

## EJERCICIOS DE RAÍCES

### RECORDAR:

- Definición de raíz n-ésima:  $\sqrt[n]{a} = x \Leftrightarrow x^n = a$
- Equivalencia con una potencia de exponente fraccionario:  

$$\sqrt[n]{x^m} = x^{m/n}$$
- Simplificación de radicales/índice común:  $\sqrt[n]{x^m \cdot x^p} = \sqrt[n]{x^m}$
- Propiedades de las raíces:
  - $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
  - $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
  - $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$
  - $\sqrt[m]{\sqrt[n]{a}} = \sqrt[m \cdot n]{a}$
- Introducir/extraer factores:  $x \cdot \sqrt[n]{a} = \sqrt[n]{x^n \cdot a}$

### 1. Calcular mentalmente, sin usar calculadora:

$\sqrt{9} =$	$\sqrt{25} =$	$\sqrt{49} =$	$\sqrt{100} =$	$\sqrt{1} =$
$\sqrt{0} =$	$\sqrt{\frac{1}{4}} =$	$\sqrt{\frac{1}{9}} =$	$\sqrt{\frac{4}{25}} =$	$\sqrt{\frac{16}{100}} =$
$\sqrt{0,25} =$	$\sqrt{0,09} =$	$\sqrt{0,0081} =$	$\sqrt{0,49} =$	$\sqrt{7^6} =$
$\sqrt{5^{24}} =$	$\sqrt{2^{10}} =$	$\sqrt{9^{-10}} =$		

### 2. Calcular mentalmente, sin usar calculadora:

$\sqrt[3]{8} =$	$\sqrt[3]{27} =$	$\sqrt[3]{64} =$	$\sqrt[3]{1000} =$	$\sqrt[3]{1331} =$
$\sqrt[3]{-1} =$	$\sqrt[3]{-8} =$	$\sqrt[3]{-27} =$	$\sqrt[3]{-1000} =$	$\sqrt[3]{-1000} =$
$\sqrt[3]{\frac{1}{8}} =$	$\sqrt[3]{\frac{1}{125}} =$	$\sqrt[3]{\frac{64}{125}} =$	$\sqrt[3]{\frac{64}{1000}} =$	$\sqrt[3]{\frac{64}{1000}} =$
$\sqrt[3]{0,125} =$	$\sqrt[3]{0,027} =$	$\sqrt[3]{0,001} =$	$\sqrt[3]{-0,216} =$	

### 3. Calcular, aplicando la definición de raíz (no vale con calculadora):

a) $\sqrt[3]{-8} = -2$ pq $(-2)^3 = -8$	b) $\sqrt{-8} =$	c) $\sqrt[6]{-1} =$	d) $\sqrt[5]{-32} =$
e) $\sqrt[4]{81} =$	f) $\sqrt{5^2} =$	g) $\sqrt[6]{2^6} =$	h) $\sqrt{\frac{625}{81}} =$
i) $\sqrt[3]{\frac{27}{64}} =$	j) $\sqrt[4]{-\frac{81}{16}} =$	k) $\sqrt[5]{3^{15}} =$	l) $\sqrt[3]{0,064} =$
m) $\sqrt{0,1} =$	n) $\sqrt{2,25} =$	o) $\sqrt{2,7} =$	

4. Hallar el valor de **k** en cada caso:

a)  $\sqrt[3]{k} = 2$  (Soluc:  $k=8$ )

b)  $\sqrt[k]{-243} = -3$  (Soluc:  $k=5$ )

c)  $\sqrt[5]{k} = \frac{2}{3}$  (Soluc:  $k=32/243$ )

d)  $\sqrt[k]{1,331} = 1,1$  (Soluc:  $k=3$ )

### POTENCIAS DE EXPONENTE FRACCIONARIO:

5. Utilizar la calculadora para hallar, con tres cifras decimales bien aproximadas:

a)  $\sqrt[4]{8} \approx 1,682$       b)  $\sqrt[5]{9}$       c)  $\sqrt[6]{25}$       d)  $\sqrt[3]{10}$

e)  $\sqrt[5]{-15}$       f)  $\sqrt[6]{-40}$       g)  $\sqrt[4]{2^3}$       h)  $\sqrt[5]{3^2}$

i)  $\sqrt[6]{5^2}$       j)  $\sqrt[8]{256}$       k)  $\sqrt[3]{64}$

6. Hallar  $\sqrt[3]{3}$  con cuatro cifras decimales bien aproximadas, razonando el error cometido.

7. Pasar a forma de raíz las siguientes potencias, y a continuación calcular (**no vale utilizar la calculadora**):

a)  $4^{1/2} = \sqrt{4} = 2$       b)  $125^{1/3}$       c)  $625^{1/4}$

d)  $8^{2/3}$       e)  $64^{5/6}$       f)  $81^{3/4}$

g)  $8^{-2/3}$       h)  $27^{-1/3}$

### RADICALES EQUIVALENTES. SIMPLIFICACIÓN DE RADICALES:

8. Simplificar los siguientes radicales, y comprobar el resultado con la calculadora cuando proceda:

a)  $\sqrt[4]{3^2} = \sqrt[4/2]{3^{2/2}} = \sqrt{3}$       b)  $\sqrt[8]{5^4}$       c)  $\sqrt[9]{27}$       d)  $\sqrt[5]{1024}$

e)  $\sqrt[6]{8}$       f)  $\sqrt[9]{64}$       g)  $\sqrt[8]{81}$       h)  $\sqrt[12]{x^9}$

i)  $\sqrt[12]{x^8}$       j)  $\sqrt[5]{x^{10}}$       k)  $\sqrt[6]{a^2b^4}$       l)  $\sqrt[10]{a^4b^6}$

m)  $\sqrt[6]{5^3}$       n)  $\sqrt[15]{2^{12}}$       o)  $\sqrt[10]{a^8}$       p)  $\sqrt[12]{x^4y^8z^4}$

q)  $\sqrt[8]{(x^2y^2)^2}$

9. Decir si los siguientes radicales son equivalentes (y comprobar después con la calculadora):

a)  $\sqrt{5}$ ,  $\sqrt[4]{25}$ ,  $\sqrt[6]{125}$ ,  $\sqrt[8]{625}$

(Soluc: NO)

b)  $\sqrt{9}$ ,  $\sqrt[3]{27}$ ,  $\sqrt[4]{81}$ ,  $\sqrt[5]{243}$

(Soluc: SÍ)

c)  $\sqrt{2}$ ,  $\sqrt[4]{4}$ ,  $\sqrt[6]{8}$ ,  $\sqrt[8]{16}$

10. Reducir los siguientes radicales a índice común y ordenarlos de menor a mayor (y comprobar el resultado con la calculadora):

a)  $\sqrt{5}$ ,  $\sqrt[5]{2^3}$ ,  $\sqrt[15]{7^2}$

b)  $\sqrt[3]{5}$ ,  $\sqrt[5]{7^3}$ ,  $\sqrt[15]{3^2}$

c)  $\sqrt[4]{3}$ ,  $\sqrt[6]{16}$ ,  $\sqrt[15]{9}$

d)  $\sqrt{2}$ ,  $\sqrt[3]{32}$ ,  $\sqrt[5]{27}$

e)  $\sqrt{2}$ ,  $\sqrt[3]{3}$ ,  $\sqrt[4]{4}$ ,  $\sqrt[5]{5}$ ,  $\sqrt[6]{6}$

f)  $\sqrt[3]{16}$ ,  $\sqrt[4]{125}$ ,  $\sqrt[6]{243}$

g)  $\sqrt[4]{31}$  y  $\sqrt[3]{13}$

i)  $\sqrt[3]{-10}$  y  $\sqrt[4]{8}$

h)  $\sqrt[3]{51}$  y  $\sqrt[9]{132650}$

**OPERACIONES CON RADICALES:**

11. Multiplicar los siguientes radicales de igual índice, y simplificar cuando sea posible:

a)  $\sqrt{2} \sqrt{32} = \sqrt{64} = 8$

b)  $\sqrt{2} \sqrt{15}$

c)  $\sqrt[3]{3} \sqrt[3]{9}$

d)  $\sqrt{2} \sqrt{8}$

e)  $\sqrt{3} \sqrt{4}$

f)  $\sqrt[3]{2} \sqrt[3]{5}$

g)  $\sqrt{12} \sqrt{6} \sqrt{50}$

h)  $\sqrt{21} \sqrt{7}$

i)  $4\sqrt{3} \cdot 2\sqrt{27}$

(Sol : 72)

12. Multiplicar los siguientes radicales de distinto índice, reduciendo previamente a índice común, y simplificar:

a)  $\sqrt{2} \sqrt[3]{32} = \sqrt{2} \sqrt[3]{2^5} = \sqrt[6]{2^3} \sqrt[6]{2^{10}} = \sqrt[6]{2^{13}}$

b)  $\sqrt[3]{2} \sqrt[4]{8}$

(Sol :  $\sqrt[12]{2^{13}}$ )

c)  $\sqrt[3]{2} \sqrt[5]{2}$

(Sol :  $\sqrt[15]{2^8}$ )

d)  $\sqrt[3]{9} \sqrt[6]{3}$

(Sol :  $\sqrt[6]{3^5}$ )

e)  $\sqrt[3]{2^2} \sqrt[4]{2}$

(Sol :  $\sqrt[12]{2^{11}}$ )

f)  $\sqrt[4]{a^3} \sqrt[6]{a^5}$

(Sol :  $\sqrt[12]{a^{19}}$ )

g)  $\sqrt[3]{2} \sqrt{3} \sqrt[4]{8}$

(Sol :  $\sqrt[12]{2^{13} 3^6}$ )

h)  $\sqrt[4]{8} \sqrt[3]{4} \sqrt{a^3}$

(Sol :  $\sqrt[12]{2^{17} a^{18}}$ )

13. Simplificar, aplicando convenientemente las propiedades de las raíces:

a)  $\frac{\sqrt{32}}{\sqrt{2}} = \sqrt{\frac{32}{2}} = \sqrt{16} = 4$

b)  $\frac{\sqrt{8}}{\sqrt{2}}$

c)  $\frac{\sqrt[3]{81}}{\sqrt[3]{9}}$

d)  $\frac{\sqrt{15}}{\sqrt{3}}$

e)  $\frac{\sqrt{3}}{\sqrt{4}}$

f)  $\frac{\sqrt[3]{16}}{\sqrt[3]{2}}$

g)  $\frac{\sqrt{256}}{\sqrt{729}}$

h)  $\frac{\sqrt{21}}{2\sqrt{7}}$

(Sol : 16/27)

(Sol :  $\sqrt{3}/2$ )

i)  $\frac{\sqrt{33}}{\sqrt{3}}$

(Sol :  $\sqrt{11}$ )

j)  $\sqrt[3]{\frac{125}{512}}$

k)  $\sqrt[4]{\frac{16}{625}}$

l)  $\frac{\sqrt{2} \cdot \sqrt{8}}{\sqrt{32}}$

(Sol :  $1/\sqrt{2}$ )

m)  $\sqrt{\frac{154}{9} + 23} - \sqrt[4]{\frac{144}{9}}$

(Sol : -5/3)

n)  $\sqrt{\left(-\frac{3}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}$

(Sol : 3)

14. ¿Cómo podríamos comprobar rápidamente que  $\frac{2\sqrt{6}}{6\sqrt{2}} = \frac{\sqrt{3}}{3}$  ? (no vale calculadora)

(Sol: multiplicando en cruz)

15. Operar los siguientes radicales de distinto índice, reduciendo previamente a índice común:

a)  $\frac{\sqrt{8}}{\sqrt[4]{2}} = \frac{\sqrt{2^3}}{\sqrt[4]{2}} = \frac{\sqrt[4]{2^6}}{\sqrt[4]{2}} = \sqrt[4]{2^5}$

b)  $\frac{\sqrt[3]{9}}{\sqrt[6]{3}}$

(Sol :  $\sqrt{3}$ )

c)  $\frac{\sqrt{2}}{\sqrt[3]{32}}$

(Sol :  $\frac{1}{\sqrt[6]{2^7}}$ )

d)  $\frac{\sqrt[4]{4}}{\sqrt[6]{8}}$

(Sol : 1)

e)  $\frac{\sqrt[3]{7^2}}{\sqrt{7}}$

(Sol :  $\sqrt[6]{7}$ )

- f)  $\frac{\sqrt{9}}{\sqrt[3]{3}}$  (Sol :  $\sqrt[3]{9}$ )
- g)  $\frac{\sqrt[5]{16}}{\sqrt{2}}$  (Sol :  $\sqrt[10]{8}$ )
- h)  $\frac{\sqrt{ab}}{\sqrt[3]{ab}}$  (Sol :  $\sqrt[6]{ab}$ )
- i)  $\frac{\sqrt[4]{a^3b^5c}}{\sqrt{ab^3c^3}}$  (Sol :  $\sqrt[4]{\frac{a}{bc^5}}$ )
- j)  $\frac{\sqrt[6]{a^3}}{\sqrt[3]{a^2}}$  (Sol :  $1/\sqrt[6]{a}$ )
- k)  $\frac{\sqrt[3]{-2000}}{3\sqrt{2}}$  (Sol : -10)
- l)  $\frac{\sqrt[3]{4} \cdot \sqrt{3}}{\sqrt[6]{12}}$  (Sol :  $\sqrt[3]{6}$ )
- m)  $\frac{\sqrt[8]{8}}{\sqrt[4]{4} \cdot \sqrt{2}}$  (Sol :  $\sqrt[8]{\frac{1}{2^5}}$ )
- n)  $\frac{\sqrt[3]{5} \cdot \sqrt{125}}{\sqrt[4]{25}} =$  (Sol :  $\sqrt[3]{625}$ )
- o)  $\frac{\sqrt[3]{2} \cdot \sqrt{3} \cdot \sqrt[12]{2}}{\sqrt[12]{18}} =$  (Sol :  $\sqrt[3]{6}$ )
- p)  $\frac{\sqrt[3]{4} \cdot \sqrt{3} \cdot \sqrt[12]{2}}{\sqrt[4]{2}} =$  (Sol :  $\sqrt{6}$ )
- q)  $\frac{\sqrt[6]{54} \cdot \sqrt[12]{27}}{\sqrt[12]{4} \cdot \sqrt[4]{12}} =$  (Sol :  $\sqrt{3/2}$ )
- r)  $\frac{\sqrt[4]{abc^2} \cdot \sqrt[12]{a^3b^5c^2}}{\sqrt[6]{a^2b^2c}} =$  (Sol :  $\sqrt[6]{ab^2c^3}$ )

16. Simplificar:

a)  $(\sqrt[3]{a^2})^6 = \sqrt[3]{a^{12}} = a^{12/3} = a^4$

b)  $(\sqrt[6]{ab^2})^2 =$  (Sol :  $\sqrt[3]{ab^2}$ )

c)  $(\sqrt{x})^3 \cdot \sqrt[3]{x} =$  (Sol :  $\sqrt[6]{x^{11}}$ )

d)  $\frac{(\sqrt[3]{2})^4}{(\sqrt[4]{2})^2} =$  (Sol :  $\sqrt[6]{2^5}$ )

e)  $\frac{\sqrt{2} (\sqrt[3]{2})^4}{(\sqrt[4]{2})^3} =$  (Sol :  $\sqrt[12]{2^{13}}$ )

f)  $\sqrt{2} (\sqrt[4]{2})^3 (\sqrt[3]{2})^2 =$  (Sol :  $\sqrt[12]{2^{23}}$ )

g)  $\frac{(\sqrt[4]{3})^5}{(\sqrt{3})^2 (\sqrt[3]{3})^4} =$  (Sol :  $\frac{1}{\sqrt[12]{3^{13}}}$ )

h)  $\sqrt{2} (\sqrt[4]{2^3 \sqrt{4}})^3 =$  (Sol :  $\sqrt[4]{2^{13}}$ )

i)  $\sqrt{\sqrt{2^6}} =$  (Sol :  $\sqrt{8}$ )

j)  $\sqrt{\sqrt{12}} =$  (Sol :  $\sqrt[4]{12}$ )

k)  $(\sqrt{\sqrt{\sqrt{2}}})^8 =$  (Sol : 2)

l)  $\sqrt[3]{\sqrt[4]{x^5 x^7}} =$  (Sol : x)

m)  $\sqrt[3]{\sqrt[4]{x^{15}}} =$  (Sol :  $\sqrt[4]{x^5}$ )

n)  $(\sqrt[3]{\sqrt[7]{\sqrt{8x^3}}})^7 =$  (Sol :  $\sqrt{2x}$ )

o)  $(\sqrt{\sqrt[3]{5}})^5 (\sqrt[4]{5})^3 =$  (Sol :  $\sqrt[12]{5^{19}}$ )

p)  $\frac{(\sqrt{x})^3}{(\sqrt[3]{4x})^6} =$  (Sol : x)

q)  $\frac{(\sqrt[3]{2})^4 \cdot (\sqrt[4]{8})^3}{\sqrt{(\sqrt[3]{4})^2}} =$  (Sol :  $\sqrt[12]{2^{35}}$ )

r)  $\frac{\sqrt[3]{a^2} \cdot (\sqrt{a^3})^3}{(\sqrt{a})^3 \cdot \sqrt[3]{a^4}} =$  (Sol :  $a^2$ )

s)  $\frac{(\sqrt{27})^3 \cdot \sqrt[3]{9}}{\sqrt[3]{81} \cdot (\sqrt{3})^3} =$  (Sol : 9)

17. Introducir convenientemente factores y simplificar:

a)  $2\sqrt{2} = \sqrt{2^2 \cdot 2} = \sqrt{2^3} = \sqrt{8}$

b)  $2\sqrt{3}$

c)  $2\sqrt{\frac{3}{2}}$  (Sol :  $\sqrt{6}$ )

d)  $3\sqrt{2}$

e)  $3\sqrt{\frac{2}{27}}$  (Sol :  $\sqrt{2/3}$ )

f)  $3^3\sqrt{3}$

g)  $6\sqrt{\frac{5}{12}}$  (Sol :  $\sqrt{15}$ )

h)  $3^4\sqrt{5}$

i)  $ab\sqrt{\frac{c}{ab^3}}$  (Sol :  $\sqrt{\frac{ac}{b}}$ )

j)  $3\sqrt{7}$

k)  $2a\sqrt{\frac{3c}{2a}}$  (Sol :  $\sqrt{6ac}$ )

l)  $\sqrt{x}\sqrt{x} =$  (Sol :  $\sqrt[4]{x^3}$ )

m)  $\sqrt{2 \cdot \sqrt[3]{2}} =$  (Sol :  $\sqrt[3]{4}$ )

n)  $\sqrt{2\sqrt{2\sqrt{2}}} =$  (Sol :  $\sqrt[8]{2^7}$ )

o)  $\sqrt{3\sqrt[3]{3\sqrt{3}}} =$

p)  $\sqrt{2 \cdot \sqrt{2} \cdot \sqrt[4]{2}} =$  (Sol : 2)

q)  $\sqrt{\sqrt[3]{2\sqrt{2} \cdot \sqrt{2}}} =$  (Sol :  $\sqrt{2}$ )

r)  $\left(\sqrt[3]{4\sqrt{2} \sqrt{2}}\right)^3 =$  (Sol : 4)

s)  $\sqrt{3\sqrt{3\sqrt[3]{3} \sqrt[3]{3}}} =$  (Sol : 3)

t)  $\left(\sqrt[3]{\sqrt{3\sqrt{3} \sqrt[3]{3}}}\right)^2 =$  (Sol :  $\sqrt[18]{3^{13}}$ )

u)  $\frac{\sqrt[3]{81} (\sqrt{3})^3}{\sqrt[3]{3\sqrt{3}} \sqrt[3]{9}} =$  (Sol : 9)

v)  $\frac{\sqrt{2\sqrt[3]{2}} \sqrt[3]{16}}{\sqrt{2\sqrt[3]{2\sqrt{2}}} \sqrt[4]{8}} =$  (Sol :  $\sqrt{2}$ )

w)  $\frac{\left(\sqrt{2 \sqrt[3]{2}}\right)^3}{\sqrt{2\sqrt{2}} \sqrt[4]{2}} =$  (Sol : 2)

x)  $\sqrt[4]{\frac{x}{y} \sqrt[3]{\frac{y}{x}}} =$  (Sol :  $\sqrt[8]{xy}$ )

$$y) \frac{(\sqrt[3]{a^2b})^2}{\sqrt{a} \sqrt[3]{a} \sqrt{b}} =$$

$$(Sol : \sqrt[12]{a^8 b})$$

$$z) \frac{(\sqrt[3]{3} \sqrt{3})^3}{\sqrt{3} \sqrt[3]{3}} =$$

$$(Sol : \sqrt[6]{3^{11}})$$

$$\alpha) \frac{\sqrt{125} (\sqrt[3]{5})^2}{\sqrt{5} \sqrt[3]{25}} =$$

$$(Sol : \sqrt[3]{5^4})$$

$$\beta) \sqrt{ab \sqrt{8ab} \sqrt{4a^2b^2}} =$$

$$(Sol : 2ab)$$

18. Realizar las siguientes operaciones de dos formas distintas, y comprobar que se obtiene el mismo resultado:

- operando, teniendo en cuenta las propiedades de las raíces
- pasando a potencia de exponente fraccionario, y aplicando a continuación las propiedades de las potencias.

$$a) \frac{1}{2} \sqrt{2} \sqrt[4]{2} =$$

$$(Sol : \frac{1}{\sqrt[4]{2}})$$

$$b) \frac{\sqrt[3]{a^2}}{a\sqrt{a}} =$$

$$(Sol : \frac{1}{\sqrt[6]{a^5}})$$

$$c) \frac{\sqrt[3]{a^2} a^3}{a^2 \sqrt{a}} =$$

$$(Sol : \sqrt[6]{a^7})$$

$$d) \sqrt{2 \sqrt[3]{2 \sqrt{2}}} =$$

$$(Sol : \sqrt[4]{8})$$

19. Extraer factores y simplificar cuando proceda:

$$a) \sqrt{8} = \sqrt{2^3} = \sqrt{2^2 \cdot 2} = 2\sqrt{2}$$

$$q) \sqrt[3]{2592}$$

$$(Sol : 6 \sqrt[3]{12})$$

$$b) \sqrt{18}$$

$$c) \sqrt{98}$$

$$r) \sqrt[5]{279936}$$

$$(Sol : 6 \sqrt[5]{36})$$

$$d) \sqrt{32}$$

$$e) \sqrt{60}$$

$$s) (\sqrt{\sqrt{2}})^{10}$$

$$(Sol : 4\sqrt{2})$$

$$f) \sqrt{72}$$

$$g) \sqrt{128}$$

$$t) \sqrt[3]{500}$$

$$(Sol : 5 \sqrt[3]{4})$$

$$h) \sqrt{162}$$

$$i) \sqrt{200}$$

$$u) \sqrt[3]{32x^4}$$

$$(Sol : 2x \sqrt[3]{4x})$$

$$j) \sqrt{12}$$

$$v) \sqrt{1936}$$

$$(Sol : 44)$$

$$k) \sqrt{27}$$

$$w) \sqrt{3,24}$$

$$(Sol : 1,8)$$

$$l) \sqrt{48}$$

$$x) \sqrt{529}$$

$$(Sol : 23)$$

$$m) \sqrt{75}$$

$$y) \sqrt{676}$$

$$(Sol : 26)$$

$$n) \sqrt{108}$$

$$z) \sqrt[3]{128a^2b^7}$$

$$(Sol : 4b^2 \sqrt[3]{2a^2b})$$

$$o) \sqrt[3]{3^4 5^5}$$

$$(Sol : 15 \sqrt[3]{75})$$

$$p) \sqrt[4]{80}$$

$$(Sol : 2 \sqrt[4]{5})$$

$$\alpha) \sqrt[3]{81a^3b^5c}$$

$$(Sol : 3ab \sqrt[3]{3b^2c})$$

$\beta) \sqrt[5]{64}$	$(\text{Sol} : 2 \sqrt[5]{2})$	$\vartheta) \frac{\sqrt{11}\sqrt{132}}{132}$	
$\gamma) \sqrt[3]{16x^6}$			$(\text{Sol} : \sqrt{3}/6)$
$\delta) \sqrt{\frac{28x^5}{75y^3}}$	$(\text{Sol} : \frac{2x^2}{5y} \sqrt{\frac{7x}{3y}})$	$\iota) \sqrt{25 + \frac{25}{4}}$	
$\epsilon) \frac{11\sqrt{132}}{132}$	$(\text{Sol} : \sqrt{33}/6)$	$\kappa) \sqrt{12} \cdot \sqrt{3} \cdot \sqrt{50} =$	$(\text{Sol} : 5\sqrt{5}/2)$
$\zeta) \frac{\sqrt{396}}{66}$	$(\text{Sol} : \sqrt{11}/11)$	$\lambda) 5 \sqrt[3]{\frac{3}{2}} \sqrt[3]{\frac{4}{81}} =$	$(\text{Sol} : \frac{5}{3} \sqrt[3]{2})$
$\eta) \sqrt{\frac{3a^2}{4}}$	$(\text{Sol} : \frac{a}{2} \sqrt{3})$		

20. Sumar los siguientes radicales, reduciéndolos previamente a radicales semejantes (Fíjate en el 1<sup>er</sup> ejemplo):

a)  $\sqrt{2} + \sqrt{8} + \sqrt{18} - \sqrt{32} = \sqrt{2} + \sqrt{2^3} + \sqrt{3^2 \cdot 2} - \sqrt{2^5} = \sqrt{2} + 2\sqrt{2} + 3\sqrt{2} - 2^2\sqrt{2} = \sqrt{2} + 2\sqrt{2} + 3\sqrt{2} - 4\sqrt{2} = 2\sqrt{2}$

b)  $\sqrt{5} + \sqrt{45} + \sqrt{180} - \sqrt{80}$   $(\text{Soluc} : 6\sqrt{5})$

c)  $\sqrt{24} - 5\sqrt{6} + \sqrt{486}$   $(\text{Soluc} : 6\sqrt{6})$

d)  $\sqrt[3]{54} - 2 \cdot \sqrt[3]{16}$   $(\text{Soluc} : -\sqrt[3]{2})$

e)  $27\sqrt{3} - 5\sqrt{27} - 9\sqrt{12}$   $(\text{Soluc} : -6\sqrt{3})$

f)  $\sqrt{75} - \sqrt{20} - \sqrt{12} + \sqrt{45}$   $(\text{Soluc} : 3\sqrt{3} + \sqrt{5})$

g)  $2\sqrt{8} + 5\sqrt{72} - 7\sqrt{18} - \sqrt{50}$  (Soluc:  $8\sqrt{2}$ )

h)  $5\sqrt[6]{256} - 2\sqrt[3]{16} - \sqrt[3]{128} =$  (Soluc:  $2\sqrt[3]{2}$ )

i)  $\sqrt{32} + 2\sqrt{3} - \sqrt{8} + \sqrt{2} - 2\sqrt{12}$  (Soluc:  $3\sqrt{2} - 2\sqrt{3}$ )

j)  $3\sqrt{24} - \frac{1}{3}\sqrt{54} + \sqrt{150}$  (Soluc:  $10\sqrt{6}$ )

k)  $5\sqrt{2} + 4\sqrt{8} + 3\sqrt{18} + 2\sqrt{32} + \sqrt{50}$  (Soluc:  $35\sqrt{2}$ )

l)  $\sqrt{20} - \frac{1}{5}\sqrt{5} + \sqrt{45}$  (Soluc:  $\frac{24}{5}\sqrt{5}$ )

m)  $2\sqrt{108} - \sqrt{75} - \sqrt{27} - \sqrt{12} - \sqrt{3}$  (Soluc:  $\sqrt{3}$ )

n)  $\sqrt{128} + 5\sqrt{12} - 2\sqrt{18} - 3\sqrt{27} - \sqrt{2} =$  (Soluc:  $\sqrt{2} + \sqrt{3}$ )

o)  $\sqrt{5} + \sqrt{\frac{45}{4}}$  (Soluc:  $\frac{5}{2}\sqrt{5}$ )

p)  $\sqrt{\frac{2}{3}} + \sqrt{\frac{18}{75}}$  (Soluc:  $\frac{8}{5}\sqrt{\frac{2}{3}}$ )

q)  $\sqrt{\frac{1}{2}} + 3\sqrt{\frac{1}{8}}$  (Soluc:  $\frac{5}{2}\sqrt{\frac{1}{2}}$ )

r)  $\sqrt{\frac{3}{16}} - 4\sqrt{12}$  (Soluc:  $-\frac{31}{4}\sqrt{3}$ )

s)  $\sqrt{\frac{5}{12}} - \sqrt{\frac{10}{6}}$  (Soluc:  $-\frac{1}{2}\sqrt{\frac{5}{3}}$ )

t)  $\sqrt{50a} - \sqrt{18a}$  (Soluc:  $2\sqrt{2a}$ )

u)  $5\sqrt{\frac{3}{4}} + \sqrt{27} - 4\sqrt{3} - \sqrt{300} =$  (Soluc:  $-\frac{17}{2}\sqrt{3}$ )

$$v) \sqrt{3} - \frac{2\sqrt{27}}{3} + \frac{5\sqrt{243}}{9}$$

(Soluc:  $4\sqrt{3}$ )

$$w) 6\sqrt[6]{4} - \frac{1}{3}\sqrt[3]{16} - 3\sqrt[3]{8} + 5\sqrt[3]{\frac{2}{27}} =$$

(Soluc:  $4\sqrt[3]{2}$ )

$$x) 2\sqrt[4]{\frac{2}{81}} - \sqrt[8]{4} + 2\sqrt[4]{32} =$$

(Soluc:  $\frac{11}{3}\sqrt[4]{2}$ )

$$y) \frac{2}{3}\sqrt[3]{16} + 2\sqrt[3]{2} - \frac{2}{3}\sqrt[3]{128} + \sqrt[3]{\frac{2}{27}} =$$

(Soluc:  $\sqrt[3]{2}$ )

$$z) \frac{3}{2}\sqrt[3]{40} - \frac{3}{2}\sqrt[3]{5} + \frac{5}{2}\sqrt[3]{320} - \frac{3}{2}\sqrt[3]{1080} + \sqrt[3]{\frac{135}{8}} =$$

(Soluc:  $4\sqrt[3]{5}$ )

$$\alpha) \frac{1}{2}\sqrt[3]{81} - \sqrt[3]{3} + \sqrt[3]{24} - \sqrt[3]{\frac{3}{8}} =$$

(Soluc:  $2\sqrt[3]{3}$ )

$$\beta) \sqrt{9x+9} - \sqrt{4x+4}$$

(Soluc:  $\sqrt{x+1}$ )

**RECORDAR LAS IGUALDADES NOTABLES:**

$(A + B)^2 = A^2 + 2AB + B^2$ $(A - B)^2 = A^2 - 2AB + B^2$ $(A + B)(A - B) = A^2 - B^2$
--

21. Calcular, dando el resultado lo más simplificado posible:

a)  $(2\sqrt{2})^2 =$

(Soluc: 8)

b)  $(3\sqrt{5})^2 =$

(Soluc: 45)

**c)**  $(1 + \sqrt{2})^2 =$  (Soluc:  $3 + 2\sqrt{2}$  )

**d)**  $(\sqrt{2} + \sqrt{3})^2 =$  (Soluc:  $5 + 2\sqrt{6}$  )

**e)**  $(\sqrt{3} - \sqrt{2})^2 =$  (Soluc:  $5 - 2\sqrt{6}$  )

**f)**  $(\sqrt{2} + 1)(\sqrt{2} - 1) =$  (Soluc: 1)

**g)**  $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) =$  (Soluc: 1)

**h)**  $(1 + \sqrt{2})(1 - \sqrt{8}) =$  (Soluc:  $-3 - \sqrt{2}$  )

**i)**  $(2 - \sqrt{3})(1 + \sqrt{12}) =$   
(Soluc:  $-4 + 3\sqrt{3}$  )

**j)**  $2\sqrt{3} \cdot 3\sqrt{2} =$  (Soluc:  $6\sqrt{6}$  )

**k)**  $2\sqrt{8} \cdot 8\sqrt{2} =$  (Soluc: 64)

**l)**  $3\sqrt{6} \cdot 2\sqrt{3} =$  (Soluc:  $18\sqrt{2}$  )

**m)**  $2\sqrt{15} \cdot 3\sqrt{15} =$  (Soluc: 90 )

**n)**  $(5\sqrt{3})^2 =$  (Soluc: 75)

**o)**  $(5 + \sqrt{3})^2 =$  (Soluc:  $28 + 10\sqrt{3}$  )

**p)**  $(5 - \sqrt{3})^2 =$  (Soluc:  $28 - 10\sqrt{3}$  )

**q)**  $(5 + \sqrt{3})(5 - \sqrt{3}) =$  (Soluc: 22)

**r)**  $(\sqrt{5} + \sqrt{3})^2 =$  (Soluc:  $8 + 2\sqrt{15}$  )

**s)**  $(\sqrt{5} - \sqrt{3})^2 =$  (Soluc:  $8 - 2\sqrt{15}$  )

**t)**  $(2\sqrt{3} + 5)^2 =$  (Soluc:  $37 + 20\sqrt{3}$  )

u)  $(3\sqrt{2} + 2\sqrt{3})^2 =$  (Soluc:  $30 + 12\sqrt{6}$  )

v)  $(2\sqrt{3} + 3\sqrt{2})(2\sqrt{3} - 3\sqrt{2}) =$  (Soluc: -6)

w)  $\sqrt{2}(\sqrt{2} - 4) =$  (Soluc:  $2 - 4\sqrt{2}$  )

x)  $(2 - \sqrt{3})\sqrt{3} =$  (Soluc:  $2\sqrt{3} - 3$  )

y)  $(3\sqrt{2} + 2)(2\sqrt{3} - \sqrt{6}) =$   
(Soluc:  $4\sqrt{6} - 2\sqrt{3}$  )

z)  $(2\sqrt{5} - 5)\sqrt{5} =$  (Soluc:  $10 - 5\sqrt{5}$  )

α)  $(\sqrt{2} - 3\sqrt{3})(\sqrt{2} + 5\sqrt{3}) =$   
(Soluc:  $-43 + 2\sqrt{6}$  )

β)  $(2\sqrt{8} + 3\sqrt{2})(3\sqrt{8} - 2\sqrt{2}) =$   
(Soluc: 56)

γ)  $(2\sqrt{5} - 5\sqrt{2})(2\sqrt{5} + 5\sqrt{2}) =$  (Soluc: -30)

δ)  $(2\sqrt{5} - 5\sqrt{2})(3\sqrt{2} + 2) =$   
(Soluc:  $-30 + 6\sqrt{10} + 4\sqrt{5} - 10\sqrt{2}$  )

ε)  $(2\sqrt{27} - 3)(1 + \sqrt{3}) =$   
(Soluc:  $1 + 3\sqrt{3}$  )

ζ)  $(3\sqrt{8} - 4\sqrt{2})(2\sqrt{2} - 5\sqrt{8}) =$   
(Soluc: -32)

η)  $(\sqrt{6} + \sqrt{5})^2 + (\sqrt{6} - \sqrt{5})^2 =$   
(Soluc: 22)

$$\theta) (\sqrt{7} + \sqrt{3})^2 (5 - \sqrt{21}) =$$

$$\iota) (3\sqrt{8} + 2\sqrt{2})(2\sqrt{8} - 3\sqrt{2}) =$$

(Soluc: 16)

$$\kappa) (2\sqrt{3} - 3\sqrt{2})^2 =$$

(Soluc:  $30 - 12\sqrt{6}$ )

$$\lambda) (\sqrt{2} + \sqrt{3} - \sqrt{5})(\sqrt{3} - \sqrt{2}) =$$

### RACIONALIZACIÓN:

22. Racionalizar denominadores, y simplificar:

$$\text{a)} \frac{1}{\sqrt{5}}$$

(Soluc:  $\frac{\sqrt{5}}{5}$ )

$$\text{b)} \frac{5}{2\sqrt{3}}$$

(Soluc:  $\frac{5\sqrt{3}}{6}$ )

$$\text{c)} \frac{5}{3\sqrt{5}}$$

(Soluc:  $\frac{\sqrt{5}}{3}$ )

$$\text{d)} \frac{\sqrt{2}}{\sqrt{3}}$$

(Soluc:  $\frac{\sqrt{6}}{3}$ )

$$\text{e)} \sqrt{\frac{3}{2}}$$

(Soluc:  $\frac{\sqrt{6}}{2}$ )

$$\text{f)} \frac{2 - \sqrt{2}}{\sqrt{7}}$$

(Soluc:  $\frac{2\sqrt{7} - \sqrt{14}}{7}$ )

$$\text{g)} \frac{2 + \sqrt{2}}{\sqrt{2}}$$

(Soluc:  $\sqrt{2} + 1$ )

$$\text{h)} \frac{4}{\sqrt{6}}$$

(Soluc:  $\frac{2\sqrt{6}}{3}$ )

$$\text{i)} \frac{1}{\sqrt{27}} =$$

(Soluc:  $\frac{\sqrt{3}}{9}$ )

$$\text{j) } \frac{3}{2\sqrt{3}} \quad (\text{Soluc: } \frac{\sqrt{3}}{2})$$

$$\text{k) } \frac{12}{\sqrt{8}} = \quad (\text{Soluc: } 3\sqrt{2})$$

$$\text{l) } \frac{\sqrt{2}-4}{3\sqrt{2}} = \quad (\text{Soluc: } \frac{1}{3} - \frac{2\sqrt{2}}{3})$$

$$\text{m) } \frac{15\sqrt{3}}{2\sqrt{5}} \quad (\text{Soluc: } \frac{3\sqrt{15}}{2})$$

$$\text{n) } \frac{\sqrt{3}+3}{2\sqrt{3}} = \quad (\text{Soluc: } \frac{1+\sqrt{3}}{2})$$

$$\text{o) } \frac{-2\sqrt{7}}{7\sqrt{2}} \quad (\text{Soluc: } -\frac{\sqrt{14}}{7})$$

$$\text{p) } \frac{\sqrt{11}}{\sqrt{12}} \quad (\text{Soluc: } \frac{\sqrt{33}}{6})$$

$$\text{q) } \left(\frac{1}{\sqrt{2}}\right)^3 \quad (\text{Soluc: } \frac{\sqrt{2}}{4})$$

$$\text{r) } \frac{(1+\sqrt{2})^2+1}{\sqrt{2}} \quad (\text{Soluc: } 2+2\sqrt{2})$$

$$\text{s) } \frac{1-(1-\sqrt{2})^2}{\sqrt{2}} \quad (\text{Soluc: } 2-\sqrt{2})$$

$$\text{t) } \frac{\sqrt{81+\frac{81}{4}}}{\sqrt{5}} \quad (\text{Soluc: } \frac{9}{2})$$

$$\text{u) } \frac{2}{\sqrt{5}} - \frac{2}{\sqrt{125}} \quad (\text{Soluc: } \frac{8\sqrt{5}}{25})$$

$$\text{v) } \left(\frac{1}{\sqrt{3}}\right)^3 \quad (\text{Soluc: } \frac{\sqrt{3}}{9})$$

$$w) \sqrt{\frac{5+\sqrt{5}}{10}}$$

$$(Soluc: \frac{\sqrt{50+10\sqrt{5}}}{10})$$

$$x) \frac{2\sqrt{6}}{6\sqrt{2}}$$

$$(Soluc: \frac{\sqrt{3}}{3})$$

$$y) \frac{3\sqrt{10}}{5\sqrt{6}}$$

$$(Soluc: \frac{\sqrt{15}}{5})$$

23. Racionalizar denominadores, y simplificar:

$$a) \frac{1}{\sqrt[3]{2}}$$

$$(Soluc: \frac{\sqrt[3]{4}}{2})$$

$$b) \frac{3}{\sqrt[5]{9}}$$

$$(Soluc: \sqrt[5]{27})$$

$$c) \frac{8}{\sqrt[6]{8}}$$

$$(Soluc: 4\sqrt{2})$$

$$d) \frac{10}{3\sqrt[4]{125}}$$

$$(Soluc: \frac{2}{3}\sqrt[4]{5})$$

$$e) \frac{\sqrt[5]{25}}{5\sqrt[3]{5}}$$

$$(Soluc: \frac{\sqrt[15]{5}}{5})$$

$$f) \frac{10}{\sqrt[5]{128}} =$$

$$(Soluc: \frac{5}{2}\sqrt[5]{8})$$

$$g) \frac{\sqrt{3}}{5\sqrt[5]{27}}$$

$$(Soluc: \frac{\sqrt[10]{3^9}}{15})$$

$$h) \frac{3\sqrt[5]{9}}{2\sqrt[3]{243}}$$

$$(Soluc: \frac{\sqrt[15]{3^{11}}}{6})$$

$$i) \frac{5\sqrt{15}}{\sqrt[3]{15}}$$

$$(Soluc: 5\sqrt[6]{15})$$

$$j) \frac{\sqrt{3}}{\sqrt[5]{9}}$$

$$(Soluc: \sqrt[10]{3})$$

k)  $\frac{\sqrt{2}}{\sqrt[3]{2}}$  (Soluc:  $\sqrt[10]{8}$ )

l)  $\frac{3}{\sqrt{\sqrt[3]{3}}}$  (Soluc:  $\sqrt[6]{243}$ )

m)  $\frac{4}{\sqrt[4]{64}}$  (Soluc:  $\sqrt{2}$ )

n)  $\frac{x}{\sqrt{x}} + \frac{x}{\sqrt[3]{x^2}}$  (Soluc:  $\sqrt{x} + \sqrt[3]{x}$ )

24. Racionalizar denominadores, y simplificar:

a)  $\frac{1+\sqrt{2}}{1-\sqrt{3}}$  (Soluc:  $-\frac{1+\sqrt{2}+\sqrt{3}+\sqrt{6}}{2}$ )

b)  $\frac{9}{\sqrt{7}-\sqrt{3}}$  (Soluc:  $\frac{9}{4}\sqrt{7} + \frac{9}{4}\sqrt{3}$ )

c)  $\frac{4(\sqrt{5}+2)}{\sqrt{5}-1}$  (Soluc:  $7+3\sqrt{5}$ )

d)  $\frac{3(\sqrt{7}+1)}{\sqrt{7}+2}$  (Soluc:  $5-\sqrt{7}$ )

e)  $\frac{\sqrt{3}+1}{\sqrt{3}-1}$  (Soluc:  $2+\sqrt{3}$ )

f)  $\frac{1+\sqrt{2}}{2-\sqrt{2}}$  (Soluc:  $2+\frac{3}{2}\sqrt{2}$ )

g)  $\frac{5-7\sqrt{3}}{1+\sqrt{3}}$  (Soluc:  $-13+6\sqrt{3}$ )

$$\text{h) } \frac{3\sqrt{2}-2\sqrt{3}}{6+\sqrt{6}} =$$

$$(\text{Soluc: } \frac{4}{5}\sqrt{2}-\frac{3}{5}\sqrt{3})$$

$$\text{i) } \frac{7}{7-\sqrt{7}}$$

$$(\text{Soluc: } \frac{7}{6}+\frac{\sqrt{7}}{6})$$

$$\text{j) } \frac{4}{\sqrt{3}+\sqrt{2}}$$

$$(\text{Soluc: } 4\sqrt{3}-4\sqrt{2})$$

$$\text{k) } \frac{\sqrt{2}+1}{3\sqrt{2}-2}$$

$$(\text{Soluc: } \frac{4}{7}+\frac{5}{14}\sqrt{2})$$

$$\text{l) } \frac{\sqrt{3}}{\sqrt{3}+\sqrt{2}}$$

$$(\text{Soluc: } 3-\sqrt{6})$$

$$\text{m) } \frac{7}{\sqrt{8}-2}$$

$$(\text{Soluc: } \frac{7}{2}+\frac{7}{2}\sqrt{2})$$

$$\text{n) } \frac{2\sqrt{3}-5}{\sqrt{3}-2} =$$

$$(\text{Soluc: } 4+\sqrt{3})$$

$$\text{o) } \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

$$(\text{Soluc: } -2-\sqrt{3})$$

$$\text{p) } \frac{\sqrt{5}+2\sqrt{3}}{2\sqrt{5}-\sqrt{3}} =$$

$$(\text{Soluc: } \frac{16}{17}+\frac{5}{17}\sqrt{15})$$

$$\text{q) } \frac{3\sqrt{2}-4}{3\sqrt{2}+4} =$$

$$(\text{Soluc: } 17-12\sqrt{2})$$

$$\text{r) } \frac{2\sqrt{8}-3\sqrt{2}}{2\sqrt{8}+3\sqrt{2}} =$$

$$(\text{Soluc: } 1/7)$$

$$\text{s)} \frac{12 - 5\sqrt{3}}{2\sqrt{3} - 3} =$$

(Soluc:  $2 + 3\sqrt{3}$  )

$$\text{t)} \frac{(\sqrt{2 + \sqrt{8}})^2}{2 - \sqrt{2}} =$$

(Soluc:  $4 + 3\sqrt{2}$  )

$$\text{u)} \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} =$$

(Soluc:  $4 + \sqrt{15}$  )

$$\text{v)} \frac{3\sqrt{5} - 4}{\sqrt{5} - 2} =$$

(Soluc:  $7 + 2\sqrt{5}$  )

$$\text{w)} \frac{24 - 13\sqrt{3}}{2\sqrt{3} - 3} =$$

(Soluc:  $-2 + 3\sqrt{3}$  )

$$\text{x)} \frac{2\sqrt{2}}{\sqrt{3} - \sqrt{2}} =$$

$$\text{y)} \frac{4 - \sqrt{6}}{\sqrt{6} - 2} =$$

(Soluc:  $1 + \sqrt{6}$  )

$$\text{z)} \frac{2 - \sqrt{8}}{2 + \sqrt{2}} =$$

(Soluc:  $4 - 3\sqrt{2}$  )

$$\alpha) \frac{-\sqrt{3} - 1}{1 - \sqrt{3}} =$$

(Soluc:  $2 + \sqrt{3}$  )

$$\beta) \frac{9 + 4\sqrt{3}}{3(4 - \sqrt{3})} =$$

(Soluc:  $\frac{48 + 25\sqrt{3}}{39}$  )

$$\gamma) \frac{\sqrt{2}+4}{2-\sqrt{2}} =$$

(Soluc:  $3\sqrt{2}+5$ )

$$\delta) \sqrt{x} + \frac{x}{2\sqrt{x}} =$$

(Soluc:  $\frac{3}{2}\sqrt{x}$ )

$$\epsilon) \frac{2\sqrt{3}-3}{2\sqrt{3}+3} + \frac{12}{\sqrt{3}} =$$

(Soluc: 7)

$$\zeta) \frac{17-9\sqrt{3}}{3\sqrt{3}-5} - \frac{9}{\sqrt{3}} =$$

(Soluc: 2)

25. ¿V o F? Razonar algebraicamente la respuesta:

$$\text{a)} \frac{5+\sqrt{3}}{5} = 1+\sqrt{3}$$

(Soluc: F)

$$\text{b)} \frac{5+\sqrt{3}}{5} = \sqrt{3}$$

(Soluc: F)

$$\text{c)} \frac{2+\sqrt{3}}{2} = 1 + \frac{\sqrt{3}}{2}$$

(Soluc: V)

$$\text{d)} \frac{5+\sqrt{2}+\sqrt{3}}{5} = 1 + \sqrt{2} + \sqrt{3}$$

(Soluc: F)

$$\text{e)} \frac{3+6\sqrt{2}}{3} = 1+2\sqrt{2}$$

(Soluc: V)

$$\text{f)} \frac{4+14\sqrt{5}}{6} = \frac{2+7\sqrt{5}}{3}$$

(Soluc: V)

$$\text{g)} (\sqrt{2} + \sqrt{3})^2 = 2 + 3 = 5$$

(Soluc: F)

$$\text{h)} \sqrt{16+9} = 4+3 = 7$$

(Soluc: F)