

**Problema 1** Sea la matriz

$$A = \begin{pmatrix} 1 & -m & 2 \\ 2 & 2 & m \\ 3 & m & 3 \end{pmatrix}$$

1. Calcular los valores de  $m$  para los que la matriz  $A$  es inversible.
2. Calcular  $A^{-1}$  para  $m = 0$ .

**Solución:**

1.

$$\begin{vmatrix} 1 & -m & 2 \\ 2 & 2 & m \\ 3 & m & 3 \end{vmatrix} = -4m^2 + 10m - 6 = 0 \implies m = 1, \quad m = \frac{3}{2}$$

Si  $m = 1$  o  $m = \frac{3}{2} \implies |A| = 0 \implies$  no existe  $A^{-1}$ .

Si  $m \neq 1$  y  $m \neq \frac{3}{2} \implies |A| \neq 0 \implies$  existe  $A^{-1}$ .

2.

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 2 & 0 \\ 3 & 0 & 3 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} -1 & 0 & 2/3 \\ 1 & 1/2 & -2/3 \\ 1 & 0 & -1/3 \end{pmatrix}$$

**Problema 2** Resolver la ecuación matricial  $XA + XB = C + I$ . Donde

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}; \quad B = \begin{pmatrix} 3 & -1 \\ 1 & -1 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

**Solución:**

$$XA + XB = C + I \implies X(A + B) = C + I \implies X = (C + I)(A + B)^{-1}$$

$$(A + B)^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & -4 \end{pmatrix}, \quad C + I = \begin{pmatrix} 2 & 0 \\ 2 & 2 \end{pmatrix}$$

$$X = (C + I)(A + B)^{-1} = \begin{pmatrix} 0 & 1 \\ 2 & 6 \end{pmatrix}$$

**Problema 3** Resolver utilizando las propiedades de los determinantes:

$$\begin{vmatrix} x & 1 & 1 & 0 \\ 1 & x & 0 & 1 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix}$$

**Solución:**

$$\begin{vmatrix} x & 1 & 1 & 0 \\ 1 & x & 0 & 1 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} = \begin{bmatrix} F_1 + F_2 + F_3 + F_4 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{vmatrix} x+2 & x+2 & x+2 & x+2 \\ 1 & x & 0 & 1 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} =$$

$$(x+2) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 0 & 1 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} = \begin{bmatrix} F_1 \\ F_2 - F_1 \\ F_3 - F_1 \\ F_4 \end{bmatrix} = (x+2) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & x-1 & -1 & 0 \\ 0 & -1 & x-1 & 0 \\ 0 & 1 & 1 & x \end{vmatrix} =$$

$$(x+2) \begin{vmatrix} x-1 & -1 & 0 \\ -1 & x-1 & 0 \\ 1 & 1 & x \end{vmatrix} = x(x+2) \begin{vmatrix} x-1 & -1 \\ -1 & x-1 \end{vmatrix} = x^2(x^2 - 4)$$