EXAM 1_1 (Real numbers)

1. Use your calculator to work out the following calculations. Express the results in scientific notation. (When you obtain results of more than 3 decimal figures, round them to three decimal places) (1 point)

a) $(5.22 \cdot 10^8) \div (6.302 \cdot 10^{-5}) =$

b) $\sqrt[3]{5.0658 \cdot 10^{-10}} =$

2. Calculate and simplify: (4.5 points)

a) $\sqrt[4]{3^3} \cdot \sqrt{3} \cdot \sqrt[6]{3^2} =$

b) $(4\sqrt{18} - 6\sqrt{8} + 8\sqrt{72}) \div \sqrt{2} =$

c) $\sqrt[6]{\dfrac{2}{3}} \cdot \sqrt[3]{\dfrac{3}{2}} =$

d) $(2\sqrt[4]{3})^2 =$

e) $(\sqrt{2} - \sqrt{3})^2 =$

f) $(5 + \sqrt{3})(5 - \sqrt{3}) =$

g) $\sqrt[3]{\sqrt{\sqrt[4]{2^3}}} =$

h) $\sqrt{7 + \sqrt{1 + \sqrt{8 + \sqrt{3 - \sqrt{4}}}}} =$

3. Write each of the following expressions as a single positive power: (1.5 points)

a) $\left(\dfrac{2}{9}\right)^{-3} \div \left(\dfrac{3}{2}\right)^3 =$

b) $\dfrac{5^5 \cdot 25^2 \cdot 5^{-2}}{125^3} =$

4. Classify according to number type and mark on the real number line the following. (Notice that some numbers may be of more than one type).(1.5 points)

a) $-\sqrt{9}$ b) 1.5

c) $\sqrt{5}$ d) $-0.\widehat{3}$

5. Sketch the graph of the given interval: (1.5 points)

a) $\left(-3, \sqrt{3}\,\right]$

b) $(-\infty, -2)$

c) $\left[\dfrac{1}{3}, +\infty\right)$

d) $\{x \in \Re / -2 < x < 5\}$

SOLUTION

1. Use your calculator to work out the following calculations. Express the results in scientific notation. (When you obtain results of more than 3 decimal figures, round them to three decimal places)

a) $\left(5.22 \cdot 10^8\right) \div \left(6.302 \cdot 10^{-5}\right) = 8.283 \cdot 10^{12}$

b) $\sqrt[3]{5.0658 \cdot 10^{-10}} = 7.972 \cdot 10^{-4}$

2. Calculate and simplify:

a) $\sqrt[4]{3^3} \cdot \sqrt{3} \cdot \sqrt[6]{3^2} = \sqrt[12]{3^9} \cdot \sqrt[12]{3^6} \cdot \sqrt[12]{3^4} = \sqrt[12]{3^{19}} = 3\sqrt[12]{3^7}$

b) $\left(4\sqrt{18} - 6\sqrt{8} + 2\sqrt{72}\right) \div \sqrt{2} = \left(4\sqrt{3^2 \cdot 2} - 6\sqrt{2^3} + 2\sqrt{3^2 \cdot 2^3}\right) \div \sqrt{2} =$

$= \left(4 \cdot 3\sqrt{2} - 6 \cdot 2\sqrt{2} + 2 \cdot 3 \cdot 2\sqrt{2}\right) \div \sqrt{2} = (12 - 12 + 12)\sqrt{2} \div \sqrt{2} = 12$

c) $\sqrt[6]{\dfrac{2}{3}} \cdot \sqrt[3]{\dfrac{3}{2}} = \sqrt[6]{\dfrac{2}{3}} \cdot \sqrt[6]{\left(\dfrac{3}{2}\right)^2} = \sqrt[6]{\dfrac{2}{3} \cdot \dfrac{3^2}{2^2}} = \sqrt[6]{\dfrac{3}{2}}$

d) $\left(2\sqrt[4]{3}\right)^2 = 2^2 \sqrt[4]{3^2} = 4\sqrt{3}$

e) $\left(\sqrt{2} - \sqrt{3}\right)^2 = \left(\sqrt{2}\right)^2 - 2\sqrt{2} \cdot \sqrt{3} + \left(\sqrt{3}\right)^2 = 2 - 2\sqrt{6} + 3 = 5 - 2\sqrt{6}$

f) $\left(5 + \sqrt{3}\right)\left(5 - \sqrt{3}\right) = 5^2 - \left(\sqrt{3}\right)^2 = 25 - 3 = 22$

g) $\sqrt[3]{\sqrt{\sqrt[4]{2^3}}} = \sqrt[24]{2^3} = \sqrt[8]{2}$

h) $\sqrt{7 + \sqrt{1 + \sqrt{8 + \sqrt{3 - \sqrt{4}}}}} = \sqrt{7 + \sqrt{1 + \sqrt{8 + \sqrt{3 - 2}}}} = \sqrt{7 + \sqrt{1 + \sqrt{8 + 1}}} =$

$= \sqrt{7 + \sqrt{1 + \sqrt{9}}} = \sqrt{7 + \sqrt{1 + 3}} = \sqrt{7 + 2} = \sqrt{9} = 3$

3. Write each of the following expressions as a single positive power:

a) $\left(\dfrac{2}{9}\right)^{-3} \div \left(\dfrac{3}{2}\right)^3 = \dfrac{9^3}{2^3} \div \dfrac{3^3}{2^3} = \dfrac{3^6 \cdot 2^3}{2^3 \cdot 3^3} = 3^3$

b) $\dfrac{5^5 \cdot 25^2 \cdot 5^{-2}}{125^3} = \dfrac{5^5 \cdot 5^4 \cdot 5^{-2}}{5^9} = \dfrac{5^7}{5^9} = \dfrac{1}{5^2}$
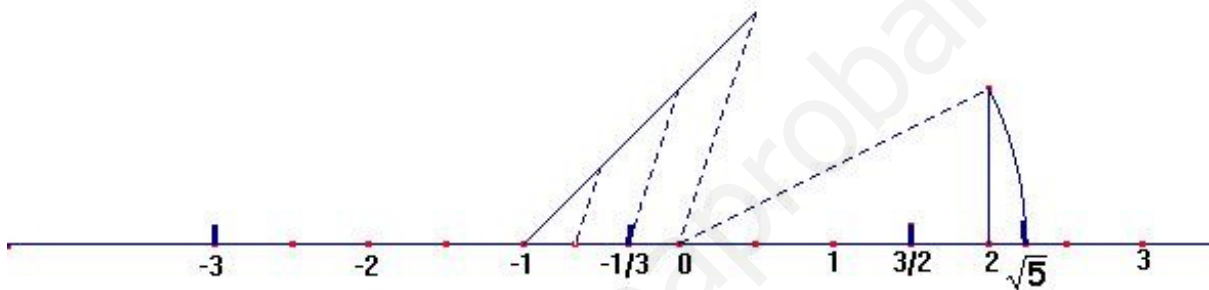
4. Classify according to number type and mark on the real number line the following. (Notice that some numbers may be of more than one type).

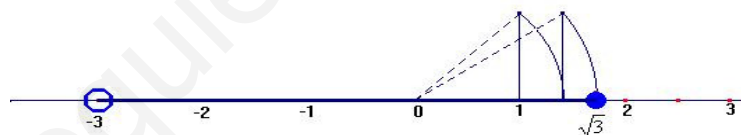   b) $-\sqrt{9} = -3$ Integer, Rational, Real

   c) $1.\overline{5}$ Rational (3/2), Real

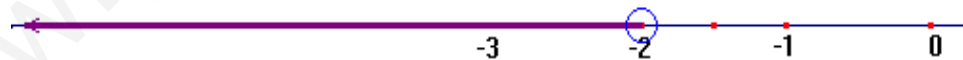   c) $\sqrt{5}$ Irrational, Real

   d) $-0.\widehat{3}$ Rational (-1/3), Real
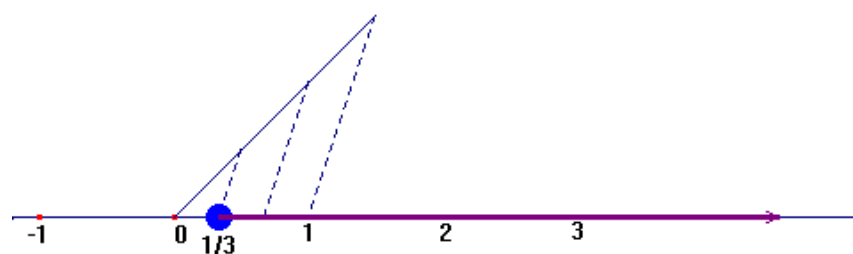

5. Sketch the graph of the given interval:

a) $\left(-3, \sqrt{3}\,\right]$

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c) $\left[\dfrac{1}{3}, +\infty\right)$

d) $\{x \in \Re / -2 < x < 5\}$