Problema 1 Sean $z_1 = 3 - 2i$ y $z_2 = -1 + 4i$. Calcular: $z_1 + z_2$, $z_1 \cdot z_2$ y $\dfrac{z_1}{z_2}$.

Solución:

- $z_1 + z_2 = 2 + 2i$
- $z_1 \cdot z_2 = 5 + 14i$
- $\dfrac{z_1}{z_2} = -\dfrac{11}{17} - \dfrac{10}{17}i$

Problema 2 Resolver la ecuación $z^3 - 2i = 0$

Solución:

$$z^3 - 2i = 0 \Longrightarrow z = \sqrt[3]{2i} = \sqrt[3]{(2)_{\pi/2}} = \begin{cases} \sqrt[3]{2}_{30^{\text{o}}} = \sqrt[3]{2}(\cos 30^{\text{o}} + i \sin 30^{\text{o}}) \\ \sqrt[3]{2}_{150^{\text{o}}} = \sqrt[3]{2}(\cos 150^{\text{o}} + i \sin 150^{\text{o}}) \\ \sqrt[3]{2}_{270^{\text{o}}} = \sqrt[3]{2}(\cos 270^{\text{o}} + i \sin 270^{\text{o}}) \end{cases}$$

Problema 3 Calcular los siguientes límites

1. $\lim\limits_{x \longrightarrow \infty} (\sqrt{x^2+1} - \sqrt{x^2+x+1})$
2. $\lim\limits_{x \longrightarrow 1} \dfrac{x^2 - 2x + 1}{x^3 - 1}$
3. $\lim\limits_{x \longrightarrow 3} \dfrac{\sqrt{x^2-5} - \sqrt{x+1}}{x - 3}$
4. $\lim\limits_{x \longrightarrow \infty} \dfrac{3x^2 - \sqrt{2x^5+1} + x - 1}{x^3 - x + 1}$

Solución:

1. $\lim\limits_{x \longrightarrow \infty} (\sqrt{x^2+1} - \sqrt{x^2+x+1}) = -\dfrac{1}{2}$
2. $\lim\limits_{x \longrightarrow 1} \dfrac{x^2 - 2x + 1}{x^3 - 1} = 0$
3. $\lim\limits_{x \longrightarrow 3} \dfrac{\sqrt{x^2-5} - \sqrt{x+1}}{x - 3} = \dfrac{5}{4}$
4. $\lim\limits_{x \longrightarrow \infty} \dfrac{3x^2 - \sqrt{2x^5+1} + x - 1}{x^3 - x + 1} = 0$

Problema 4 Calcular la derivada de las siguientes funciones

1. $y = (x^3 - 2x^2 + 1)^8$
2. $y = e^{x^2-3x+1}$
3. $y = e^{2x} \cdot (x^2 + x - 1)$
4. $y = \dfrac{\sin(x^2)}{e^{2x}}$
5. $y = \ln\left(\dfrac{\sin x}{x}\right)$

Solución:

1. $y = (x^3 - 2x^2 + 1)^8 \Longrightarrow y' = 8(x^3 - 2x^2 + 1)^7(3x^2 - 4x)$
2. $y = e^{x^2-3x+1} \Longrightarrow y' = (2x - 3)e^{x^2-3x+1}$
3. $y = e^{2x} \cdot (x^2 + x - 1) \Longrightarrow y' = 2e^{2x} \cdot (x^2 + x - 1) + e^{2x} \cdot (2x + 1)$
4. $y = \dfrac{\sin(x^2)}{e^{2x}} \Longrightarrow y' = \dfrac{2x\cos(x^2)e^{2x} - 2e^{2x}\sin(x^2)}{(e^{2x})^2}$
5. $y = \ln\left(\dfrac{\sin x}{x}\right) \Longrightarrow y' = \dfrac{\cos x}{\sin x} - \dfrac{1}{x}$

Problema 5 Calcular las rectas tangente y normal de la siguiente función

$$f(x) = \frac{x^2 + 2}{x^2 - 1}$$

en $x = 2$

Solución:

$f'(x) = \dfrac{-6x}{(x^2 - 1)^2} \Longrightarrow f'(2) = -\dfrac{4}{3}$ y $f(2) = 2$

Recta Tangente: $y - 2 = -\dfrac{4}{3}(x - 2)$

Recta Normal: $y - 2 = \dfrac{3}{4}(x - 2)$