IES MURILLO

Maths 3rd ESO

2nd term General Exam

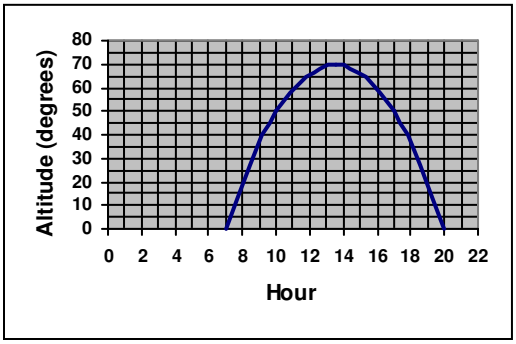
1.- The following graph shows the altitude of the sun on the horizon, in degrees, on a certain day.

a) What time is sunrise? And sunset?

b) When is the altitude increasing? And decreasing?

c) What time does the sun reach maximum height above the horizon?

d) How many hours of daylight were there this day?



2.- Simplify:

a) $\frac{x^3 - x}{2x^2 + 2x} =$

b) $\frac{x^2 - 6x + 9}{3x^2 - 9x} =$

3.- Complete:

a) $9x^2 - \quad + \quad = (\quad - 2)^2$

b) $y^2 - 25x^2 = (\quad)(\quad)$

c) $(3 + \quad)^2 = \quad + 12x^2 +$

4.- Solve: $2(x-1)^2 + 4x - (x+1)^2 - x^2 = 1 - 2x$

5.- Solve: $\frac{2x}{3} - \frac{x-2}{5} = 1 - \frac{(x-3)^2}{15}$

6.- The measure of the unequal side of an isosceles triangle is 21 cm. If the perimeter is 87 cm, how long are the other sides?

7.- Bob is five times as old as Carlos and in 8 years' time he will be three times as old as him. How old is each one?

8.- Two cyclists leave at 9 in the morning from two towns (A and B) located at 150 km away from each other. The cyclist that leaves from A goes at 34 km/h and the one that leaves from B at 26 km/h. How far from A and B will they meet up? When will they meet up?

9.- A hotel has double rooms and single rooms. In total there are 58 rooms and 100 beds. How many rooms are there of each type?

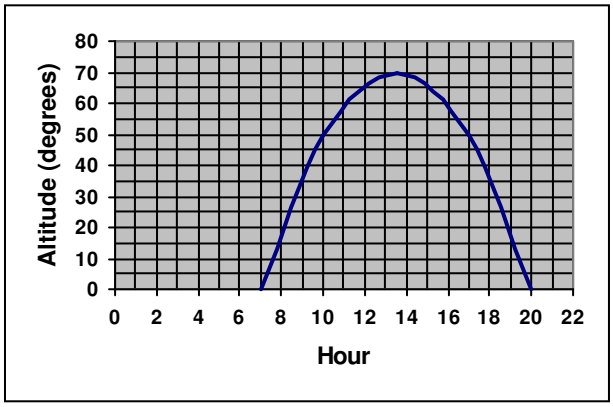
10.- Solve the following system by graphing and by another method (substitution or addition):

$$\left.\begin{array}{l} 3 + 4x = 2y \\ 2x - y = -1 \end{array}\right\}$$

(1 point each question)

SOLUTION

1.- The following graph shows the altitude of the sun on the horizon, in degrees, on a certain day.



c) What time is sunrise? And sunset?
Sunrise is at 7 h
Sunset is at 20 h

d) When is the altitude increasing? And decreasing?
The altitude is increasing from 7 to 13:30
It is decreasing from 13:30 to 20

c) What time does the sun reach maximum height above the horizon?
At 13:30 approximately

d) How many hours of daylight were there this day?
20 - 7 = 13 hours of daylight

2.- Simplify:

a) $\frac{x^3-x}{2x^2+2x}=\frac{x(x^2-1)}{2x(x+1)}=\frac{x(x+1)(x-1)}{2x(x+1)}=\frac{x-1}{2}$

b) $\frac{x^2-6x+9}{3x^2-9x}=\frac{(x-3)^2}{3x(x-3)}=\frac{x-3}{3x}$

3.- Complete:

a) $9x^2-12x+4=(3x-2)^2$

b) $y^2-25x^2=(y-5x)(y+5x)$

c) $\left(3+2x^2\right)^2=9+12x^2+4x^4$

4.- $2(x-1)^2+4x-(x+1)^2-x^2=1-2x$

$2(x^2-2x+1)+4x-(x^2+2x+1)-x^2=1-2x$

$2x^2-4x+2+4x-x^2-2x-1-x^2=1-2x\rightarrow -2x+1=1-2x\rightarrow 0x=0$

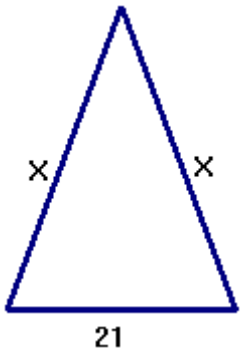
INFINITE SOLUTIONS (IT IS AN IDENTITY)

5.- $\frac{2x}{3}-\frac{x-2}{5}=1-\frac{(x-3)^2}{15}\rightarrow\frac{10x}{15}-\frac{3(x-2)}{15}=\frac{15}{15}-\frac{(x^2-6x+9)}{15}$

$10x-3x+6=15-x^2+6x-9\rightarrow x^2+10x-3x-6x+6-15+9=0$

$x^2+x=0\rightarrow x(x+1)=0\rightarrow\begin{cases}x=0\\x+1=0\rightarrow x=-1\end{cases}$

6.- The measure of the unequal side of an isosceles triangle is 21 cm. If the perimeter is 87 cm, how long are the other sides?



$$\text{Perimeter} = x + x + 21 = 87$$
$$2x + 21 = 87$$
$$2x = 66$$
$$x = \frac{66}{2} = 33$$

The other sides are 33 cm long

7.- Bob is five times as old as Carlos and in 8 years' time he will be three times as old as him. How old is each one?

	Now	In 8 years' time
Bob	5x	5x+8
Carlos	x	x+8

$5x + 8 = 3(x + 8) \Rightarrow 5x + 8 = 3x + 24 \Rightarrow 5x - 3x = 24 - 8 \Rightarrow 2x = 16 \Rightarrow x = 8$

Carlos is 8 years old and Bob is 40 years old

8.- Two cyclists leave at 9 in the morning to from two towns (A and B) located at 150 km away from each other. The cyclist that leaves from A goes at 34 km/h and the one that leaves from B at 26 km/h. How far from A and B will they meet up? When will they meet up?

	Cyclist from A	Cyclist from B
Velocity	34 km/h	26 km/h
Distance	x	150 - x
time	t	t

$$t = \frac{x}{34}; \quad t = \frac{150 - x}{26} \Rightarrow \frac{x}{34} = \frac{150 - x}{26} \Rightarrow 26x = 34(150 - x)$$

$$26x = 5100 - 34x \Rightarrow 26x + 34x = 5100 \Rightarrow 60x = 5100 \Rightarrow x = \frac{5100}{60} = 85$$

They meet up at 85 km from A and 65 km from B

$$t = \frac{x}{34} = \frac{85}{34} = 2.5 \Rightarrow 2\text{h}30\text{m}$$

They meet up at 11:30 a.m.

9.- A hotel has double rooms and single rooms. In total there are 58 rooms and 100 beds. How many rooms are there of each type?

Double rooms – x Single rooms – y

$$\left.\begin{array}{l} x+y=58 \\ 2x+y=100 \end{array}\right\} \rightarrow \left.\begin{array}{l} -x-y=-58 \\ 2x+y=100 \end{array}\right\} \rightarrow x=42$$

$$42+y=58 \rightarrow y=58-42=16$$

Solution: There are 42 double rooms and 16 single rooms in the hotel

10.-

$$\left.\begin{array}{l} 3+4x=2y \\ 2x-y=-1 \end{array}\right\} \rightarrow \left.\begin{array}{l} 4x-2y=-3 \\ 2x-y=-1 \end{array}\right\}$$ Substitution:

$$\left.\begin{array}{l} 2x+1=y \\ 4x-2(2x+1)=2 \end{array}\right\} \rightarrow 0x=4$$ No solution

Graphing:

$$\left.\begin{array}{l} y=\dfrac{4x+3}{2} \\ y=2x+1 \end{array}\right\}$$ parallel lines, inconsistent system