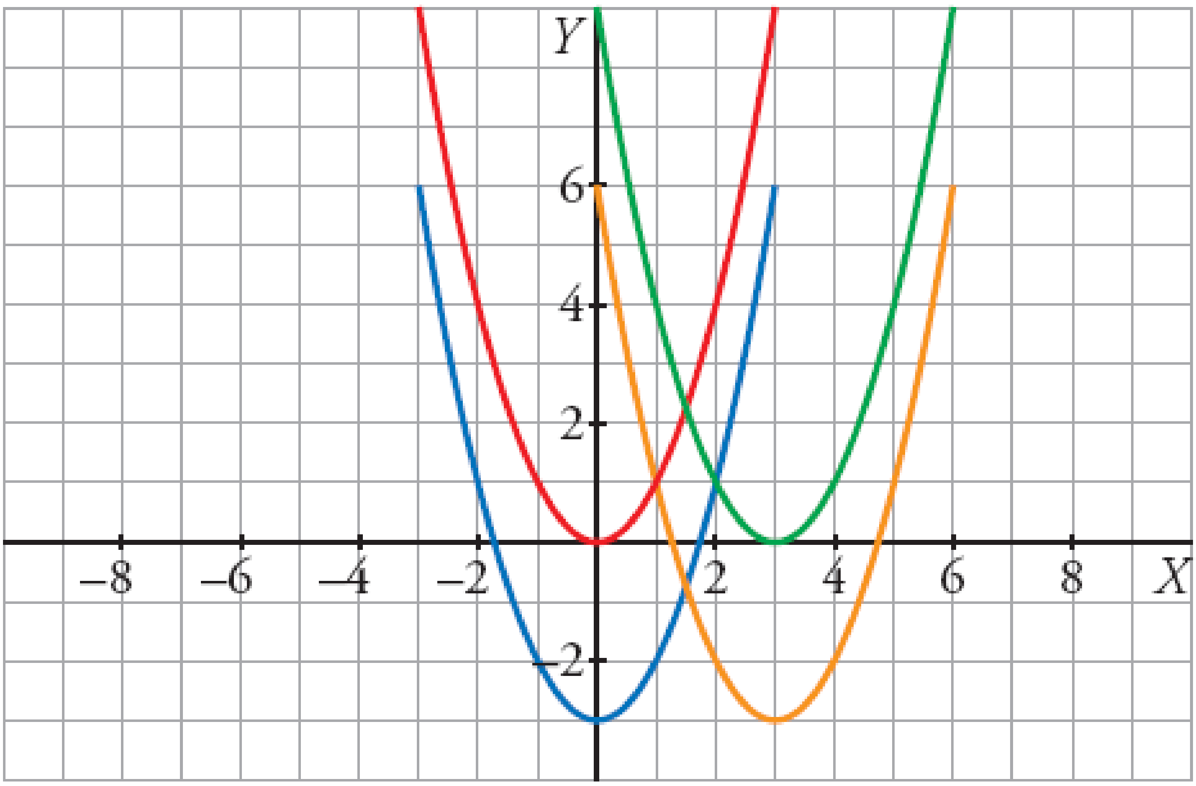
- The graph of the quadratic functions $y=ax^2+bx+c$ is a parabola.
- Quadratic functions are continuous in $\mathbb{R}$.
- The axis of symmetry of quadratic functions is parallel to the y-axis.
- If two quadratic functions have the same "*a*", the corresponding parabolas are equal, but they are placed in different positions.
- If *a*>0, the parabola opens upward (**concave** function).
  If *a*<0, the parabola opens downward (**convex** function).
- The greater $|a|$, the slimmer the parabola will be.

Your Turn



1. Match each parabola with its equation:

   a) $y=x^2$ b) $y=x^2-3$ c) $y=(x-3)^2$ d) $y=x^2-6x+6$



2. The parabola $y=ax^2+bx+c$ passes through (0,0). What is the value of c?
   Apart from that, the parabola passes through the points (1,3) and (4,6). Find the values of a and b and graph the function.

3. Calculate the axes intercepts of the parabola $y=x^2-5x+4$ and represent its graph.

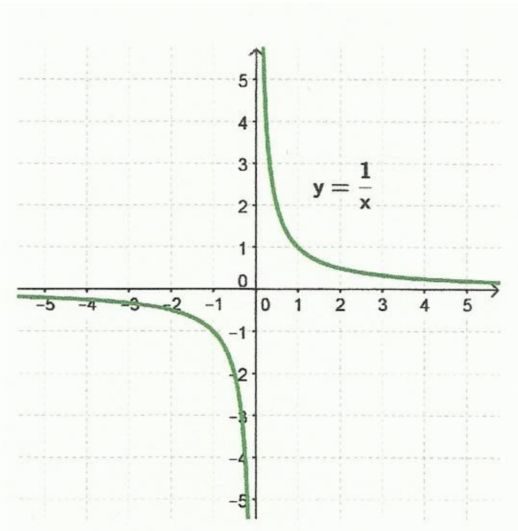
Inversely Proportional Functions:

If the variables x and y are inversely proportional, then the functional dependence between them is represented by the equation:

$$\boxed{y=\frac{k}{x}}$$, where *k* is a constant.

The function $y=\frac{1}{x}$:

Look at the graph of the function $y=\frac{1}{x}$:



The domain is $\mathbb{R}-\{0\}$.

Its graph is a hyperbola.

Its has two branches.

If we focus on the branch for x>0:

- As x increases, then y decreases to 0.
- As x drops to 0, then y increases to $+\infty$.

The x-axis and y-axis are **asymptotes** of the function.

NOTE: Asymptote is a line that a graph gets closer and closer to, but never touches or crosses it.

The function $y=\frac{k}{x}$:

The functions $y=\frac{k}{x}$ are called inversely proportional functions.

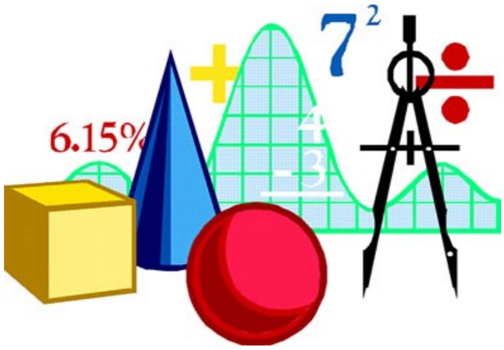
Examples: Represent the graph of the functions:

a) $y=\frac{5}{x}$ $y=\frac{-1}{x}$ $y=\frac{-4}{x}$

Their graphs are similar to the graph of $y=\frac{1}{x}$, that is:

- The domain of this functions is $\mathbb{R}-\{0\}$.
- They are hyperbolas whose asymptotes are the coordinates axes.
- If k>0, the function decreases.
  If k>0, the function increases.

Your Turn



1. Graph the following functions, and find the their domain, range and asymptotes:

a) $y=\frac{2}{x}$ b) $y=\frac{2}{x-3}$

c) $y=\frac{2}{x+3}$

d) $y=\frac{2}{x}+1$

e) $y=\frac{2}{x}-1$

f) $y=\frac{2}{x-3}+1$

2. Complete these sentences:

If $a>0$, the graph of the function $y=\frac{k}{x-a}$ is the graph of the function $y=\frac{k}{x}$ moved _________ units to the _________.

If $a>0$, the graph of the function $y=\frac{k}{x+a}$ is the graph of the function $y=\frac{k}{x}$ moved _________ units to the _________.

If $b>0$, the graph of the function $y=\frac{k}{x}-b$ is the graph of the function $y=\frac{k}{x}$ moved _________ _____ units.

If $b<0$, the graph of the function $y=\frac{k}{x}+b$ is the graph of the function $y=\frac{k}{x}$ moved _________ _____ units.

Keywords:

Linear function=función lineal (función polinómica de primer grado)
slope=pendiente
y-intercept= ordenada en el origen
constant function=función constante
directly proportional function= función de proporcionalidad directa
identity function= función identidad
parabola=parábola
quadratic function=función cuadrática
vertex=vértice
axes of symmetry=eje de simetría
concave function= función cóncava
convex function=función convexa
inversely proportional function=función de proporcionalidad inversa
hyperbola=hipérbola
asymptotes=asíntotas