IES MURILLO

Maths 4th ESO

EXAM 1_3 (Algebra - Equations)

1. Solve the following: (1.5 points)

$$\left.\begin{array}{l} y^2 - 2y + 1 = x \\ \sqrt{x} + y = 5 \end{array}\right\}$$

2. Solve the following: (4 points)

a) $\frac{1}{x^2 - x} - \frac{1}{x - 1} = 0$

b) $x - \sqrt{3x - 5} = 3$

c) $2x^3 + 5x^2 + x - 2 = 0$

d) $\frac{2}{x^2 - 9} = \frac{x^2 - 16}{72}$

3. Mr. B. has $20,000 to invest. He invests part at 6%, the rest at 7%, and he earns $1,280 interest. How much did he invest at each rate? (1.5 p)

4. The Lakers scored a total of 80 points in a basketball game against the Bulls. The Lakers made a total of 37 two-points and three-points baskets. How many three-points shots did the Lakers make? How many two-points shots did the Lakers make? (1.5 p)

5. The length of a rectangle is 7 metres less than twice the width. Find the dimensions if the area is 60 square metres. (1.5 p)

SOLUTION

1. $\left.\begin{matrix} y^2-2y+1=x \\ \sqrt{x}+y=5 \end{matrix}\right\} \rightarrow y=5-\sqrt{x} \rightarrow (5-\sqrt{x})^2-2(5-\sqrt{x})+1=x$

$25-2\cdot 5\sqrt{x}+x-10+2\sqrt{x}+1=x \rightarrow 25-10\sqrt{x}+2\sqrt{x}-10+1=0$

$16=8\sqrt{x} \rightarrow 2=\sqrt{x} \rightarrow x=4 \Rightarrow y=5-\sqrt{x}=5-2=3$ Solution: x = 4, y = 3

2. Solve the following:

a) $\frac{1}{x^2-x}-\frac{1}{x-1}=0 \rightarrow \frac{1}{x(x-1)}-\frac{x}{x(x-1)}=\frac{0}{x(x-1)} \rightarrow 1-x=0 \Rightarrow x=1$

But x = 1 is not a solution. Denominator cancels $\frac{1}{0}-\frac{1}{-1}=0$

So, it doesn't have any solution

b) $x-\sqrt{3x-5}=3 \rightarrow x-3=\sqrt{3x-5} \rightarrow (x-3)^2=\left(\sqrt{3x-5}\right)^2$

$x^2-6x+9=3x-5 \Rightarrow x^2-9x+14=0 \rightarrow x=\frac{9\pm\sqrt{81-56}}{2}=\begin{cases}7\\2\end{cases}$

Checking: $\begin{cases}7-\sqrt{21-5}=3 \rightarrow 7-4=3 \text{ YES}\\ 2-\sqrt{6-5}=3 \rightarrow 2-2\neq 3 \quad \text{NO}\end{cases}$ Solution x = 7

c) $2x^3+5x^2+x-2=0$ $\quad Factors(2)=1,-1,2,-2$

$P(1)=2+5x^2+1-2\neq 0$; $P-(1)=-2+5x^2-1-2=0$

Ruffini

	2	+5	+1	-2
-1		-2	-3	+2
	2	+3	-2	0

$2x^2+3x-2=0$

$x=\frac{-3\pm\sqrt{9+16}}{4}=\begin{cases}\frac{1}{2}\\-2\end{cases}$

$2x^3+5x^2+x-2=0 \rightarrow 2(x+1)(x+2)\left(x-\frac{1}{2}\right)=0 \Rightarrow$ Solution : $\begin{cases}x_1=-1\\x_2=-2\\x_3=\frac{1}{2}\end{cases}$

d) $\frac{2}{x^2-9}=\frac{x^2-16}{72} \rightarrow 144=(x^2-16)(x^2-9) \rightarrow 144=x^4-16x^2-9x^2+144$

$x^4-25x^2=0 \Rightarrow x^2(x^2-25)=0 \Rightarrow \begin{cases}x^2=0 \rightarrow x=0\\ x^2-25=0 \rightarrow x=\pm 5\end{cases}$

3. Mr. B. has $20,000 to invest. He invests part at 6%, the rest at 7%, and he earns $1,280 interest. How much did he invest at each rate?

Investment at 6% - x $\quad x+y=20000$

Investment at 7% - y $\quad \frac{6}{100}x+\frac{7}{100}y=1280$

$$\left.\begin{array}{l} x+y=20000 \\ 0.06x+0.07y=1280 \end{array}\right\} \rightarrow y=20000-x \Rightarrow 0.06x+0.07(20000-x)=1280$$

$$0.06x+1400-0.07x=1280 \Rightarrow -0.01x=-120 \Rightarrow x=12000$$

He invested $12000 at 6% and $8000 at 7%

4. The Lakers scored a total of 80 points in a basketball game against the Bulls. The Lakers made a total of 37 two-points and three-points baskets. How many three-points shots did the Lakers make? How many two-points shots did the Lakers make?

Two-points shots – x $\quad x+y=37$

Three-points shots – y $\quad 2x+3y=80$

$$\left.\begin{array}{l} x+y=37 \\ 2x+3y=80 \end{array}\right\} \rightarrow \left.\begin{array}{r} -2x-2y=-74 \\ 2x+3y=80 \end{array}\right\} \Rightarrow y=6 \rightarrow x=37-6=31$$

Answer: They made 6 three-points shots and 31 two-points shots

5. The length of a rectangle is 7 metres less than twice the width. Find the dimensions if the area is 60 square metres.

Width x Length y

$$\left.\begin{array}{l} y=2x-7 \\ x\cdot y=60 \end{array}\right\} \Rightarrow x\cdot(2x-7)=60 \rightarrow 2x^2-7x-60=0 \rightarrow x=\frac{7\pm\sqrt{49+480}}{4}=\begin{cases}\frac{15}{2}\\ -4\end{cases}$$

So $x=\frac{15}{2} \Rightarrow y=2\cdot\frac{15}{2}-7=8$

Answer: lenght 8 metres and width 7.5 metres