

Problema 1 Calcular los siguientes límites:

1. $\lim_{x \rightarrow 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$
2. $\lim_{x \rightarrow 0} (\sin x)^x$
3. $\lim_{x \rightarrow \infty} (\sqrt{3x^2 - 1} - \sqrt{3x^2 - x + 2})$
4. $\lim_{x \rightarrow 0} \frac{x \sin x}{\cos x - 1}$

Solución:

1. $\lim_{x \rightarrow 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right) = \frac{1}{2}$
2. $\lim_{x \rightarrow 0} (\sin x)^x = 1$
3. $\lim_{x \rightarrow \infty} (\sqrt{3x^2 - 1} - \sqrt{3x^2 - x + 2}) = \frac{\sqrt{3}}{6}$
4. $\lim_{x \rightarrow 0} \frac{x \sin x}{\cos x - 1} = -2$

Problema 2 Calcular las siguientes integrales:

1. $\int \frac{x}{\sqrt{x^2 + 1}} dx$
2. $\int \frac{\cos x}{\sin x} dx$
3. $\int x^3 \ln x dx$
4. $\int e^x \sin 2x dx$
5. $\int \frac{x^3 - x + 3}{x^2 + x - 2} dx$

Solución:

1. $\int \frac{x}{\sqrt{x^2 + 1}} dx = \sqrt{x^2 + 1} + C$
2. $\int \frac{\cos x}{\sin x} dx = \ln(\sin x) + C$

$$3. \int x^3 \ln x \, dx = \frac{x^4(4 \ln x - 1)}{16} + C$$

$$4. \int e^x \sin 2x \, dx = e^x \left(\frac{\sin(2x)}{5} - \frac{2 \cos(2x)}{5} \right) + C$$

$$5. \int \frac{x^3 - x + 3}{x^2 + x - 2} \, dx = \frac{x^2}{2} - x + \ln(x^2 + x - 2) + C$$

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