IES MURILLO

Maths 4th ESO

EXAM 2_1 (Algebra: Inequalities, Word Problems)

1. Solve the following inequality and draw the solutions on the number line:

$$\frac{3-x}{4}-\frac{x+1}{6}<2-\frac{2-x}{3}$$ (1 point)

2. Solve the following systems of inequalities: (3 points)

a) $$\left.\begin{array}{l}\frac{x-15}{2}\le 5-2x\\ 2-x<\frac{1-x}{2}\end{array}\right\}$$

b) $$\left.\begin{array}{l}2x-3(x+1)\ge 2\\ -2(x-2)+5x<1\end{array}\right\}$$

c) $$\left.\begin{array}{l}x-2y\ge 2-y\\ -3x<5+y\end{array}\right\}$$

3. Solve: (1.5 points)

$$\left.\begin{array}{l}(1+x^2)y^2=5\\ 4x-y=0\end{array}\right\}$$

4. Some kilograms of olive oil, which costs €5/kg, are combined with sunflower oil which costs €2.5/kg, to obtain 40 kg of a mixture which costs E3.25/kg How many kilograms of each type will we have to put into the mixture? (1.5 points)

5. A furniture shop sold a total of 315 sofas and tables. A sofa sold for €2300 and a table for €890 euro. The total sales were € 401 610. How many tables were sold? (1.5 points)

6. The area of a rectangle is 91 cm^2 and its perimeter is 40 cm. Find the base and the height of the rectangle. (1.5 points)

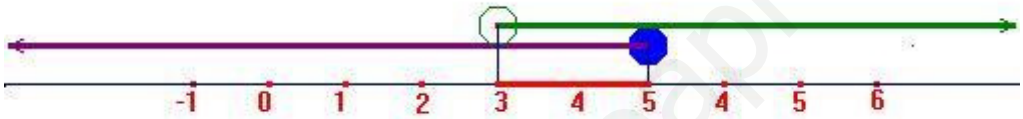
SOLUTION

1. $\frac{3-x}{4}-\frac{x+1}{6}<2-\frac{2-x}{3}\rightarrow\frac{9-3x}{12}-\frac{2x+2}{12}<\frac{24}{12}-\frac{8-4x}{12}$

$9-3x-2x-2<24-8+4x\rightarrow -9x<9\rightarrow 9x>-9\rightarrow x>-1$

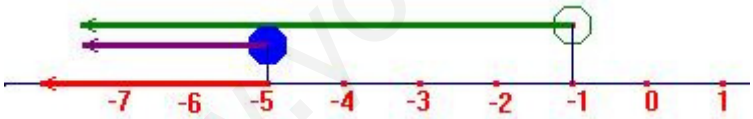
Solution: $(-1,+\infty)$

2. a) $\left.\begin{array}{l}\frac{x-15}{2}\le 5-2x\\ 2-x<\frac{1-x}{2}\end{array}\right\}\rightarrow\left.\begin{array}{l}x-15\le 10-4x\\ 4-2x<1-x\end{array}\right\}\rightarrow\left.\begin{array}{l}x+4x\le 10+15\\ -2x+x<1-4\end{array}\right\}\rightarrow\left.\begin{array}{l}x\le 5\\ x>3\end{array}\right\}$



Solution: $(3,5]$

b) $\left.\begin{array}{l}2x-3(x+1)\ge 2\\ -2(x-2)+5x<1\end{array}\right\}\rightarrow\left.\begin{array}{l}2x-3x-3\ge 2\\ -2x+4+5x<1\end{array}\right\}\rightarrow\left.\begin{array}{l}-x\ge 5\\ 3x<-3\end{array}\right\}\rightarrow\left.\begin{array}{l}x\le -5\\ x<-1\end{array}\right\}$



Solution: $(-\infty,-5]$

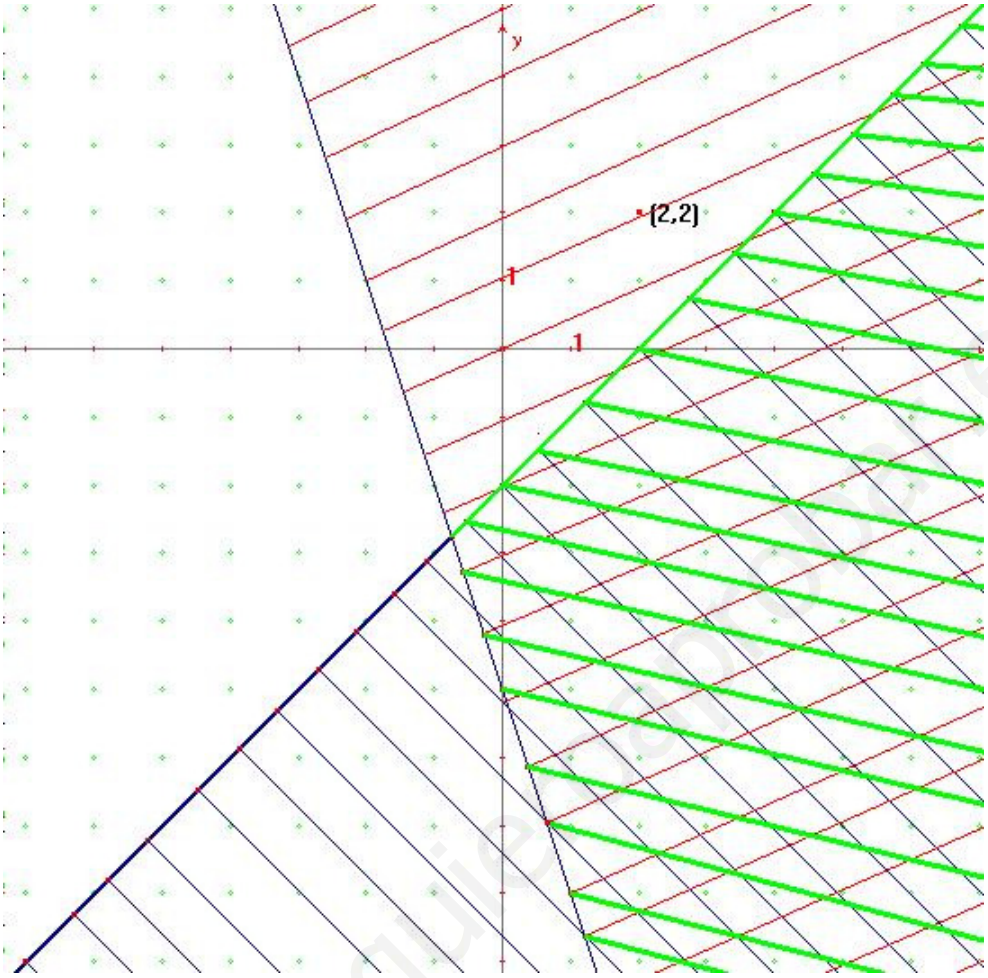
c) $\left.\begin{array}{l}x-2y\ge 2-y\\ -3x<5+y\end{array}\right\}\rightarrow\left\{\begin{array}{l}x-2y=2-y\\ -3x=5+y\end{array}\right.\rightarrow\left\{\begin{array}{l}y=x-2\\ y=-3x-5\end{array}\right.$

$y=x-2$	x	0	2
	y	-2	0

$y=-3x-5$	x	0	-2
	y	-5	1

Point (2,2) $\left.\begin{array}{l}2-2\cdot 2\ge 2-2\\ -3\cdot 2<5+2\end{array}\right\}\rightarrow\left\{\begin{array}{l}-2\ge 0\rightarrow NO\\ -6<7\rightarrow SI\end{array}\right.$

Solution: GREEN



3. $\left.\begin{array}{l}(1+x^2)y^2=5\\ 4x-y=0\end{array}\right\}\rightarrow\left.\begin{array}{l}y^2+x^2y^2=5\\ y=4x\end{array}\right\}\rightarrow(4x)^2+x^2(4x)^2=5$

$16x^4+16x^2-5=0\rightarrow z=x^2\rightarrow 16z^2+16z-5=0$

$$z=\frac{-16\pm\sqrt{256+320}}{32}=\frac{-16\pm 24}{32}=\begin{cases}\frac{1}{4}\rightarrow x=\pm\sqrt{\frac{1}{4}}=\pm\frac{1}{2}\\ -\frac{5}{4}\rightarrow x=\pm\sqrt{-\frac{5}{4}}\rightarrow \text{NO}\end{cases}$$

$$x=\frac{1}{2}\rightarrow y=4\cdot\frac{1}{2}=2;\ x=-\frac{1}{2}\rightarrow y=4\cdot-\frac{1}{2}=-2$$

Solution: $\begin{cases} x = \frac{1}{2}; \ y = 2 \\ x = -\frac{1}{2}; \ y = -2 \end{cases}$

4. Some kilograms of olive oil, which costs €5/kg, are combined with sunflower oil which costs €2.5/kg, to obtain 40 kg of a mixture which costs E3.25/kg How many kilograms of each type will we have to put into the mixture?

	Olive oil	Sunflower oil	Mixture
kilograms	x	40 - x	40
Price/kg	5	2.5	3.25

Equation:

$$5x + 2.5(40 - x) = 3.25 \cdot 40 \Rightarrow 5x + 100 - 2.5x = 130 \rightarrow 2.5x = 30 \rightarrow x = 12$$

Answer- We needed 12 kg of olive oil and 28 kg of sunflower oil

5. A furniture shop sold a total of 315 sofas and tables. A sofa sold for €2300 and a table for €890 euro. The total sales were € 401 610. How many tables were sold?

Number of sofas x; Number of tables y

$$\left.\begin{matrix} x + y = 315 \\ 2300x + 890y = 401610 \end{matrix}\right\} \rightarrow \left.\begin{matrix} y = 315 - x \\ 230x + 89y = 40161 \end{matrix}\right\}$$

$$230x + 89(315 - x) = 40161 \rightarrow 230x - 89x = 40161 - 28035$$

$$141x = 12126 \rightarrow x = 86 \rightarrow y = 315 - x = 315 - 86 = 229$$

Answer: They have sold 86 sofas and 229 tables

6. The area of a rectangle is 91 cm^2 and its perimeter is 40 cm. Find the base and the height of the rectangle.

Base x Height y

Equations: $\left.\begin{matrix} x \cdot y = 91 \\ 2x + 2y = 40 \end{matrix}\right\} \rightarrow \left.\begin{matrix} x \cdot y = 91 \\ y = \frac{40 - 2x}{2} = 20 - x \end{matrix}\right\} x(20 - x) = 91$

$$x^2 - 20x + 91 = 0 \rightarrow x = \frac{20 \pm \sqrt{400 - 364}}{2} = \begin{cases} 13 \rightarrow y = 20 - 13 = 7 \\ 7 \rightarrow y = 20 - 7 = 13 \end{cases}$$

Answer: Base 13 cm and height 7 cm