Problema 1 Sea la matriz

$$A = \begin{pmatrix} 3 & m & 0 \\ m & 2 & 2 \\ -m & 3 & 4 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} 3 & m & 0 \\ m & 2 & 2 \\ -m & 3 & 4 \end{vmatrix} = -6(m^2 - 1) = 0 \Longrightarrow m = \pm 1$$

Si $m = 1$ o $m = -1 \Longrightarrow |A| = 0 \Longrightarrow$ no existe A^{-1}.

Si $m \neq 1$ y $m \neq -1 \Longrightarrow |A| \neq 0 \Longrightarrow$ existe A^{-1}.

2.

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 3 & 4 \end{pmatrix} \Longrightarrow A^{-1} = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -3/2 & 1 \end{pmatrix}$$

Problema 2 Resolver la ecuación matricial $AX - X + B = C$. Donde

$$A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 4 \\ -1 & 3 \end{pmatrix}; \quad C = \begin{pmatrix} 3 & 7 \\ 1 & 0 \end{pmatrix}$$

Solución:

$$AX - X + B = C \Longrightarrow X = (A - I)^{-1}(C - B)$$

$$A - I = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}, \quad (A - I)^{-1} = \begin{pmatrix} 1/2 & -1/2 \\ 0 & 1 \end{pmatrix}$$

$$C - B = \begin{pmatrix} 3 & 7 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 4 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 2 & -3 \end{pmatrix}$$

$$X = X = (A - I)^{-1}(C - B) = \begin{pmatrix} 1/2 & -1/2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ 2 & -3 \end{pmatrix}$$

Problema 3 Resolver, utilizando las propiedades de los determinantes, calcular:

$$\begin{vmatrix} 1 & x & 1 & x \\ x & 1 & x & 1 \\ 1 & 1 & x & x \\ x & x & 1 & 1 \end{vmatrix}$$

Solución:

$$\begin{vmatrix} 1 & x & 1 & x \\ x & 1 & x & 1 \\ 1 & 1 & x & x \\ x & x & 1 & 1 \end{vmatrix} = \begin{bmatrix} \mathrm{F}_1+\mathrm{F}_2+\mathrm{F}_3+\mathrm{F}_4 \\ \mathrm{F}_2 \\ \mathrm{F}_3 \\ \mathrm{F}_4 \end{bmatrix} = \begin{vmatrix} 2x+2 & 2x+2 & 2x+2 & 2x+2 \\ x & 1 & x & 1 \\ 1 & 1 & x & x \\ x & x & 1 & 1 \end{vmatrix} =$$

$$2(x{+}1)\begin{vmatrix} 1 & 1 & 1 & 1 \\ x & 1 & x & 1 \\ 1 & 1 & x & x \\ x & x & 1 & 1 \end{vmatrix} = \begin{bmatrix} \mathrm{C}_1 \\ \mathrm{C}_2-\mathrm{C}_1 \\ \mathrm{C}_3-\mathrm{C}_1 \\ \mathrm{C}_4-\mathrm{C}_1 \end{bmatrix} = 2(x{+}1)\begin{vmatrix} 1 & 0 & 0 & 0 \\ x & 1-x & 0 & 1-x \\ 1 & 0 & x-1 & x-1 \\ x & 0 & 1-x & 1-x \end{vmatrix} =$$

$$2(x+1)\begin{vmatrix} 1-x & 0 & 1-x \\ 0 & x-1 & x-1 \\ 0 & 1-x & 1-x \end{vmatrix} = -2(x-1)(x+1)\begin{vmatrix} x-1 & x-1 \\ 1-x & 1-x \end{vmatrix} = 0$$