3rd TERM GENERAL EXAM

Name:_____

Remember: in each question, write the steps you have taken to reach the solution. (1 point each question)

1. Work out and simplify:

   a) $\dfrac{2}{3}\cdot\dfrac{6}{5}-\dfrac{6}{5}\cdot\dfrac{3}{2}=$ (0,5p)   b) $\dfrac{2}{3}\cdot\left(\dfrac{6}{5}-\dfrac{6}{5}\div\dfrac{3}{2}\right)=$ (0.5p)

2. Calculate, using the rules for powers :

   a) $\left(\dfrac{2^5\cdot 4^2}{8^3}\right)^2=$ (0.5p)   b) $\dfrac{x^{-2}}{x^{-1}}\cdot\left(\dfrac{x^2 y}{x^3 y^2}\right)^2=$ (0.5p)

3. Solve:

   a) $(2x-1)^2=(x-1)^2+8$ (0.5p)   b) $\dfrac{5x-16}{6}-\dfrac{x+1}{3}+\dfrac{x+8}{12}=0$ (0.5p)

4. Work out and simplify:

   a) $\dfrac{2}{x-2}-\dfrac{2}{x+2}=$ (0.5p)   b) $\left(\dfrac{1}{x}\div\dfrac{1}{x+1}\right)\cdot\dfrac{x}{2}=$ (0.5p)
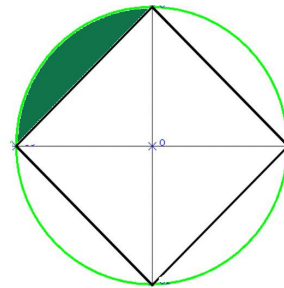
5. Calculate the sum to 50 terms of an arithmetic progression whose first and ninth terms are 3 and 35.

6. A cone of radius 6 cm has a total surface area of 324π cm^2. Calculate the volume of the cone.

7. A square of side 10cm is drawn inside a circle.
   a) Find the diameter of the circle.
   b) Find the shaded area.

8. Halla las dimensiones de un rectángulo del que conocemos su perímetro, 34 m y su área 60 m^2.

9. Hace dos años compré un juego de ordenador y un equipo de música por 260€. Los acabo de vender por un total de 162 €, habiendo perdido el 30% con el juego y el 40% con el equipo de música ¿Cuánto me costó cada cosa?

10. Solve by graphing and using another method the simultaneous equation:

$$\left.\begin{array}{l}3x+2y=7\\5x-y=16\end{array}\right\}$$

SOLUTION

1. Work out and simplify:

a) $\dfrac{2}{3}\cdot\dfrac{6}{5}-\dfrac{6}{5}\cdot\dfrac{3}{2}=\dfrac{2\cdot6}{3\cdot5}-\dfrac{6\cdot3}{5\cdot2}=\dfrac{2\cdot2}{5}-\dfrac{3\cdot3}{5}=\dfrac{4}{5}-\dfrac{9}{5}=-\dfrac{5}{5}=-1$

b) $\dfrac{2}{3}\cdot\left(\dfrac{6}{5}-\dfrac{6}{5}\div\dfrac{3}{2}\right)=\dfrac{2}{3}\cdot\left(\dfrac{6}{5}-\dfrac{6\cdot2}{3\cdot5}\right)=\dfrac{2}{3}\cdot\left(\dfrac{6}{5}-\dfrac{2\cdot2}{5}\right)=\dfrac{2}{3}-\dfrac{2}{5}=\dfrac{10-6}{15}=\dfrac{4}{15}$

2. Calculate, using the rules for powers :

a) $\left(\dfrac{2^5\cdot4^2}{8^3}\right)^2=\left(\dfrac{2^5\cdot2^4}{2^9}\right)^2=\left(2^0\right)^2=2^0=1$

b) $\dfrac{x^{-2}}{x^{-1}}\cdot\left(\dfrac{x^2y}{x^3y^2}\right)^2=\dfrac{x}{x^2}\cdot\dfrac{x^4y^2}{x^6y^4}=\dfrac{x^5y^2}{x^8y^4}=\dfrac{1}{x^3y^2}$

3. Solve:

a) $(2x-1)^2-(x-1)^2=8\rightarrow 4x^2-4x+1-\left(x^2-2x+1\right)=8$

$4x^2-4x+1-x^2+2x-1=8\rightarrow 3x^2-2x-8=0$

$x=\dfrac{2\pm\sqrt{4+96}}{6}=\dfrac{2\pm10}{6}=\begin{cases}2\\[4pt]-\dfrac{8}{6}=-\dfrac{4}{3}\end{cases}$

b) $\dfrac{5x-16}{6}-\dfrac{x+1}{3}+\dfrac{x+8}{12}=0\rightarrow\dfrac{2(5x-16)}{12}-\dfrac{4(x+1)}{12}+\dfrac{x+8}{12}=0$

$10x-32-4x-4+x+8=0\rightarrow 7x=32+4-8\rightarrow 7x=28\rightarrow x=4$

4. Work out and simplify:

a) $\dfrac{2}{x-2}-\dfrac{2}{x+2}=\dfrac{2(x+2)}{(x+2)(x-2)}-\dfrac{2(x-2)}{(x+2)(x-2)}=\dfrac{2x+4}{x^2-4}-\dfrac{2x-4}{x^2-4}=\dfrac{8}{x^2-4}$

b) $\left(\dfrac{1}{x}\div\dfrac{1}{x+1}\right)\cdot\dfrac{x}{2}=\dfrac{x+1}{x}\cdot\dfrac{x}{2}=\dfrac{(x+1)x}{2x}=\dfrac{x+1}{2}$
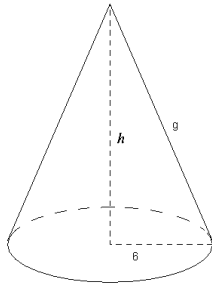
5. Calculate the sum to 50 terms of an arithmetic progression whose first and ninth terms are 3 and 35.

$a_1=3;\quad a_9=35\rightarrow a_9=a_1+8d\Rightarrow35=3+8d\rightarrow8d=32\rightarrow d=4$

$S_{50}=\dfrac{(a_1+a_{50})\cdot50}{2}$, we need to calculate $a_{50}=a_1+49d=3+49\cdot4=199$

6. $S_{50}=\dfrac{(a_1+a_{50})\cdot50}{2}=\dfrac{(3+199)\cdot50}{2}=202\cdot25=5050$

6. A cone of radius 6 cm has a total surface area of 324π cm². Calculate the volume of the cone.

$$A = \pi r^2 + 2\pi r g \rightarrow 324\pi = 36\pi + 12\pi g \rightarrow 324 = 36 + 12g$$

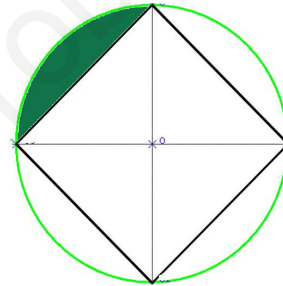$$324 - 36 = 12g \rightarrow 12g = 288 \rightarrow g = \frac{288}{12} = 24$$

Pitágoras: $g^2 = h^2 + r^2 \rightarrow h^2 = 24^2 - 6^2 = 540$

$$h = \sqrt{540} = \sqrt{2^2 \cdot 3^3 \cdot 5} = 6\sqrt{15} \text{ cm}$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi 6^2 \cdot 6\sqrt{15} = \frac{216\sqrt{15}}{3}\pi = 72\sqrt{15}\,\pi \text{ cm}^3$$

7. A square of side 10cm is drawn inside a circle.
   a) Find the diameter of the circle.
   b) Find the shaded area.

a) Pythagoras: $d^2 = 10^2 + 10^2 \rightarrow d^2 = 100 + 100 = 200$

$$d = \sqrt{200} = 10\sqrt{2} \text{ cm} \rightarrow r = \frac{d}{2} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

b) $A = \frac{1}{4}\left(A_{Circle} - A_{Square}\right) = \frac{1}{4}\left(\pi r^2 - l^2\right)$

$$A = \frac{1}{4}\left(\pi\left(5\sqrt{2}\right)^2 - 10^2\right) = \frac{50\pi - 100}{4} = \left(\frac{25}{2}\pi - 25\right)\text{cm}^2$$

8. Halla las dimensiones de un rectángulo del que conocemos su perímetro, 34 m y su área 60 m². Largo x, Ancho y

$$\left.\begin{array}{l}2x + 2y = 34 \\ x \cdot y = 60\end{array}\right\} \rightarrow 2y = 34 - 2x \rightarrow y = 17 - x \Rightarrow x(17 - x) = 60$$

$$17x - x^2 = 60 \rightarrow x^2 - 17x + 60 = 0 \rightarrow x = \frac{17 \pm \sqrt{289 - 240}}{2} = \begin{cases}\frac{17+7}{2} = 12 \\ \frac{17-7}{2} = 5\end{cases}$$

Si x = 12 y = 5 , y si x= 5 Y = 12
Solución: las dimensiones son 12 m de largo por 5 m de ancho

9. Hace dos años compré un juego de ordenador y un equipo de música por 260€. Los acabo de vender por un total de 162 €, habiendo perdido el 30% con el juego y el 40% con el equipo de música ¿Cuánto me costó cada cosa?

Precio juego x, Precio equipo de música y

$$\left.\begin{array}{l} x+y=260 \\ \dfrac{70}{100}x+\dfrac{60}{100}y=162 \end{array}\right\} \rightarrow \left.\begin{array}{l} y=260-x \\ 7x+6y=1620 \end{array}\right\} \rightarrow 7x+6(260-x)=1620$$

$$7x+1560-6x=1620 \Rightarrow x=1620-1560=60$$
$$y=260-x=260-60=200$$

Solución: El juego costó 60 euros y el equipo de música 200 euros

10. Solve by graphing and using another method the simultaneous equation:

$$\left.\begin{array}{l} 3x+2y=7 \\ 5x-y=16 \end{array}\right\} \rightarrow \left\{\begin{array}{l} y=\dfrac{7-3x}{2} \\ y=5x-16 \end{array}\right. \rightarrow \dfrac{7-3x}{2}=5x-16 \rightarrow 7-3x=10x-32$$

$$13x=39 \rightarrow x=\dfrac{39}{13}=3 \Rightarrow y=5x-16=5\cdot3-16=15-16=-1 \quad \text{Solution: x=3, y=-1}$$

Graphically: We plot both straight lines