EXAM UNIT 3 (SEQUENCES)

Remember: in each question, write the steps you have taken to reach the solution.

1) For each of the following sequences, determine whether it is arithmetic or geometric. Find the tenth and twentieth terms, and the sum of the first 20 terms. (4 points)

a. 3, -1, -5, …

b. 32, 16, 8,...

c. 2, 14, 26, …

d. 3, 9, 27,…

2) Find the sum to 100 terms of an arithmetic progression whose fourth and sixth terms are 10 and 16. (1.25 points)

3) Find the sum to 20 terms of a geometric progression whose first term is 5 and the common ratio 1/3. Find the sum of all the terms. (1.5 points)

4) A culture of bacteria doubles every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?(1.25 points)

5) Find the sum of the 50 first multiples of 7. (1.25 points)

6) The sum of all terms of a geometric progression is 4, and the first term is 3. Find the common ratio. (1.25 points)

SOLUTIONS

1) For each of the following sequences, determine whether it is arithmetic or geometric. Find the tenth and twentieth terms, and the sum of the first 20 terms.

a. 3, -1, -5, … It is an AP, with $d=-4$

$a_n=a_1+(n-1)d \rightarrow a_{10}=3+9\times(-4)=-33$ and $a_{20}=3+19\times(-4)=-73$

$S_n=\frac{(a_1+a_n)n}{2} \rightarrow S_{20}=\frac{(3-73)\times 20}{2}=-700$

b. 32, 16, 8,... It is a GP, with $r=\frac{1}{2}$

$a_n=a_1\cdot r^{n-1} \rightarrow a_{10}=32\cdot\left(\frac{1}{2}\right)^9=\frac{2^5}{2^9}=\frac{1}{2^4}=\frac{1}{16}$ and $a_{20}=32\cdot\left(\frac{1}{2}\right)^{19}=6.1\times10^{-5}$

$S_n=\frac{a_n r-a_1}{r-1} \rightarrow S_{20}=\frac{6.1\times10^{-5}\times\frac{1}{2}-32}{\frac{1}{2}-1}=\frac{3.05\times10^{-5}-32}{-\frac{1}{2}}=63.9999$

c. 2, 14, 26, … It is an A.P. with $d=12$

$a_n=a_1+(n-1)d \rightarrow a_{10}=2+9\times12=110$ and $a_{20}=2+19\times12=230$

$S_n=\frac{(a_1+a_n)n}{2} \rightarrow S_{20}=\frac{(2+230)\times 20}{2}=2320$

d. 3, 9, 27,... It is a GP, with $r=3$

$a_n=a_1\cdot r^{n-1} \rightarrow a_{10}=3\cdot3^9=3^{10}$ and $a_{20}=3\cdot3^{19}=3^{20}$

$S_n=\frac{a_n r-a_1}{r-1} \rightarrow S_{20}=\frac{3^{20}\times3-3}{3-1}=\frac{3^{21}-3}{2}=5230176600$

2) Find the sum to 100 terms of an arithmetic progression whose fourth and sixth terms are 10 and 16.

$a_4=10,\ a_6=16 \rightarrow a_6=a_4+2d \rightarrow 16=10+2d \rightarrow d=3$

$a_4=a_1+3d \rightarrow 10=a_1+9 \rightarrow a_1=1$

$a_{100}=a_1+99d \rightarrow a_{100}=1+99\times3=298$

$S_n=\frac{(a_1+a_n)n}{2} \rightarrow S_{100}=\frac{(1+298)\times100}{2}=14950$

3) Find the sum to 20 terms of a geometric progression whose first term is 5 and the common ratio 1/3. Find the sum of all the terms.

$a_{20}=5\cdot\left(\frac{1}{3}\right)^{19}=4\times10^{-9}$

$$S_n = \frac{a_n r - a_1}{r-1} \rightarrow S_{20} = \frac{4\times10^{-9}\times\left(1/3\right)-5}{\frac{1}{3}-1} = \frac{-4.99999999}{-\frac{2}{3}} = 7.4999999$$

$$S = \frac{a_1}{1-r} \rightarrow S = \frac{5}{1-\frac{1}{3}} = 5 \div \frac{2}{3} = 7.5$$

4) A culture of bacteria doubles every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?

G.P: 500, 1000, 2000,..... $a_1 = 500;\quad r = 2$

24 hours are 12 times 2 hours $\rightarrow a_{12} = 500\times2^{11} = 1024000$ bacteria

5) Find the sum of the 50 first multiples of 7.

7, 14, 21,... multiples of 7, it is an A.P. with $a_1 = 7;\quad d = 7$

$a_{50} = a_1 + 49d \rightarrow a_{50} = 7 + 49\times7 = 350$

$$S_n = \frac{(a_1 + a_n)n}{2} \rightarrow S_{50} = \frac{(7+350)\times50}{2} = 8925$$

6) The sum of all terms of a geometric progression is 4, and the first term is 3. Find the common ratio.

$$S = \frac{a_1}{1-r} \rightarrow 4 = \frac{3}{1-r} \rightarrow 4(1-r) = 3 \rightarrow 4-4r = 3 \rightarrow 4-3 = 4r \rightarrow r = \frac{1}{4}$$