Problema 1 Discutir y resolver por el método de Gauss los siguientes sistemas:

$$\left\{\begin{array}{rrr} x- & y & =2 \\ -x+ & 2y+ & 3z=-1 \\ x+ & y+ & 6z=4 \end{array}\right. ; \quad \left\{\begin{array}{rrr} x- & 2y+ & z=1 \\ & y- & z=1 \\ 3x+ & 2y+ & z=-1 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrr} x- & y & =2 \\ -x+ & 2y+ & 3z=-1 \\ x+ & y+ & 6z=4 \end{array}\right. \text{Sistema Compatible Indeterminado} \Longrightarrow \left\{\begin{array}{l} x=3-3\lambda \\ y=1-3\lambda \\ z=\lambda \end{array}\right.$$

$$\left\{\begin{array}{rrr} x- & 2y+ & z=1 \\ & y- & z=1 \\ 3x+ & 2y+ & z=-1 \end{array}\right. \text{Sistema Compatible Determinado} \Longrightarrow \left\{\begin{array}{l} x=1 \\ y=-1 \\ z=-2 \end{array}\right.$$

Problema 2 Resolver las ecuaciones:

1. $\log(2-x)-\log(x-1)=2$
2. $\log(3-x^2)-\log x=1+\log(x+2)$
3. $4^{x^2-1}\cdot 2^{x+5}=32^{x+1}$
4. $4^{x-2}-2^{x+1}-3=0$

Solución:

1. $\log(2-x)-\log(x-1)=2 \Longrightarrow \ln\dfrac{2-x}{x-1}=\log 100 \Longrightarrow$

   $101x=102 \Longrightarrow x=\dfrac{102}{101}.$

2. $\log(3-x^2)-\log x=1+\log(x+2) \Longrightarrow \log\dfrac{3-x^2}{x}=\log 10(x+2) \Longrightarrow$
   $11x^2+20x-3=0 \Longrightarrow x=0,139,\ \ x=-1,958(\text{no vale}).$

3. $$4^{x^2-1}\cdot 2^{x+5}=32^{x+1} \Longrightarrow x^2-4x-2=0 \Longrightarrow \left\{\begin{array}{l} x=2,4142 \\ x=-0,414 \end{array}\right.$$

4. $$4^{x-2}-2^{x+1}-3=0 \Longrightarrow t^2-32t-48=0 \Longrightarrow \left\{\begin{array}{l} t=33,435 \Longrightarrow x=5,063 \\ t=-1,435 \ \text{ no vale} \end{array}\right.$$

Problema 3 Resolver los siguientes sistemas:

$$\left\{\begin{array}{c} x\cdot y=2 \\ 3x+2y=7 \end{array}\right. ; \quad \left\{\begin{array}{c} x^2+2y^2=6 \\ x-2y=0 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{c} x\cdot y=2 \\ 3x+2y=7 \end{array}\right. \Longrightarrow \left\{\begin{array}{c} x=1,\ \ y=2 \\ x=4/3,\ y=3/2 \end{array}\right.$$

$$\left\{\begin{array}{c} x^2+2y^2=6 \\ x-2y=0 \end{array}\right. \Longrightarrow \left\{\begin{array}{c} x=2,\ \ y=1 \\ x=2,\ y=-1 \end{array}\right.$$

Problema 4 Resolver las inecuaciones siguientes:

1. $\dfrac{2x+1}{3}-\dfrac{x-1}{6}\leq 1+\dfrac{2x+3}{2}$
2. $\dfrac{x^2+2x-15}{x^2+x-2}\geq 0$
3. $\dfrac{x^2-x-6}{x^2+x-2}\leq 0$

Solución:

1. $\dfrac{2x+1}{3}-\dfrac{x-1}{6}\leq 1+\dfrac{2x+3}{2}\Longrightarrow [-4,\infty)$
2. $\dfrac{x^2+2x-15}{x^2+x-2}\geq 0\Longrightarrow (-\infty,-5]\cup(-2,1)\cup[-4,\infty)$
3. $\dfrac{x^2-x-6}{x^2+x-2}\leq 0\Longrightarrow (-\infty,-2)\cup(-2,1)\cup[3,\infty)$