Problema 1 Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty} \dfrac{5x^2+x-1}{x^3+2}$

2. $\lim\limits_{x\longrightarrow\infty} \dfrac{3x^3+x+3}{5x^3-x^2+2}$

3. $\lim\limits_{x\longrightarrow\infty} \dfrac{\sqrt{4x^2-1}}{3x+2}$

4. $\lim\limits_{x\longrightarrow\infty} \dfrac{\sqrt{4x^2-1}}{-x^2+1}$

5. $\lim\limits_{x\longrightarrow 1} \dfrac{2x^7+3x^6-6x^4+1}{x^5+4x^4-7x^3+x+1}$

6. $\lim\limits_{x\longrightarrow 2} \dfrac{x^4-x^3+2x-12}{2x^3-3x^2-x-2}$

7. $\lim\limits_{x\longrightarrow 4} \dfrac{\sqrt{2x^2+3}-\sqrt{8x+3}}{x-4}$

8. $\lim\limits_{x\longrightarrow 5} \dfrac{\sqrt{2x^2-1}-\sqrt{8x+9}}{x-5}$

Solución:

1. $\lim\limits_{x\longrightarrow\infty} \dfrac{5x^2+x-1}{x^3+2} = 0$

2. $\lim\limits_{x\longrightarrow\infty} \dfrac{3x^3+x+3}{5x^3-x^2+2} = \dfrac{3}{5}$

3. $\lim\limits_{x\longrightarrow\infty} \dfrac{\sqrt{4x^2-1}}{3x+2} = \dfrac{2}{3}$

4. $\lim\limits_{x\longrightarrow\infty} \dfrac{\sqrt{4x^2-1}}{-x^2+1} = -\infty$

5. $\lim\limits_{x\longrightarrow 1} \dfrac{2x^7+3x^6-6x^4+1}{x^5+4x^4-7x^3+x+1} = 8$

6. $\lim\limits_{x\longrightarrow 2} \dfrac{x^4-x^3+2x-12}{2x^3-3x^2-x-2} = 2$

7. $\lim_{x \longrightarrow 4} \dfrac{\sqrt{2x^2+3} - \sqrt{8x+3}}{x-4} = \dfrac{4\sqrt{35}}{35}$

8. $\lim_{x \longrightarrow 5} \dfrac{\sqrt{2x^2-1} - \sqrt{8x+9}}{x-5} = \dfrac{6}{7}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{3x^3+2x^2-x+1}$
2. $y = \ln(4x^3+5)$
3. $y = (3x^2-2)^{10}$
4. $y = (x^2+x-2)(3x^3-x^2+1)$
5. $y = \frac{4x^2+1}{3x-2}$
6. $y = x^2e^{2x}$

Solución:

1. $y = e^{3x^3+2x^2-x+1} \Longrightarrow y' = (9x^2+4x-1)e^{3x^3+2x^2-x+1}$
2. $y = \ln(4x^3+5) \Longrightarrow y' = \dfrac{12x^2}{4x^3+5}$
3. $y = (3x^2-2)^{10} \Longrightarrow y' = 10(3x^2-2)^9(6x)$
4. $y = (x^2+x-2)(3x^3-x^2+1) \Longrightarrow y' = (2x+1)(3x^3-x^2+1) + (x^2+x-2)(9x^2-2x)$
5. $y = \dfrac{4x^2+1}{3x-2} \Longrightarrow y' = \dfrac{8x(3x-2)-(4x^2+1)3}{(3x-2)^2}$
6. $y = x^2e^{2x} \Longrightarrow y' = 2xe^{2x} + 2x^2e^{2x}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{2x^2-x+1}{x^2+2}$ en el punto $x = 0$.
2. $f(x) = \dfrac{4x^2-1}{x+1}$ en el punto $x = 1$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = \frac{1}{2}$ e $y - b = m(x - a)$

$$f'(x) = \frac{x^2 + 6x - 2}{(x^2 + 2)^2} \Longrightarrow m = f'(1) = -\frac{1}{2}$$

   Recta Tangente: $y - \frac{1}{2} = -\frac{1}{2}(x - 1)$

   Recta Normal: $y - \frac{1}{2} = 2(x - 1)$

2. $b = f(a) \Longrightarrow b = f(0) = \frac{3}{2}$ e $y - b = m(x - a)$

$$f'(x) = \frac{4x^2 + 8x + 1}{(x + 1)^2} \Longrightarrow m = f'(1) = \frac{13}{2}$$

   Recta Tangente: $y - \frac{3}{2} = \frac{13}{2}x$

   Recta Normal: $y - \frac{3}{2} = -\frac{2}{13}x$