Problema 1 Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty}\dfrac{5x^3+2x^2-3x+1}{2x^5+3x^2-2}$

2. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{5x^2-x+1}{x^2+x-2}\right)^{x^2-1}$

3. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{x^2+x+2}{x^2-1}\right)^{2x-1}$

4. $\lim\limits_{x\longrightarrow\infty}\dfrac{\sqrt{5x^2+3}}{-x^2+4}$

5. $\lim\limits_{x\longrightarrow 1}\dfrac{6x^5+2x^4-7x^3+x^2-x-1}{4x^5+7x^4-12x^3+2x-1}$

6. $\lim\limits_{x\longrightarrow 2}\dfrac{2x^3+x^2-12x+4}{x^3+5x^2-15x+2}$

7. $\lim\limits_{x\longrightarrow 3}\dfrac{\sqrt{4x^2+1}-\sqrt{10x+7}}{x-3}$

8. $\lim\limits_{x\longrightarrow 5}\dfrac{\sqrt{2x^2-3}-\sqrt{9x+2}}{x-5}$

Solución:

1. $\lim\limits_{x\longrightarrow\infty}\dfrac{5x^3+2x^2-3x+1}{2x^5+3x^2-2}=0$

2. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{5x^2-x+1}{x^2+x-2}\right)^{x^2-1}=\infty$

3. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{x^2+x+2}{x^2-1}\right)^{2x-1}=e^2$

4. $\lim\limits_{x\longrightarrow\infty}\dfrac{\sqrt{5x^2+3}}{-x^2+4}=0$

5. $\lim\limits_{x\longrightarrow 1}\dfrac{6x^5+2x^4-7x^3+x^2-x-1}{4x^5+7x^4-12x^3+2x-1}=\dfrac{9}{7}$

6. $\lim\limits_{x\longrightarrow 2}\dfrac{2x^3+x^2-12x+4}{x^3+5x^2-15x+2}=\dfrac{16}{17}$

7. $\lim_{x\longrightarrow 3} \dfrac{\sqrt{4x^2+1}-\sqrt{10x+7}}{x-3} = \dfrac{7\sqrt{37}}{37}$

8. $\lim_{x\longrightarrow 5} \dfrac{\sqrt{2x^2-3}-\sqrt{9x+2}}{x-5} = \dfrac{11\sqrt{47}}{94}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{2x^3-4x^2+x-1}$
2. $y = \ln(3x^4+1)$
3. $y = (5x^2+x-3)^{15}$
4. $y = (2x^2-3x+1)(x^3+5x^2-2)$
5. $y = \frac{3x^2-2x+1}{5x-8}$
6. $y = \ln\frac{x^2-5}{x+3}$

Solución:

1. $y = e^{2x^3-4x^2+x-1} \Longrightarrow y' = (6x^2-8x+1)e^{2x^3-4x^2+x-1}$
2. $y = \ln(3x^4+1) \Longrightarrow y' = \dfrac{12x^3}{3x^4+1}$
3. $y = (5x^2+x-3)^{15} \Longrightarrow y' = 15(5x^2+x-3)^{14}(10x+1)$
4. $y = (2x^2-3x+1)(x^3+5x^2-2) \Longrightarrow y' = (4x-3)(x^3+5x^2-2) + (2x^2-3x+1)(3x^2+10x)$
5. $y = \dfrac{3x^2-2x+1}{5x-8} \Longrightarrow y' = \dfrac{(6x-2)(5x-8)-(3x^2-2x+1)5}{(5x-8)^2}$
6. $y = \ln\dfrac{x^2-5}{x+3} = \ln(x^2-5)-\ln(x+3) \Longrightarrow y' = \dfrac{2x}{x^2-5} - \dfrac{1}{x+3}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{5x^2-1}{x^2+2}$ en el punto $x = 1$.
2. $f(x) = \dfrac{x^2+3}{2x-1}$ en el punto $x = 0$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = \frac{4}{3}$ e $y - b = m(x - a)$

$$f'(x) = -\frac{22x}{(x^2+2)^2} \Longrightarrow m = f'(1) = \frac{22}{9}$$

Recta Tangente: $y - \frac{4}{3} = \frac{22}{9}(x - 1)$

Recta Normal: $y - \frac{4}{3} = -\frac{9}{22}(x - 1)$

2. $b = f(a) \Longrightarrow b = f(0) = -3$ e $y - b = m(x - a)$

$$f'(x) = \frac{2(x^2 - x - 3)}{(2x-1)^2} \Longrightarrow m = f'(0) = -6$$

Recta Tangente: $y + 3 = -6x$

Recta Normal: $y + 3 = \frac{1}{6}x$