



3. Ejercicios de refuerzo: operaciones combinadas con matrices cuadradas

Soluciones

1 Dadas las matrices:

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 0 \\ 0 & 2 & -2 \end{pmatrix} \quad B = \begin{pmatrix} -2 & 2 & 2 \\ 1 & 3 & -1 \\ -1 & 1 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

Efectúa:

a) $(A + B) \cdot C$

b) $(A \cdot C^{-1} + B \cdot C^{-1}) \cdot C$

c) $A \cdot C^{-1} \cdot B + C$

d) $(A \cdot B + B^2) \cdot C^{-1}$

e) $(B + A) \cdot C^{-1} + B \cdot C^{-1}$

Resolución

$$\text{a) } (A + B) \cdot C = \left[\begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 0 \\ 0 & 2 & -2 \end{pmatrix} + \begin{pmatrix} -2 & 2 & 2 \\ 1 & 3 & -1 \\ -1 & 1 & 1 \end{pmatrix} \right] \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 4 \\ 2 & 4 & -1 \\ -1 & 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 0 \\ 2 & -1 & -2 \\ -1 & -1 & -4 \end{pmatrix}$$

$$\text{b) } (A \cdot C^{-1} + B \cdot C^{-1}) \cdot C = (A \cdot C^{-1} \cdot C + B \cdot C^{-1} \cdot C) = A \cdot I + B \cdot I = A + B = \begin{pmatrix} 1 & 1 & 4 \\ 2 & 4 & -1 \\ -1 & 3 & -1 \end{pmatrix}$$

c) Hallamos C^{-1} :

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} (1.^{\circ}) \\ (3.^{\circ}) \\ (2.^{\circ}) \end{array} \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 \end{array} \right) \begin{array}{l} (1.^{\circ}) \\ (2.^{\circ}) \\ (-1) \cdot (3.^{\circ}) \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{array} \right) \rightarrow C^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\begin{aligned} A \cdot C^{-1} \cdot B + C &= \begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 0 \\ 0 & 2 & -2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} -2 & 2 & 2 \\ 1 & 3 & -1 \\ -1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \\ &= \begin{pmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} -2 & 2 & 2 \\ 1 & 3 & -1 \\ -1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \\ &= \begin{pmatrix} -6 & 10 & 6 \\ -2 & 6 & 2 \\ 2 & 5 & -2 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} -5 & 10 & 7 \\ -2 & 6 & 3 \\ 2 & 5 & -2 \end{pmatrix} \end{aligned}$$



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$$d) (A \cdot B + B^2) \cdot C^{-1} = (A + B) \cdot B \cdot C^{-1} = \begin{pmatrix} 1 & 1 & 4 \\ 2 & 4 & -1 \\ -1 & 3 & -1 \end{pmatrix} \begin{pmatrix} -2 & 2 & 2 \\ 1 & 3 & -1 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 1 & 4 \\ 2 & 4 & -1 \\ -1 & 3 & -1 \end{pmatrix} \begin{pmatrix} -2 & -4 & 2 \\ 1 & 2 & 3 \\ -1 & -2 & 1 \end{pmatrix} = \begin{pmatrix} -5 & -10 & 9 \\ 1 & 2 & 15 \\ 6 & 12 & 6 \end{pmatrix}$$

$$e) (B + A) \cdot C^{-1} + B \cdot C^{-1} = 2B \cdot C^{-1} + A \cdot C^{-1} = 2 \cdot \begin{pmatrix} -2 & -4 & 2 \\ 1 & 2 & 3 \\ -1 & -2 & 1 \end{pmatrix} + \begin{pmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 0 \end{pmatrix} = \begin{pmatrix} -1 & -7 & 5 \\ 3 & 5 & 7 \\ -2 & -2 & 2 \end{pmatrix}$$