Función derivada 1

Nivel: bachillerato

Calcula la 1ª y 2ª derivada, simplificando el resultado, de

a) $f(x)=x^3-2x^2+x+1$

b) $f(x)=2x^3-8x+1$

c) $f(x)=\left(x+1\right)^2\left(x-2\right)$

d) $f(x)=\dfrac{x^3}{\left(1+x\right)^2}$

e) $f(x)=\dfrac{x^2}{1+x}$

f) $f(x)=\dfrac{1}{x^2+x-2}$

g) $f(x)=\dfrac{x^2+1}{x^2-1}$

h) $f(x)=\dfrac{2}{x^2-1}$

i) $f(x)=\dfrac{x^2}{\left(x-1\right)^2}$

j) $f(x)=\dfrac{x+3}{x-2}$

k) $f(x)=\dfrac{x^2-1}{x^2+1}$

l) $f(x)=\dfrac{16}{x^2\left(x-1\right)}$

m) $f(x)=\dfrac{x}{x^2-1}$

n) $f(x)=\dfrac{x^2}{x-1}$

o) $f(x)=\dfrac{x^2-1}{x}$

p) $f(x)=\dfrac{x^3}{x^2-1}$

q) $f(x)=\dfrac{x^2-6x+5}{x-3}$

r) $f(x)=x^4-6x^2$

s) $f(x)=-x^3+6x^2-9x+8$

t) $f(x)=\left(x^5-3x^2\right)^3\cdot\left(2-5x^3\right)^4\cdot\left(4x+7x^3\right)^6$

RECUERDA

$y = x^n \to y' = n \cdot x^{n-1}$

$y = k \cdot f \to y' = k \cdot f'$

$y = f \pm g \to y' = f' \pm g'$

$y = f \cdot g \to y' = f' \cdot g + f \cdot g'$

$y = \dfrac{f}{g} \to y' = \dfrac{f' \cdot g - f \cdot g'}{g^2}$

$y = f^n \to y' = n \cdot f^{n-1} \cdot f'$.

1º

a) $f(x) = x^3 - 2x^2 + x + 1$.

$f'(x) = 3x^2 - 2 \cdot 2x + 1 = 3x^2 - 4x + 1$

$f''(x) = 3 \cdot 2x - 4 = 6x - 4$

b) $f(x) = 2x^3 - 8x + 1$

$f'(x) = 2 \cdot 3x^2 - 8 = 6x^2 - 8$.

$f''(x) = 6 \cdot 2x = 12x$.

c) COMO PRODUCTO.

$y = (x+1)^2 \cdot (x-2)$

$\left.\begin{array}{l} f = (x+1)^2 \to f' = 2 \cdot (x+1) \\ g = (x-2) \to g' = 1. \end{array}\right\} \Rightarrow$

$y' = 2 \cdot (x+1) \cdot (x-2) + (x+1)^2 \cdot 1$.

para hacer la 2ª derivada lo más sencillo es operar.

$y' = 2(x^2 - 2x + x - 2) + x^2 + 2x + 1 = 3x^2 - 3$

$y'' = 3 \cdot 2x = 6x$.

DESARROLLANDO.

$y = (x^2 + 2x + 1) \cdot (x-2) = x^3 - 2x^2 + 2x^2 - 4x + x - 2 = x^3 - 3x - 2$

$y' = 3x^2 - 3$

$y'' = 3 \cdot 2x = 6x$.

(d) $y = \dfrac{x^3}{(1+x)^2}$

Recuerda $y = \dfrac{f}{g} \rightarrow y' = \dfrac{f' \cdot g - f \cdot g'}{g^2}$ $\quad y = f^n \rightarrow y' = n \cdot f^{n-1} \cdot f'$

En nuestro caso:

$f = x^3 \longrightarrow f' = 3x^2$

$g = (1+x)^2 \longrightarrow g' = 2(1+x)^1 \cdot 1. \quad \Rightarrow$

$\boxed{y'}$

$$y' = \frac{3x^2 \cdot (1+x)^2 - x^3 \cdot 2 \cdot (1+x)}{[(1+x)^2]^2} = \frac{3x^2 \cdot (1+x)^2 - 2x^3(1+x)}{(1+x)^4}$$

$$= \frac{x^2 \cdot (1+x) \cdot [3(1+x) - 2x]}{(1+x)^4} = \frac{x^2 \cdot [3 + 3x - 2x]}{(1+x)^3}$$

$$\boxed{y' = \frac{x^2 \cdot (3+x)}{(1+x)^3} = \frac{3x^2 + x^3}{(1+x)^3}} \quad (*)$$

$\boxed{y''}$

$f = 3x^2 + x^3 \longrightarrow f' = 6x + 3x^2$

$g = (1+x)^3 \longrightarrow g' = 3 \cdot (1+x)^2 \cdot 1.$

$$y'' = \frac{(6x+3x^2) \cdot (1+x)^3 - (3x^2 + x^3) \cdot 3 \cdot (1+x)^2}{[(1+x)^3]^2}$$

$$= \frac{(1+x)^2 \cdot [(6x+3x^2) \cdot (1+x) - 3 \cdot (3x^2+x^3)]}{(1+x)^6} =$$

$$= \frac{6x + 6x^2 + 3x^2 + 3x^3 - 9x^2 - 3x^3}{(1+x)^4} \Rightarrow \boxed{y'' = \frac{6x}{(1+x)^4}}$$

(*) se han efectuado las operaciones del numerador porque es MÁS sencillo derivar una suma que un producto.

(e) Recuerda

$$y=\frac{f}{g} \longrightarrow y'=\frac{f'\cdot g-f\cdot g'}{g^2}$$

$$y=\frac{x^2}{1+x}$$

$$\left.\begin{array}{l} f=x^2 \longrightarrow f'=2x \\ g=1+x \longrightarrow g'=1 \end{array}\right\}$$

$\boxed{y'}$

$$\boxed{y'}=\frac{2x\cdot(1+x)-x^2\cdot 1}{(1+x)^2}=\frac{2x+2x^2-x^2}{(1+x)^2}=\boxed{\frac{2x-x^2}{(1+x)^2}}$$

$\boxed{y''}$

$$\left.\begin{array}{l} f=2x-x^2 \longrightarrow f'=2-2x \\ g=(1+x)^2 \longrightarrow g'=2\cdot(1+x)^1\cdot 1 \end{array}\right\}$$

$$y''=\frac{(2-2x)\cdot(1+x)^2-(2x-x^2)\cdot 2\cdot(1+x)}{[(1+x)^2]^2}=$$

$$=\frac{(1+x)\cdot[(2-2x)\cdot(1+x)-2\cdot(2x-x^2)]}{(1+x)^4}$$

$$=\frac{2+2x-2x-2x^2-4x+2x^2}{(1+x)^3} \Rightarrow$$

$$\boxed{y''=\frac{2}{(1+x)^3}}$$

(f) $y = \dfrac{1}{x^2+x-2}$

Recuerda $y = \dfrac{1}{f} \rightarrow y' = \dfrac{-f'}{f^2}$

En nuestro caso

$f = x^2+x-2 \longrightarrow f' = 2x+1$

$$\Rightarrow \boxed{y' = \frac{-(2x+1)}{(x^2+x-2)^2}}$$

$y = \dfrac{f}{g} \rightarrow y' = \dfrac{f' \cdot g - f \cdot g'}{g^2}$

$f = -(2x+1) \longrightarrow f' = -2$

$g = (x^2+x-2)^2 \longrightarrow g' = 2 \cdot (x^2+x-2) \cdot (2x+1)$

$$y'' = \frac{-2 \cdot (x^2+x-2)^2 - [-(2x+1)] \cdot 2 \cdot (x^2+x-2) \cdot (2x+1)}{[(x^2+x-2)^2]^2}$$

$$= \frac{(x^2+x-2) \cdot [-2(x^2+x-2) + 2 \cdot (2x+1)^2]}{(x^2+x-2)^4} =$$

$$= \frac{-2x^2-2x+4+8x^2+8x+2}{(x^2+x-2)^3}$$

$$\Rightarrow \boxed{y'' = \frac{6x^2+6x+6}{(x^2+x-2)^3}}$$

(g) $y = \frac{f}{g} \rightarrow y' = \frac{f' \cdot g - f \cdot g'}{g^2}$.

$y = \frac{x^2+1}{x^2-1}$

$\boxed{y'}$

$$\left.\begin{array}{l} f = x^2+1 \longrightarrow f' = 2x \\ g = x^2-1 \longrightarrow g' = 2x \end{array}\right\}$$

$$y' = \frac{2x \cdot (x^2-1) - (x^2+1) \cdot 2x}{(x^2-1)^2} = \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2-1)^2}$$

$$\Rightarrow \boxed{y' = \frac{-4x}{(x^2-1)^2}}$$

$\boxed{y''}$

$$\left.\begin{array}{l} f = -4x \longrightarrow f' = -4 \\ g = (x^2-1)^2 \longrightarrow g' = 2 \cdot (x^2-1)^1 \cdot 2x \end{array}\right\}$$

$$y'' = \frac{-4 \cdot (x^2-1)^2 - (-4x) \cdot 2 \cdot (x^2-1) \cdot 2x}{[(x^2-1)^2]^2} =$$

$$= \frac{(x^2-1) \cdot [-4 \cdot (x^2-1) + 16x^2]}{(x^2-1)^4} = \frac{-4x^2 + 4 + 16x^2}{(x^2-1)^3}$$

$$\Rightarrow \boxed{y'' = \frac{12x^2+4}{(x^2-1)^3}}$$

h) $y = \frac{k}{f} \rightarrow y' = \frac{-kf'}{f^2}$

$y = \frac{2}{x^2-1}$

$f = x^2 - 1 \rightarrow f' = 2x \Rightarrow \boxed{y' = \frac{-2 \cdot 2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}}$

Fórmula : $y = \frac{f}{g} \rightarrow y' = \frac{f' \cdot g - f \cdot g'}{g^2}$

$$\left.\begin{array}{l} f = -4x \longrightarrow f' = -4 \\ g = (x^2-1)^2 \longrightarrow g' = 2 \cdot (x^2-1)^1 \cdot 2x . \end{array}\right\}$$

$$y'' = \frac{-4 \cdot (x^2-1)^2 - (-4x) \cdot 2 \cdot (x^2-1) \cdot 2x}{[(x^2-1)^2]^2} =$$

$$= \frac{(x^2-1) \cdot [-4 \cdot (x^2-1) + 16x^2]}{(x^2-1)^4}$$

$$= \frac{-4x^2 + 4 + 16x^2}{(x^2-1)^3}$$

$$\Rightarrow \boxed{y'' = \frac{12x^2+4}{(x^2-1)^3}}$$

(i) $y = \dfrac{x^2}{(x-1)^2}$

$$y = \frac{f}{g} \rightarrow y' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$\boxed{y'}$

$$\left.\begin{array}{l} f = x^2 \longrightarrow f' = 2x \\ g = (x-1)^2 \longrightarrow g' = 2 \cdot (x-1)^1 \cdot 1 \end{array}\right\}$$

$$y' = \frac{2x \cdot (x-1)^2 - x^2 \cdot 2 \cdot (x-1)}{[(x-1)^2]^2} = \frac{(x-1) \cdot [2x \cdot (x-1) - 2x^2]}{(x-1)^4}$$

$$y' = \frac{2x^2 - 2x - 2x^2}{(x-1)^3} \Rightarrow \boxed{y' = \frac{-2x}{(x-1)^3}}$$

$\boxed{y''}$

$$\left.\begin{array}{l} f = -2x \longrightarrow f' = -2 \\ g = (x-1)^3 \longrightarrow g' = 3 \cdot (x-1)^2 \cdot 1 \end{array}\right\}$$

$$y'' = \frac{-2 \cdot (x-1)^3 - (-2x) \cdot 3 \cdot (x-1)^2}{[(x-1)^3]^2}$$

$$= \frac{(x-1)^2 \cdot [-2 \cdot (x-1) + 6x]}{(x-1)^6} = \frac{-2x + 2 + 6x}{(x-1)^4}$$

$$\Rightarrow \boxed{y'' = \frac{4x+2}{(x-1)^4}}$$

j) Recuerda

$$y=\frac{f}{g}\longrightarrow y'=\frac{f'\cdot g-f\cdot g'}{g^2}$$

$$y=\frac{x+3}{x-2}$$

$$\left.\begin{array}{l} f=x+3\longrightarrow f'=1 \\ g=x-2\longrightarrow g'=1 \end{array}\right\}$$

$$y'=\frac{1\cdot(x-2)-(x+3)\cdot 1}{(x-2)^2}=\frac{x-2-x-3}{(x-2)^2}\Rightarrow \boxed{y'=\frac{-5}{(x-2)^2}}$$

$$y=\frac{k}{f}\rightarrow y'=\frac{-kf'}{f^2}$$

$$\left. f=(x-2)^2\longrightarrow f'=2\cdot(x-2)^1\cdot 1.\ \right\}\Rightarrow$$

$$\boxed{y''=\frac{-(-5)\cdot 2\cdot(x-1)}{\left[(x-1)^2\right]^2}=\frac{10}{(x-1)^3}}$$

Nota: la derivada de $y'=\frac{-5}{(x-2)^2}$ se podría haber trabajado como una potencia.

$$y'=-5\cdot(x-2)^{-2}\rightarrow y''=-5\cdot-2\cdot(x-2)^{-3}=10\cdot(x-2)^{-3}=\frac{10}{(x-2)^3}$$

(k) Recuerda : $y = \frac{f}{g} \rightarrow y' = \frac{f' \cdot g - f \cdot g'}{g^2}$

$$y = \frac{x^2-1}{x^2+1}$$

$$\left.\begin{array}{l} f = x^2-1 \longrightarrow f' = 2x \\ g = x^2+1 \longrightarrow g' = 2x \end{array}\right\}$$

$$y' = \frac{2x \cdot (x^2+1) - (x^2-1) \cdot 2x}{(x^2+1)^2} = \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}$$

$$\Rightarrow \boxed{y' = \frac{4x}{(x^2+1)^2}}$$

$$\left.\begin{array}{l} f = 4x \longrightarrow f' = 4 \\ g = (x^2+1)^2 \longrightarrow g' = 2 \cdot (x^2+1)^1 \cdot 2x \end{array}\right\}$$

$$y'' = \frac{4 \cdot (x^2+1)^2 - 4x \cdot 2 \cdot (x^2+1) \cdot 2x}{[(x^2+1)^2]^2}$$

$$= \frac{(x^2+1) \cdot [4 \cdot (x^2+1) - 16x^2]}{(x^2+1)^4}$$

$$= \frac{4x^2+4-16x^2}{(x^2+1)^3}$$

$$\Rightarrow \boxed{y'' = \frac{4-12x^2}{(x^2+1)^3}}$$

ⓔ Recuerda $y=\frac{f}{g} \longrightarrow y'=\frac{f'\cdot g-f\cdot g'}{g^2}$

$y=\frac{k}{f} \longrightarrow y'=\frac{-kf'}{f^2}$.

Preparamos la función

$$y=\frac{16}{x^2\cdot(x-1)}=\frac{16}{x^3-x}$$

Es más sencillo derivar la suma (x^3-x) que el producto $x^2\cdot(x-1)$.

$$f=x^3-x \longrightarrow f'=3x^2-1$$

$$y'=\frac{-16\cdot(3x^2-1)}{(x^3-x)^2}=\frac{-48x^2+16}{(x^3-x)^2}.$$

$$\left.\begin{array}{l} f=-48x^2+16 \longrightarrow f'=-96x \\ g=(x^3-x)^2 \longrightarrow g'=2\cdot(x^3-x)^1\cdot(3x^2-1). \end{array}\right\}$$

$$y''=\frac{-96x\cdot(x^3-x)^2-(-48x^2+16)\cdot 2\cdot(x^3-x)\cdot(3x^2-1)}{\left[(x^3-x)^2\right]^2}$$

$$=\frac{(x^3-x)\cdot\left[-96x\cdot(x^3-x)-2\cdot(-48x^2+16)\cdot(3x^2-1)\right]}{(x^3-x)^4}$$

$$=\frac{-96x^4+96x^2+288x^4-96x^2-96x^2+32}{(x^3-x)^3}$$

$$\boxed{y''=\frac{292x^4-96x^2+32}{(x^3-x)^3}}$$

m Recuerda $y=\frac{f}{g} \rightarrow y'=\frac{f'\cdot g-f\cdot g'}{g^2}$.

$$y=\frac{x}{x^2-1}$$

$$\left.\begin{array}{l} f=x \longrightarrow f'=1 \\ g=x^2-1 \longrightarrow g'=2x \end{array}\right\}$$

$$y'=\frac{1\cdot(x^2-1)-x\cdot 2x}{(x^2-1)^2}=\frac{x^2-1-2x^2}{(x^2-1)^2} \Rightarrow \boxed{y'=\frac{-1-x^2}{(x^2-1)^2}}$$

$$\left.\begin{array}{l} f=-1-x^2 \longrightarrow f'=-2x \\ g=(x^2-1)^2 \longrightarrow g'=2\cdot(x^2-1)^1\cdot 2x \end{array}\right\}$$

$$y''=\frac{-2x\cdot(x^2-1)^2-(-1-x^2)\cdot 2\cdot(x^2-1)\cdot 2x}{\left[(x^2-1)^2\right]^2}=$$

$$=\frac{(x^2-1)\cdot\left[-2x\cdot(x^2-1)+4x\cdot(1+x^2)\right]}{(x^2-1)^4}$$

$$=\frac{-2x^3+2x+4x+8x^3}{(x^2-1)^3} \Rightarrow$$

$$\boxed{y''=\frac{6x^3+6x}{(x^2-1)^3}=\frac{6x\cdot(x^2+1)}{(x^2-1)^3}}$$

n) Recuerda $y = \frac{f}{g} \rightarrow y' = \frac{f' \cdot g - f \cdot g'}{g^2}$

$$y = \frac{x^2}{x-1}$$

$$\left.\begin{aligned} f = x^2 &\longrightarrow f' = 2x \\ g = x-1 &\longrightarrow g' = 1 \end{aligned}\right\}$$

$$y' = \frac{2x \cdot (x-1) - x^2 \cdot 1}{(x-1)^2} = \frac{2x^2 - 2x - x^2}{(x-1)^2} \Rightarrow$$

$$\boxed{y' = \frac{x^2 - 2x}{(x-1)^2} = \frac{x \cdot (x-2)}{(x-1)^2}}$$

$$\left.\begin{aligned} f = x^2 - 2x &\longrightarrow f' = 2x - 2 \\ g = (x-1)^2 &\longrightarrow g' = 2 \cdot (x-1) \end{aligned}\right\}$$

$$y'' = \frac{(2x-2) \cdot (x-1)^2 - (x^2 - 2x) \cdot 2 \cdot (x-1)}{[(x-1)^2]^2}$$

$$= \frac{(x-1) \cdot [(2x-2) \cdot (x-1) - 2(x^2 - 2x)]}{(x-1)^4}$$

$$= \frac{2x^2 - 4x + 2 - 2x^2 + 4x}{(x-1)^3} \Rightarrow$$

$$\boxed{y'' = \frac{2}{(x-1)^3}}$$

① $y=\dfrac{x^2-1}{x}=\dfrac{x^2}{x}-\dfrac{1}{x}=x-\dfrac{1}{x}$

$$\boxed{y'=1-\frac{-1}{x^2}=1+\frac{1}{x^2}=\frac{x^2+1}{x^2}}$$

como cociente:

$$\left.\begin{array}{l} f=x^2-1 \longrightarrow f'=2x \\ g=x \longrightarrow g'=1 \end{array}\right\}$$

$$y'=\frac{2x\cdot x-(x^2-1)\cdot 1}{x^2}=\frac{2x^2-x^2+1}{x^2}=\frac{x^2+1}{x^2}.$$

Para hacer la 2ª derivada preparamos la 1ª.

$$y'=\frac{x^2+1}{x^2}=1+\frac{1}{x^2}$$

$$\boxed{y''=-2x^{-3}=\frac{-2}{x^3}}$$

Como cociente:

$$\left.\begin{array}{l} f=x^2+1 \longrightarrow f'=2x \\ g=x^2 \longrightarrow g'=2x \end{array}\right\}$$

$$y''=\frac{2x\cdot x^2-(x^2+1)\cdot 2x}{(x^2)^2}=\frac{2x^3-2x^3-2x}{x^4}=\frac{-2x}{x^4}=\frac{-2}{x^3}.$$

(P) Recuerda $y=\dfrac{f}{g} \rightarrow y'=\dfrac{f'\cdot g-f\cdot g'}{g^2}$

$$\boxed{y=\frac{x^3}{x^2-1}}$$

$$\left.\begin{array}{l} f=x^3 \longrightarrow f'=3x^2 \\ g=x^2-1 \longrightarrow g'=2x \end{array}\right\}$$

$$y'=\frac{3x^2\cdot(x^2-1)-x^3\cdot 2x}{(x^2-1)^2}=\frac{3x^4-3x^2-2x^4}{(x^2-1)^2}$$

$$\Rightarrow \boxed{y'=\frac{x^4-3x^2}{(x^2-1)^2}=\frac{x^2\cdot(x^2-3)}{(x^2-1)^2}}$$

$$\left.\begin{array}{l} f=x^4-3x^2 \longrightarrow f'=4x^3-6x \\ g=(x^2-1)^2 \longrightarrow g'=2\cdot(x^2-1)^1\cdot 2x \end{array}\right\}$$

$$y''=\frac{(4x^3-6x)\cdot(x^2-1)^2-(x^4-3x^2)\cdot 2\cdot(x^2-1)\cdot 2x}{\left[(x^2-1)^2\right]^2}$$

$$=\frac{(x^2-1)\cdot\left[(4x^3-6x)\cdot(x^2-1)-4x\cdot(x^4-3x^2)\right]}{(x^2-1)^4}$$

$$=\frac{4x^5-4x^3-6x^3+6x-4x^5+12x^3}{(x^2-1)^3}$$

$$\Rightarrow \boxed{y''=\frac{2x^3+6x}{(x^2-1)^3}=\frac{2x\cdot(x^2+3)}{(x^2-1)^3}}$$

(9) Recuerda $y = \dfrac{f}{g} \rightarrow y' = \dfrac{f' \cdot g - f \cdot g'}{g^2}$

$$y = \frac{x^2 - 6x + 5}{x - 3}$$

$$\left.\begin{array}{l} f = x^2 - 6x + 5 \longrightarrow f' = 2x - 6 \\ g = x - 3 \longrightarrow g' = 1. \end{array}\right\}$$

$$y' = \frac{(2x-6)(x-3) - (x^2 - 6x + 5) \cdot 1}{(x-3)^2} = \frac{2x^2 - 6x - 6x + 18 - x^2 + 6x - 5}{(x-3)^2}$$

$$\boxed{y' = \frac{x^2 - 6x + 13}{(x-3)^2}}$$

$$\left.\begin{array}{l} f = x^2 - 6x + 13 \longrightarrow f' = 2x - 6 \\ g = (x-3)^2 \longrightarrow g' = 2 \cdot (x-3)^1 \cdot 1 \end{array}\right\}$$

$$y'' = \frac{(2x-6) \cdot (x-3)^2 - (x^2 - 6x + 13) \cdot 2 \cdot (x-3)}{[(x-3)^2]^2}$$

$$= \frac{(x-3) \cdot [(2x-6) \cdot (x-3) - 2 \cdot (x^2 - 6x + 13)]}{(x-3)^4}$$

$$= \frac{2x^2 - 6x - 6x + 18 - 2x^2 + 12x - 26}{(x-3)^3}$$

$$\Rrightarrow \boxed{y'' = \frac{-8}{(x-3)^3}}$$

(r) $f(x) = x^4 - 6x^2$

$f'(x) = 4x^3 - 12x$

$f''(x) = 12x^2 - 12$

(s) $f(x) = -x^3 + 6x^2 - 9x + 8$

$f'(x) = -3x^2 + 12x - 9$

$f''(x) = -6x + 12$

(t) $f(x) = (x^5 - 3x^2)^3 \cdot (2-5x^3)^4 \cdot (4x+7x^3)^6$

Sólo haré la 1ª derivada (al verla te podrías imaginar la razón de NO hacer la 2ª).

RECUERDA.

$y = f \cdot g \cdot h \rightarrow y' = f' \cdot g \cdot h + f \cdot g' \cdot h + f \cdot g \cdot h'$

$y = f^n \longrightarrow y' = n \cdot f^{n-1} \cdot f'$

$$f'(x) = \boxed{3 \cdot (x^5 - 3x^2)^2 \cdot (5x^4 - 6x)} \cdot (2 - 5x^3)^4 \cdot (4x + 7x^3)^6 +$$
$$(x^5 - 3x^2)^3 \cdot \boxed{4 \cdot (2 - 5x^3)^3 \cdot (-15x^2)} \cdot (4x + 7x^3)^6 +$$
$$(x^5 - 3x^2)^3 \cdot (2 - 5x^3)^4 \cdot \boxed{6 \cdot (4x + 7x^3)^5 \cdot (4 + 21x^2)}$$