1st TERM GENERAL EXAM

Remember: in each question, write the steps you have taken to reach the solution. (1 point each question)

1) Work out and simplify:

a) $-\dfrac{1}{5}\cdot\left(2-\dfrac{2}{3}+\dfrac{3}{4}\right)-\left(\dfrac{1}{2}\right)^{2}=$

b) $\dfrac{5}{12}-1-\dfrac{1}{3}+\dfrac{5}{6}\div\dfrac{1}{2}=$

2) Write each of the following expressions as a single positive power:

a) $\dfrac{\left(ab^{3}\right)^{2}\cdot b^{-2}\cdot b^{5}\cdot a^{2}}{b^{4}\left(a^{-2}\right)^{-2}}=$

b) $\left(\dfrac{2}{5}\right)^{-2}\div\left(\dfrac{2}{5}\right)^{-4}=$

3) Calculate and express your result using standard form:

a) $\left(3.15\cdot10^{7}\right)\div\left(3\cdot10^{-3}\right)=$

b) $\left(1.5\cdot10^{4}+2.3\cdot10^{5}\right)\cdot2\cdot10^{-2}=$

4) Complete:

$\sqrt{-\dfrac{1}{9}}=$	$\sqrt{\dfrac{1}{9}}=$	$\sqrt[3]{-27}=$
$\sqrt[3]{(-7)^{3}}=$	$(-3)^{-2}=$	$-5^{2}=$

5) Classify the following numbers and express the rational numbers as fractions:

$$-1.5,\quad 0.010010001....,\quad -\sqrt{5},\quad -0.2\hat{3},\quad 0.181818......,\quad \dfrac{6}{3}$$

6) Donovan took a math test and got 35 correct and 10 incorrect answers. What was the percentage of incorrect answers? (Round to the nearest hundredth)

7) Work out and simplify:

a) $\dfrac{x^{2}-4}{x^{2}-4x+4}\cdot\dfrac{3x-6}{(x+2)^{2}}=$

b) $\dfrac{x^{2}-9}{4x^{2}+12x}\div\dfrac{x-3}{2x^{3}}=$

8) Solve the following equations:

a) $\dfrac{x-1}{6}-\dfrac{x-3}{2}=-1$

b) $3(x-3)^{2}-(x+2)(x-2)=31$

9) Solve: $\dfrac{2x}{3}-\dfrac{x+2}{5}=1-(x-3)^{2}$

10) Translate each sentence into an equation:

a) Seven times the sum of two different numbers gives ten.

b) Twice a number plus three is equal to twenty.

SOLUTION

1) Work out and simplify:

a) $-\dfrac{1}{5}\cdot\left(2-\dfrac{2}{3}+\dfrac{3}{4}\right)-\left(\dfrac{1}{2}\right)^2 = -\dfrac{1}{5}\cdot\left(\dfrac{24-8+9}{12}\right)-\dfrac{1}{4} = -\dfrac{1}{5}\cdot\dfrac{25}{12}-\dfrac{1}{4} = -\dfrac{5}{12}-\dfrac{1}{4} =$

$= -\dfrac{5}{12}-\dfrac{3}{12} = -\dfrac{8}{12} = -\dfrac{2}{3}$

b) $\dfrac{5}{12}-1-\dfrac{1}{3}+\dfrac{5}{6}\div\dfrac{1}{2} = \dfrac{5}{12}-1-\dfrac{1}{3}+\dfrac{5}{3} = \dfrac{5}{12}-\dfrac{12}{12}+\dfrac{16}{12} = \dfrac{9}{12} = \dfrac{3}{4}$

2) Write each of the following expressions as a single positive power:

a) $\dfrac{\left(ab^3\right)^2\cdot b^{-2}\cdot b^5\cdot a^2}{b^4\left(a^{-2}\right)^{-2}} = \dfrac{a^2 b^6\cdot b^3\cdot a^2}{b^4\cdot a^4} = \dfrac{b^9\cdot a^4}{b^4\cdot a^4} = b^5$

b) $\left(\dfrac{2}{5}\right)^{-2}\div\left(\dfrac{2}{5}\right)^{-4} = \left(\dfrac{2}{5}\right)^{-2-(-4)} = \left(\dfrac{2}{5}\right)^2$

3) Calculate and express your result using standard form:

a) $\left(3.15\cdot10^7\right)\div\left(3\cdot10^{-3}\right) = 1.05\cdot10^{7+3} = 1.05\cdot10^{10}$

b) $\left(1.5\cdot10^4 + 2.3\cdot10^5\right)\cdot2\cdot10^{-2} = \left(1.5\cdot10^4 + 23\cdot10^4\right)\cdot2\cdot10^{-2} = 24.5\cdot10^4\cdot2\cdot10^{-2} =$

$= 24.5\cdot10^4\cdot2\cdot10^{-2} = 49\cdot10^2 = 4.9\cdot10^3$

4) Complete:

$\sqrt{-\dfrac{1}{9}}=$ It doesn't exist	$\sqrt{\dfrac{1}{9}}=\pm\dfrac{1}{3}$	$\sqrt[3]{-27}=-3$
$\sqrt[3]{(-7)^3}=-7$	$(-3)^{-2}=\dfrac{1}{(-3)^2}=\dfrac{1}{9}$	$-5^2=-25$

5) Classify the following numbers and express the rational numbers as fractions:

$$-1.5,\quad 0.010010001...,\quad -\sqrt{5},\quad -0.23\hat{3},\quad 0.181818......,\quad \dfrac{6}{3}$$

-1.5 rational number, negative $\rightarrow -1.5 = -\dfrac{15}{10} = -\dfrac{3}{2}$

$0.010010001....$ irrational number, positive

$-\sqrt{5}$ irrational number, negative

$-0.23\hat{3}$ rational number, negative

$N = 0.23\hat{3} \rightarrow \left.\begin{array}{l} 100N = 23.333.. \\ 10N = 2.333.. \end{array}\right\} \rightarrow 90N = 21 \Rightarrow N = -\dfrac{21}{90} = -\dfrac{7}{30}$

$0.181818......$ rational number, positive

$N = 0.1818... \rightarrow \left.\begin{array}{l} 100N = 18.1818.. \\ N = 0.1818.. \end{array}\right\} \rightarrow 99N = 18 \Rightarrow N = \dfrac{18}{99} = \dfrac{2}{11}$

$\dfrac{6}{3}$ Natural number

6) Donovan took a math test and got 35 correct and 10 incorrect answers. What was the percentage of incorrect answers? (Round to the nearest hundredth)

35 correct and 10 incorrect, total 45

$$\frac{10}{x} = \frac{45}{100} \Rightarrow 1000 = 45x \Rightarrow x = \frac{1000}{45} = \frac{200}{9} = 22.2222.... \approx 22.22$$

Solution: The percentage of incorrect answer was 22.22%

7) Work out and simplify:

a) $\dfrac{x^2-4}{x^2-4x+4} \cdot \dfrac{3x-6}{(x+2)^2} = \dfrac{(x+2)(x-2)}{(x-2)^2} \cdot \dfrac{3(x-2)}{(x+2)^2} = \dfrac{3(x+2)(x-2)^2}{(x+2)^2(x-2)^2} = \dfrac{3}{x+2}$

b) $\dfrac{x^2-9}{4x^2+12x} \div \dfrac{x-3}{2x^3} = \dfrac{(x+3)(x-3)}{4x(x+3)} \cdot \dfrac{2x^3}{(x-3)} = \dfrac{2x^3(x+3)(x-3)}{2 \cdot 2x(x+3)(x-3)} = \dfrac{x^2}{2}$

8) Solve the following equations:

a) $\dfrac{x-1}{6} - \dfrac{x-3}{2} = -1 \rightarrow \dfrac{x-1}{6} - \dfrac{3x-9}{6} = -\dfrac{6}{6} \Rightarrow x-1-3x+9 = -6$

$x - 3x = -6-9+1 \Rightarrow -2x = -14 \Rightarrow x = 7$

b) $3(x-3)^2 - (x+2)(x-2) = 31 \Rightarrow 3(x^2-6x+9) - (x^2-4) = 31$

$3x^2 - 18x + 27 - x^2 + 4 = 31 \Rightarrow 2x^2 - 18x = 0 \Rightarrow x(2x-18) = 0 \Rightarrow \begin{cases} x = 0 \\ 2x-18 = 0 \rightarrow x = 9 \end{cases}$

9) Solve: $\dfrac{2x}{3} - \dfrac{x+2}{5} = 1 - (x-3)^2 \rightarrow \dfrac{10x}{15} - \dfrac{3x+6}{15} = \dfrac{5(1-x^2+6x-9)}{15}$

$10x - 3x - 6 = 15 - 15x^2 + 90x - 135 \rightarrow 15x^2 - 83x + 114 = 0$

$x = \dfrac{83 \pm \sqrt{83^2 - 4 \cdot 15 \cdot 114}}{2 \cdot 15} = \dfrac{83 \pm \sqrt{49}}{30} = \dfrac{83 \pm 7}{30} = \begin{cases} \dfrac{83+7}{30} = 3 \\ \dfrac{83-7}{30} = \dfrac{76}{30} = \dfrac{38}{15} \end{cases}$

10) Translate each sentence into an equation:

a) Seven times the sum of two different numbers gives ten. $7(x+y) = 10$

b) Twice a number plus three is equal to twenty. $2x+3 = 20$