

ECUACIONES EXPONENCIALES

1) $2^{x+4} = 64$

$$2^{x+4} = 2^6 \Rightarrow x + 4 = 6 \Rightarrow \boxed{x=2}$$

2) $5^{x-3} = 125$

$$5^{x-3} = 5^3 \Rightarrow x - 3 = 3 \Rightarrow \boxed{x=6}$$

3) $2^{x^2-3x} = \frac{1}{4}$

$$2^{x^2-3x} = 2^{-2} \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow x = \frac{3 \pm \sqrt{3^2 - 4 \cdot 2}}{2} = \frac{3 \pm 1}{2} \Rightarrow \begin{cases} x_1 = \frac{3+1}{2} = 2 \Rightarrow \boxed{x=2} \\ x_2 = \frac{3-1}{2} = 1 \Rightarrow \boxed{x=1} \end{cases}$$

4) $3^{x^2+3} = 9^{2x}$

$$3^{x^2+3} = 3^{4x} \Rightarrow x^2 + 3 = 4x \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow x = \frac{4 \pm \sqrt{4^2 - 4 \cdot 3}}{2} = \frac{4 \pm 2}{2} \Rightarrow \begin{cases} x_1 = \frac{4+2}{2} = 3 \Rightarrow \boxed{x=3} \\ x_2 = \frac{4-2}{2} = 1 \Rightarrow \boxed{x=1} \end{cases}$$

5) $3^{1-x^2} = \frac{1}{27}$

$$3^{1-x^2} = 3^{-3} \Rightarrow 1 - x^2 = -3 \Rightarrow x^2 = 4 \Rightarrow \boxed{x = \pm 2}$$

6) $3^{x-1} = \left(\frac{1}{3}\right)^{-2x-1}$

$$3^{x-1} = 3^{2x+1} \Rightarrow x - 1 = 2x + 1 \Rightarrow \boxed{x = -2}$$

7) $8^{x-1} = 4^{3x+1}$

$$2^{3(x-1)} = 2^{2(3x+1)} \Rightarrow 3(x-1) = 2(3x+1) \Rightarrow 3x - 3 = 6x + 2 \Rightarrow 3x = -5 \Rightarrow \boxed{x = -\frac{5}{3}}$$

8) $7^{x^2-5x+6} = 1$

$$x^2 - 5x + 6 = 0 \Rightarrow x = \frac{5 \pm \sqrt{5^2 - 4 \cdot 6}}{2} = \frac{5 \pm 1}{2} \Rightarrow \begin{cases} x_1 = \frac{5+1}{2} = 3 \Rightarrow \boxed{x=3} \\ x_2 = \frac{5-1}{2} = 2 \Rightarrow \boxed{x=2} \end{cases}$$

$$9) (4^{3-x})^{2-x} = 1$$

$$(3-x)(2-x) = 0 \Rightarrow \begin{cases} 3-x=0 \Rightarrow \boxed{x=3} \\ 2-x=0 \Rightarrow \boxed{x=2} \end{cases}$$

$$10) 2^{2-x} = \frac{1}{8}$$

$$2^{2-x} = 2^{-3} \Rightarrow 2-x = -3 \Rightarrow \boxed{x=5}$$

$$11) 5^x \cdot 2^{x-1} = 0,05$$

$$5^x \cdot \frac{2^x}{2} = \frac{5}{100} \Rightarrow 10^x = \frac{10}{100} = 10^{-1} \Rightarrow \boxed{x=-1}$$

$$12) 5^{2x-1} = 25^{x^2-\frac{1}{4}}$$

$$5^{2x-1} = 5^{2\left(x^2-\frac{1}{4}\right)} \Rightarrow 2x-1 = 2\left(x^2-\frac{1}{4}\right) \Rightarrow 2x-1 = 2x^2-\frac{1}{2} \Rightarrow 4x^2-4x+1 = 0 \Rightarrow (2x-1) = 0 \Rightarrow \boxed{x=\frac{1}{2}}$$

$$13) 4^{x+3} = \frac{1}{2^x}$$

$$2^{2(x+3)} = 2^{-x} \Rightarrow 2(x+3) = -x \Rightarrow 2x+6+x = 0 \Rightarrow 3x+6 = 0 \Rightarrow \boxed{x=-2}$$

$$14) \sqrt[3]{2^{7-x}} = 4$$

$$\sqrt[3]{2^{7-x}} = 4 \Rightarrow 2^{7-x} = 4^3 \Rightarrow 2^{7-x} = 2^6 \Rightarrow 7-x = 6 \Rightarrow \boxed{x=1}$$

$$15) 3^{x-2} = \frac{1}{9^{1+x}}$$

$$3^{x-2} = 3^{-2(1+x)} \Rightarrow x-2 = -2(1+x) \Rightarrow x-2 = -2-2x \Rightarrow 3x = 0 \Rightarrow \boxed{x=0}$$

$$16) \left(\frac{2}{7}\right)^3 = \left(\frac{7}{2}\right)^{1-x^2}$$

$$\left(\frac{2}{7}\right)^3 = \left(\frac{2}{7}\right)^{x^2-1} \Rightarrow 3 = x^2 - 1 \Rightarrow x^2 = 4 \Rightarrow \boxed{x=\pm 2}$$

$$17) \left(\frac{3}{5}\right)^{x^2+x} = \left(\frac{125}{27}\right)^{x+1}$$

$$\left(\frac{3}{5}\right)^{x^2+x} = \left(\frac{3}{5}\right)^{-3(x+1)} \Rightarrow x^2+x = -3(x+1) \Rightarrow x^2+x = -3x-3 \Rightarrow x^2+4x-3=0$$

$$x = \frac{-4 \pm \sqrt{(-4)^2 - 4 \cdot 3}}{2} = \frac{-4 \pm 2}{2} \Rightarrow \begin{cases} x_1 = \frac{-4+2}{2} = -1 \Rightarrow \boxed{x=-1} \\ x_2 = \frac{-4-2}{2} = -3 \Rightarrow \boxed{x=-3} \end{cases}$$

$$18) 4^{\sqrt{x+1}} = 2^{\sqrt{x+1}+2}$$

$$2^{2\sqrt{x+1}} = 2^{\sqrt{x+1}+2} \Rightarrow 2\sqrt{x+1} = \sqrt{x+1} + 2 \Rightarrow \sqrt{x+1} = 2 \Rightarrow x+1 = 4 \Rightarrow \boxed{x=3}$$

$$19) \sqrt{\sqrt{3} + \sqrt{12}} = \left(\frac{1}{3}\right)^{x-2}$$

$$\sqrt{\sqrt{3} + 2\sqrt{3}} = \left(\frac{1}{3}\right)^{x-2} \Rightarrow \sqrt{3\sqrt{3}} = \left(\frac{1}{3}\right)^{x-2} \Rightarrow \sqrt{\sqrt{3^3}} = \left(\frac{1}{3}\right)^{x-2} \Rightarrow \sqrt[4]{3^3} = 3^{2-x} \Rightarrow 3^{\frac{3}{4}} = 3^{2-x} \Rightarrow \frac{3}{4} = 2-x \Rightarrow$$

$$\boxed{x = \frac{5}{4}}$$

$$20) 4^x \cdot 5^{x-1} = 1600$$

$$4^x \cdot \frac{5^x}{5} = 1600 \Rightarrow 4^x \cdot \frac{5^x}{5} = 1600 \Rightarrow 20^x = 8000 \Rightarrow 20^x = 20^3 \Rightarrow \boxed{x=3}$$

$$21) 10^{x^2+x-2} = 1$$

$$x^2+x-2=0 \Rightarrow x = \frac{-1 \pm \sqrt{(-1)^2 - 4 \cdot (-2)}}{2} = \frac{-1 \pm 3}{2} \Rightarrow \begin{cases} x_1 = \frac{-1+3}{2} = 1 \Rightarrow \boxed{x=1} \\ x_2 = \frac{-1-3}{2} = -2 \Rightarrow \boxed{x=-2} \end{cases}$$

$$22) 4^x = 8^{2x-3}$$

$$2^{2x} = 2^{3(2x-3)} \Rightarrow 2x = 3(2x-3) \Rightarrow 2x = 6x-9 \Rightarrow 4x = 9 \Rightarrow \boxed{x = \frac{9}{4}}$$

$$23) 4^x - 2^x = 2$$

$$2^{2x} - 2^x - 2 = 0, \text{ realizamos el cambio } y = 2^x \Rightarrow y^2 = 2^{2x}$$

$$y^2 - y - 2 = 0 \Rightarrow y = \frac{1 \pm \sqrt{1^2 - 4 \cdot (-2)}}{2} = \frac{1 \pm 3}{2} \Rightarrow \begin{cases} x_1 = \frac{1+3}{2} = 2 \Rightarrow 2^x = 2 \Rightarrow \boxed{x=1} \\ x_2 = \frac{1-3}{2} = -1 \Rightarrow 2^x = -1!!! \end{cases}$$

24) $4^x - 5 \cdot 2^x + 4 = 0$

$2^{2x} - 5 \cdot 2^x + 4 = 0$, realizamos el cambio $y = 2^x \Rightarrow y^2 = 2^{2x}$

$$y^2 - 5y + 4 = 0 \Rightarrow y = \frac{5 \pm \sqrt{5^2 - 4 \cdot 4}}{2} = \frac{5 \pm 3}{2} \Rightarrow \begin{cases} x_1 = \frac{5+3}{2} = 4 \Rightarrow 2^x = 2^2 \Rightarrow \boxed{x=2} \\ x_2 = \frac{5-3}{2} = 1 \Rightarrow 2^x = 1 \Rightarrow \boxed{x=0} \end{cases}$$

25) $3^{x-1} + 3^{x-2} + 3^{x-3} = 117$

$3^{x-3}(3^2 + 3 + 1) = 117 \Rightarrow 3^{x-3} = 3^2 \Rightarrow x - 3 = 2 \Rightarrow \boxed{x=5}$

26) $4^{x-1} + 2^{x+2} = 48$

$\frac{4^x}{4} + 4 \cdot 2^x - 48 = 0 \Rightarrow 2^{2x} + 16 \cdot 2^x - 192 = 0 \Rightarrow$ realizamos el cambio $y = 2^x \Rightarrow y^2 = 2^{2x}$

$$y^2 + 16y - 192 = 0$$

$$\Rightarrow y = \frac{-16 \pm \sqrt{(-16)^2 - 4 \cdot (-192)}}{2} = \frac{-16 \pm 32}{2} \Rightarrow \begin{cases} x_1 = \frac{-16+32}{2} = 8 \Rightarrow 2^x = 2^3 \Rightarrow \boxed{x=3} \\ x_2 = \frac{-16-32}{2} = -24 \Rightarrow 2^x = -24 !!! \end{cases}$$

27) $3^{x+1} + 3^x + 3^{x+2} = 39$

$3^x(3 + 1 + 3^2) = 39 \Rightarrow 3^x = 3 \Rightarrow \boxed{x=1}$

28) $4^x - 6 \cdot 2^x + 8 = 0$

$2^{2x} - 6 \cdot 2^x + 8 = 0$, realizamos el cambio $y = 2^x \Rightarrow y^2 = 2^{2x}$

$$y^2 - 6y + 8 = 0 \Rightarrow y = \frac{6 \pm \sqrt{6^2 - 4 \cdot 8}}{2} = \frac{6 \pm 2}{2} \Rightarrow \begin{cases} x_1 = \frac{6+2}{2} = 4 \Rightarrow 2^x = 2^2 \Rightarrow \boxed{x=2} \\ x_2 = \frac{6-2}{2} = 2 \Rightarrow 2^x = 2 \Rightarrow \boxed{x=1} \end{cases}$$

29) $5^{2x+1} - 4 \cdot 5^{x+1} - 25 = 0$

$5 \cdot 5^{2x} - 20 \cdot 5^x - 25 = 0 \Rightarrow 5^{2x} - 4 \cdot 5^x - 5 = 0$, realizamos el cambio $y = 5^x \Rightarrow y^2 = 5^{2x}$

$$y^2 - 4y - 5 = 0 \Rightarrow y = \frac{4 \pm \sqrt{4^2 - 4 \cdot (-5)}}{2} = \frac{4 \pm 6}{2} \Rightarrow \begin{cases} x_1 = \frac{4+6}{2} = 5 \Rightarrow 5^x = 5 \Rightarrow \boxed{x=1} \\ x_2 = \frac{4-6}{2} = -1 \Rightarrow 5^x = -1 !!! \end{cases}$$

30) $3^x + 9^{x-1} - 18 = 0$

$\frac{3^{2x}}{9} + 3^x - 18 = 0 \Rightarrow 3^{2x} + 9 \cdot 3^x - 162 = 0$, realizamos el cambio $y = 3^x \Rightarrow y^2 = 3^{2x}$

$$y^2 + 9y - 162 = 0 \Rightarrow y = \frac{-9 \pm \sqrt{(-9)^2 - 4 \cdot (-162)}}{2} = \frac{-9 \pm 27}{2} \Rightarrow \begin{cases} x_1 = \frac{-9+27}{2} = 9 \Rightarrow 3^x = 3^2 \Rightarrow \boxed{x=2} \\ x_2 = \frac{-9-27}{2} = -18 \Rightarrow 3^x = -18 !!! \end{cases}$$

$$31) 2^x - 5 \cdot 2^{-x} + 4 \cdot 2^{-3x} = 0$$

$$2^x - \frac{5}{2^x} + \frac{4}{2^{3x}} = 0 \Rightarrow 2^{4x} - 5 \cdot 2^{2x} + 4 = 0 \Rightarrow \text{realizamos el cambio } y = 2^{2x} = 4^x \Rightarrow y^2 = 2^{4x} = 4^{2x}$$

$$y^2 - 5y + 4 = 0 \Rightarrow y = \frac{5 \pm \sqrt{5^2 - 4 \cdot 4}}{2} = \frac{5 \pm 3}{2} \Rightarrow \begin{cases} x_1 = \frac{5+3}{2} = 4 \Rightarrow 4^x = 4 \Rightarrow \boxed{x=1} \\ x_2 = \frac{5-3}{2} = 1 \Rightarrow 4^x = 1 \Rightarrow \boxed{x=0} \end{cases}$$

$$32) 4^{x-1} - 2^{x+2} = 128$$

$$\frac{4^x}{4} - 4 \cdot 2^x - 128 = 0 \Rightarrow 2^{2x} - 16 \cdot 2^x - 512 = 0 \Rightarrow \text{realizamos el cambio } y = 2^x \Rightarrow y^2 = 2^{2x}$$

$$y^2 - 16y - 512 = 0 \Rightarrow y = \frac{16 \pm \sqrt{16^2 - 4 \cdot (-512)}}{2} = \frac{16 \pm 48}{2} \Rightarrow \begin{cases} x_1 = \frac{16+48}{2} = 32 \Rightarrow 2^x = 2^5 \Rightarrow \boxed{x=5} \\ x_2 = \frac{16-48}{2} = -16 \Rightarrow 2^x = -16 !!! \end{cases}$$

$$33) 4^{2x} - 10 \cdot 4^x + 16 = 0$$

$$\text{Realizamos el cambio } y = 4^x \Rightarrow y^2 = 4^{2x}$$

$$y^2 - 10y + 16 = 0 \Rightarrow y = \frac{10 \pm \sqrt{10^2 - 4 \cdot 16}}{2} = \frac{10 \pm 6}{2} \Rightarrow \begin{cases} x_1 = \frac{10+6}{2} = 8 \Rightarrow 2^{2x} = 2^3 \Rightarrow \boxed{x = \frac{3}{2}} \\ x_2 = \frac{10-6}{2} = 2 \Rightarrow 2^{2x} = 2 \Rightarrow \boxed{x = \frac{1}{2}} \end{cases}$$

$$34) 2^{x+3} + 4^{x+1} - 320 = 0$$

$$8 \cdot 2^x + 4 \cdot 2^{2x} - 320 = 0 \Rightarrow 2^{2x} + 2 \cdot 2^x - 80 = 0 \Rightarrow \text{realizamos el cambio } y = 2^x \Rightarrow y^2 = 2^{2x}$$

$$y^2 + 2y - 80 = 0 \Rightarrow y = \frac{-2 \pm \sqrt{(-2)^2 - 4 \cdot (-80)}}{2} = \frac{-2 \pm 18}{2} \Rightarrow \begin{cases} x_1 = \frac{-2+18}{2} = 8 \Rightarrow 2^x = 2^3 \Rightarrow \boxed{x=3} \\ x_2 = \frac{-2-18}{2} = -10 \Rightarrow 2^x = -10 !!! \end{cases}$$

$$35) 7 \cdot 2^x - 5 \cdot 2^{x+1} + 6 \cdot 2^{x-1} = 0$$

$$2^{x-1} (14 - 20 + 6) = 0 \Rightarrow 0 \cdot 2^{x-1} = 0 \Rightarrow 0 = 0 \Rightarrow \boxed{\text{Infinitas soluciones}}$$

$$36) 3^x + 3^{-x+1} + 4 = 0$$

$$3^x + \frac{3}{3^x} + 4 = 0 \Rightarrow 3^{2x} + 4 \cdot 3^x + 3 = 0 \Rightarrow \text{realizamos el cambio } y = 3^x \Rightarrow y^2 = 3^{2x}$$

$$y^2 + 4y + 3 = 0 \Rightarrow y = \frac{-4 \pm \sqrt{(-4)^2 - 4 \cdot 3}}{2} = \frac{-4 \pm 2}{2} \Rightarrow \begin{cases} x_1 = \frac{-4+2}{2} = -1 \Rightarrow 3^x = -1 !!! \\ x_2 = \frac{-4-2}{2} = -3 \Rightarrow 3^x = -3 !!! \end{cases} \Rightarrow \boxed{\text{No tiene}}$$

$$37) 5^{2x} - 2 \cdot 5^x - 15 = 0$$

Realizamos el cambio $y = 5^x \Rightarrow y^2 = 5^{2x}$

$$y^2 - 2y - 15 = 0 \Rightarrow y = \frac{2 \pm \sqrt{2^2 - 4 \cdot (-15)}}{2} = \frac{2 \pm 8}{2} \Rightarrow \begin{cases} x_1 = \frac{2+8}{2} = 5 \Rightarrow 5^x = 5 \Rightarrow \boxed{x=1} \\ x_2 = \frac{2-8}{2} = -3 \Rightarrow 5^x = -3!!! \end{cases}$$

$$38) 3^{2x+2} - 18 \cdot 3^x + 9 = 0$$

$9 \cdot 3^{2x} - 18 \cdot 3^x + 9 = 0 \Rightarrow 3^{2x} - 2 \cdot 3^x + 1 = 0$, realizamos el cambio $y = 3^x \Rightarrow y^2 = 3^{2x}$

$$y^2 - 2y + 1 = 0 \Rightarrow (y - 1)^2 = 0 \Rightarrow y = 1 \Rightarrow 3^x = 3^0 \Rightarrow \boxed{x=0}$$

$$39) 5^{2x+1} - 3 \cdot 5^{2x-1} = 550$$

$$5^{2x-1} \cdot (25 - 3) = 550 \Rightarrow 5^{2x-1} = 25 \Rightarrow 5^{2x-1} = 5^2 \Rightarrow 2x - 1 = 2 \Rightarrow 2x = 3 \Rightarrow \boxed{x = \frac{3}{2}}$$

$$40) 6^x - 9 \cdot 6^{-x} + 8 = 0$$

$6^x - \frac{9}{6^x} + 8 = 0 \Rightarrow 6^{2x} + 8 \cdot 6^x - 9 = 0$, realizamos el cambio $y = 6^x \Rightarrow y^2 = 6^{2x}$

$$y^2 + 8y - 9 = 0 \Rightarrow y = \frac{-8 \pm \sqrt{(-8)^2 - 4 \cdot (-9)}}{2} = \frac{-8 \pm 10}{2} \Rightarrow \begin{cases} x_1 = \frac{-8+10}{2} = 1 \Rightarrow 6^x = 1 \Rightarrow \boxed{x=0} \\ x_2 = \frac{-8-10}{2} = -9 \Rightarrow 6^x = -9!!! \end{cases}$$