

EXAM 3_2 (Statistics - Algebra)

Name: _____

1. The histogram below shows the heights (in cm) distribution of 30 people. (1 point)

- How many people have heights between 159.5 and 169.5 cm?
- How many people have heights less than 159.5 cm?
- How many people have heights more than 169.5 cm?
- What percentage of people have heights between 149.5 and 179.5 cm?



2. The number of books read in a given months by each of the students in a group of 4°ESO are listed below: (2 points)

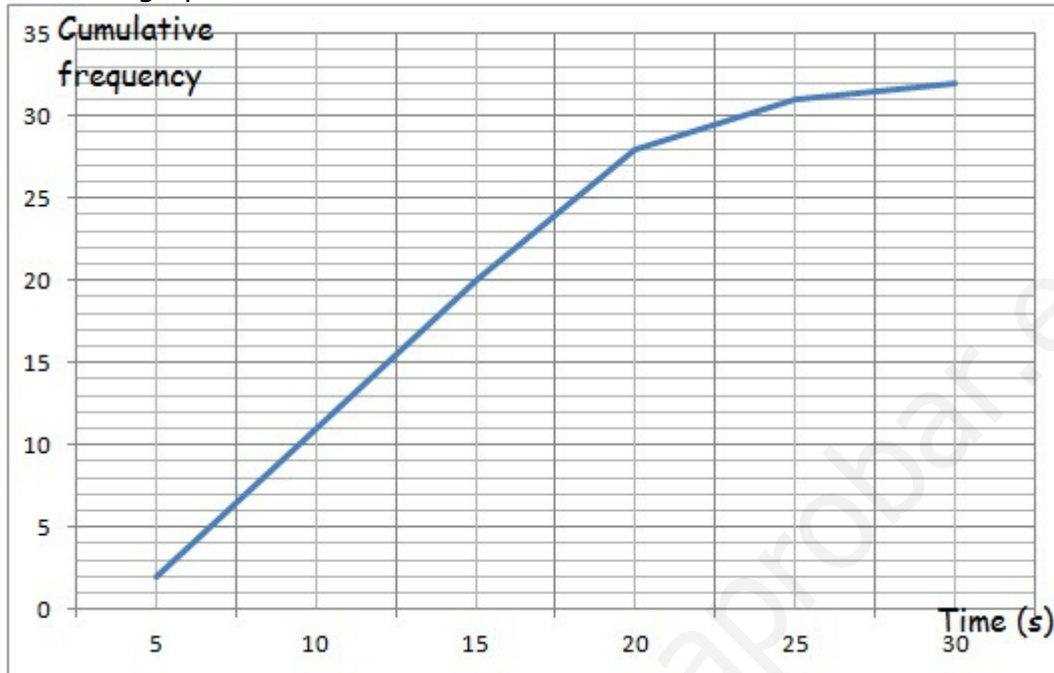
Number	Frequency
0	5
1	10
2	13
3	7
4	4
5	1

Calculate:

- Range, Mode and Median
- Mean and Standard deviation
- Percentiles 15 and 90

3. The following graph shows the cumulative frequency curve for time taken for students to solve a puzzle. (2 points)

From the graph estimate:



- The median time.
- The time at the lower quartile and at the upper quartile.
- After how many seconds were 70% of the puzzles made?
- Using this data draw a box and whisker plot.

4. Solve the following equations:

(3 points)

a) $6x^3 - 15x^2 + 12x - 3 = 0$

b) $1 - \sqrt{x-3} = x - 8$

c) $\frac{3x+1}{x-2} - \frac{x^2}{x^2-4} = \frac{x}{x+2} - 4$

5. Solve the simultaneous equations:

(2 points)

a) $\left. \begin{aligned} (x^2 + 1)y^2 &= 5 \\ 4x - y &= 0 \end{aligned} \right\}$

b) $\left. \begin{aligned} y^2 - 2y + 1 &= x \\ \sqrt{x} + y &= 5 \end{aligned} \right\}$

SOLUTION

1. The histogram below shows the heights (in cm) distribution of 30 people.

- How many people have heights between 159.5 and 169.5 cm? 7
- How many people have heights less than 159.5 cm? 15
- How many people have heights more than 169.5 cm? 8
- What percentage of people have heights between 149.5 and 179.5 cm?

21 of 30, so the percentage is 70%



2. The number of books read in a given months by each of the students in a group of 4^oESO are listed below:

x_i	f_i	F_i	$x_i f_i$	$x_i^2 f_i$
0	5	5	0	0
1	10	15	10	10
2	13	28	26	52
3	7	35	21	63
4	4	39	16	64
5	1	40	5	25
			78	214

Calculate:

- a) Range, Mode and Median: $r=5-0=5$ range 5 ; Mode 2 books
 Median: $40/2=20$, so the median is 2 books

- b) Mean and Standard deviation

$$\bar{x} = \frac{78}{40} = 1.95 \text{ books ; } \sigma = \sqrt{\frac{214}{40} - 1.95^2} = 1.24 \text{ books}$$

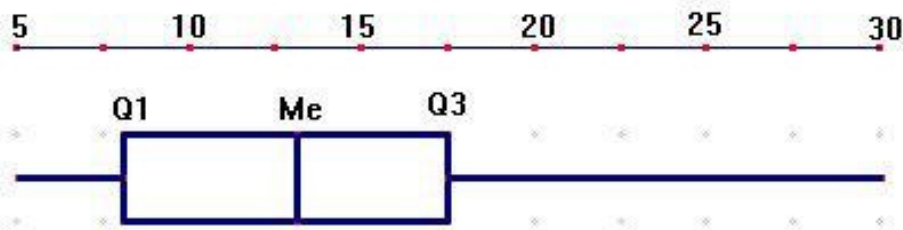
- c) Percentiles 15 and 90 $P_{15} \rightarrow \frac{15N}{100} = 6 \rightarrow 1 \text{ book}$; $P_{90} \rightarrow \frac{90N}{100} = 36 \rightarrow 4 \text{ books}$

3. The following graph shows the cumulative frequency curve for time taken for students to solve a puzzle.

From the graph estimate:



- a) The median time. $N=32$, half is 16, so the median time is 13 seconds
 b) The time at the lower quartile and at the upper quartile.
 $Q_1 \rightarrow 32/4 = 8 \rightarrow Q_1 = 8.5s$; $Q_3 \rightarrow 3 \cdot 32/4 = 24 \rightarrow Q_3 = 17.5s$
 c) After how many seconds were 70% of the puzzles made?
 $70\% \text{ of } 32 = 22.4 \rightarrow \text{about } 16.5 \text{ seconds}$
 d) Using this data draw a box and whisker plot.



4. Solve the following equations:

a) $6x^3 - 15x^2 + 12x - 3 = 0$, $P(1) = 6 - 15 + 12 - 3 = 0$

$$\begin{array}{r|rrrr} 1 & 6 & -15 & +12 & -3 \\ & & +6 & -9 & +3 \\ \hline & 6 & -9 & +3 & 0 \end{array}$$

$$6x^2 - 9x + 3 = 0 \rightarrow 2x^2 - 3x + 1 = 0$$

$$x = \frac{3 \pm \sqrt{1}}{4} = \left\langle \frac{1}{2} \right\rangle$$

Solution: $x = 1, x = \frac{1}{2}$

b) $1 - \sqrt{x-3} = x - 8 \rightarrow -\sqrt{x-3} = x - 8 - 1 \rightarrow (-\sqrt{x-3})^2 = (x-9)^2$

$$x - 3 = x^2 - 18x + 81 \rightarrow x^2 - 19x + 84 = 0 \rightarrow x = \frac{19 \pm \sqrt{25}}{2} = \left\langle \frac{12}{7} \right\rangle$$

Checking: $x = 12 \rightarrow 1 - \sqrt{12-3} = 12 - 8 \rightarrow 1 - 3 = 4$ NO

$$x = 7 \rightarrow 1 - \sqrt{7-3} = 7 - 8 \rightarrow 1 - 2 = -1$$
 YES

Solution $x = 7$

c) $\frac{3x+1}{x-2} - \frac{x^2}{x^2-4} = \frac{x}{x+2} - 4 \rightarrow \text{LCD} = (x+2)(x-2)$

$$\frac{(3x+1)(x+2)}{x^2-4} - \frac{x^2}{x^2-4} = \frac{x(x-2)}{x^2-4} - \frac{4(x^2-4)}{x^2-4} \rightarrow 3x^2 + 7x + 2 - x^2 = x^2 - 2x - 4x^2 + 16$$

$$3x^2 + 7x + 2 - x^2 - x^2 + 2x + 4x^2 - 16 = 0 \rightarrow 5x^2 + 9x - 14 = 0 \rightarrow x = \frac{-9 \pm \sqrt{361}}{10} = \left\langle \frac{1}{-5}, -\frac{14}{5} \right\rangle$$

5. Solve the simultaneous equations:

a) $\left. \begin{array}{l} (x^2 + 1)y^2 = 5 \\ 4x - y = 0 \end{array} \right\} \rightarrow y = 4x \Rightarrow (x^2 + 1)(4x)^2 = 5 \rightarrow 16x^4 + 16x^2 - 5 = 0$

$$z = x^2 \rightarrow 16z^2 + 16z - 5 = 0 \rightarrow z = \frac{-16 \pm \sqrt{576}}{32} = \frac{-16 \pm 24}{32} = \left\langle \frac{1}{4}, -\frac{5}{2} \right\rangle$$

$$z = x^2 \rightarrow \left\langle \begin{array}{l} x = \sqrt{\frac{1}{4}} = \pm \frac{1}{2} \\ x = \sqrt{-\frac{5}{2}} \text{ NO} \end{array} \right. \rightarrow \left\{ \begin{array}{l} x = \frac{1}{2} \rightarrow y = 4 \cdot \frac{1}{2} = 2 \\ x = -\frac{1}{2} \rightarrow y = -4 \cdot \frac{1}{2} = -2 \end{array} \right. \rightarrow \text{solution} \left\{ \begin{array}{l} \left(\frac{1}{2}, 2 \right) \\ \left(-\frac{1}{2}, -2 \right) \end{array} \right.$$

b)

$$\left. \begin{array}{l} y^2 - 2y + 1 = x \\ \sqrt{x} + y = 5 \end{array} \right\} \sqrt{y^2 - 2y + 1} + y = 5 \rightarrow \sqrt{(y-1)^2} + y = 5 \rightarrow y - 1 + y = 5 \rightarrow 2y = 6 \rightarrow y = 3$$

$$x = y^2 - 2y + 1 = 9 - 6 + 1 = 4 \rightarrow \text{solution } (4, 3)$$