

Examen de Matemáticas 2º Bachillerato (CN)

Problema 1 Sea la matriz

$$A = \begin{pmatrix} m & 2 & -1 \\ m & 3 & 2 \\ 1 & m & -4 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} m & 2 & -1 \\ m & 3 & 2 \\ 1 & m & -4 \end{vmatrix} = -(3m^2 + 4m - 7) = 0 \implies m = 1, m = -\frac{7}{3}$$

Si $m = 1$ o $m = -7/3 \implies |A| = 0 \implies$ no existe A^{-1} .

Si $m \neq 1$ y $m \neq -7/3 \implies |A| \neq 0 \implies$ existe A^{-1} .

2.

$$A = \begin{pmatrix} 0 & 2 & -1 \\ 0 & 3 & 2 \\ 1 & 0 & -4 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} -12/7 & 8/7 & 1 \\ 2/7 & 1/7 & 1 \\ -3/7 & 2/7 & 0 \end{pmatrix}$$

Problema 2 Resolver la ecuación matricial $AX - B = -C + X$. Donde

$$A = \begin{pmatrix} 5 & 1 \\ 2 & 0 \end{pmatrix}; B = \begin{pmatrix} -1 & 3 \\ 2 & -5 \end{pmatrix}; C = \begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix}$$

Solución:

$$AX - B = -C + X \implies X = (A - I)^{-1}(B - C)$$

$$A - I = \begin{pmatrix} 5 & 1 \\ 2 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & -1 \end{pmatrix}, (A - I)^{-1} = \begin{pmatrix} 1/6 & 1/6 \\ 1/3 & -2/3 \end{pmatrix}$$

$$B - C = \begin{pmatrix} -1 & 3 \\ 2 & -5 \end{pmatrix} - \begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -4 & 4 \\ 1 & -5 \end{pmatrix}$$

$$X = (A - I)^{-1}(B - C) = \begin{pmatrix} 1/6 & 1/6 \\ 1/3 & -2/3 \end{pmatrix} \begin{pmatrix} -4 & 4 \\ 1 & -5 \end{pmatrix} = \begin{pmatrix} -1/2 & -1/6 \\ -2 & 14/3 \end{pmatrix}$$

Problema 3 Resolver utilizando las propiedades de los determinantes:

$$\begin{vmatrix} x & 1 & 0 & 1 \\ 1 & x & 1 & 0 \\ 0 & 1 & x & 1 \\ 1 & 0 & 1 & x \end{vmatrix}$$

Solución:

$$\begin{vmatrix} x & 1 & 0 & 1 \\ 1 & x & 1 & 0 \\ 0 & 1 & x & 1 \\ 1 & 0 & 1 & x \end{vmatrix} = \begin{bmatrix} F_1 + F_2 + F_3 + F_4 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{vmatrix} x+2 & x+2 & x+2 & x+2 \\ 1 & x & 1 & 0 \\ 0 & 1 & x & 1 \\ 1 & 0 & 1 & x \end{vmatrix} =$$

$$(x+2) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 1 & 0 \\ 0 & 1 & x & 1 \\ 1 & 0 & 1 & x \end{vmatrix} = \begin{bmatrix} F_1 \\ F_2 - F_1 \\ F_3 \\ F_4 - F_1 \end{bmatrix} = (x+2) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & x-1 & 0 & -1 \\ 0 & 1 & x & 1 \\ 0 & -1 & 0 & x-1 \end{vmatrix} =$$

$$(x+2) \begin{vmatrix} x-1 & 0 & -1 \\ 1 & x & 1 \\ -1 & 0 & x-1 \end{vmatrix} = x(x+2) \begin{vmatrix} x-1 & -1 \\ -1 & x-1 \end{vmatrix} = x^2(x^2 - 4)$$