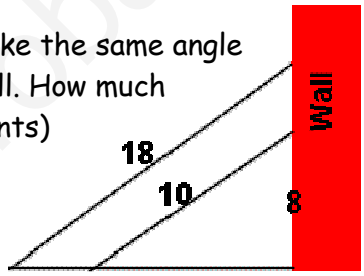
GEOMETRY 2

1. Calculate the sum to 31 terms of an arithmetic progression whose first and tenth terms are 3 and 18, respectively. (1.25 points)

2. The sum of all terms of a geometric progression is 2, and the first term is 5/3. Find the common ratio. (1.25 points)

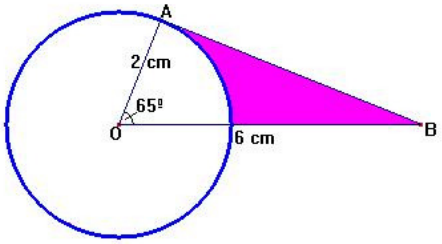
1. Two triangles are similar. The sides of the first triangle are 7, 9, and 11 metres. The smallest side of the second triangle is 21 metres. Find the perimeter of the second triangle. (1.5 p)

3. Two ladders are leaned against a wall such that they make the same angle with the ground. The 10 m ladder reaches 8m up the wall. How much further up the wall does the 18m ladder reach? (1.5 points)

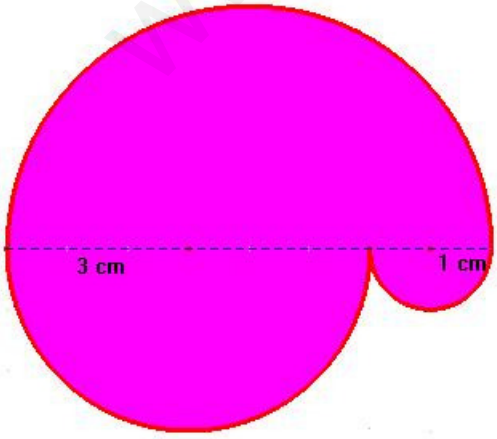


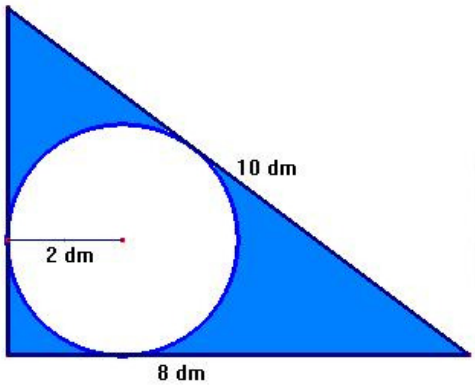
4. A statue that is 12 ft tall casts a shadow that is 15 ft long. Find the length of the shadow that a 8 ft cardboard box casts. (1.5 points)

5. Calculate: (1.5 points)
   a) The length of AB
   b) The area of the shaded part.



6. Find the area and perimeter of the shaded part in each diagram: (1.5 points)





SOLUTION

2. Calculate the sum to 31 terms of an arithmetic progression whose first and tenth terms are 3 and 18, respectively.

AP $a_1 = 3; a_{10} = 18$ $a_n = a_1 + (n-1)d \rightarrow a_{10} = 3 + 9d$

$18 = 3 + 9d \rightarrow 9d = 15 \rightarrow d = \frac{15}{9} = \frac{5}{3}$, $a_{31} = 3 + 30\left(\frac{5}{3}\right) = 53$

$S_{31} = \frac{(a_1 + a_{31})31}{2} = \frac{(3+53)31}{2} = 868$

3. The sum of all terms of a geometric progression is 2, and the first term is 5/3. Find the common ratio. (1.25 points)

GP $a_1 = \frac{5}{3}, S = 2 \rightarrow S = \frac{a_1}{1-r} \rightarrow 2 = \frac{\frac{5}{3}}{1-r} \rightarrow 2(1-r) = \frac{5}{3}$

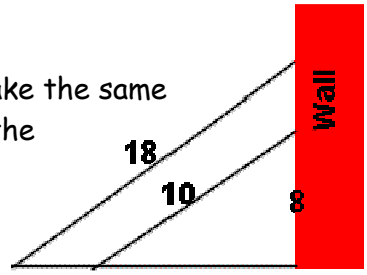
$2 - 2r = \frac{5}{3} \rightarrow 2 - \frac{5}{3} = 2r \rightarrow \frac{1}{3} = 2r \rightarrow r = \frac{1}{6}$

4. Two triangles are similar. The sides of the first triangle are 7, 9, and 11 metres. The smallest side of the second triangle is 21 metres. Find the perimeter of the second triangle. (1.5 p)

$r = \frac{21}{7} = 3 \rightarrow p = 7 + 9 + 11 = 27$

Perimeter of the second triangle: $27 \times 3 = 81$ metres

5. Two ladders are leaned against a wall such that they make the same angle with the ground. The 10 m ladder reaches 8m up the wall. How much further up the wall does the 18m ladder reach?

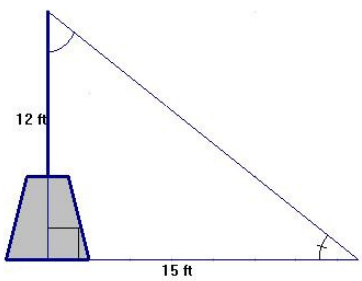


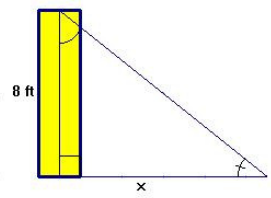
The right triangles are similar (congruent angles), so

$\frac{18}{10} = \frac{x}{8} \rightarrow 18 \times 8 = 10x \rightarrow x = \frac{144}{10} = 14.4$ metres

Solution: 14.4 – 8 = 6.4 metres

6. A statue that is 12 ft tall casts a shadow that is 15 ft long. Find the length of the shadow that a 8 ft cardboard box casts.

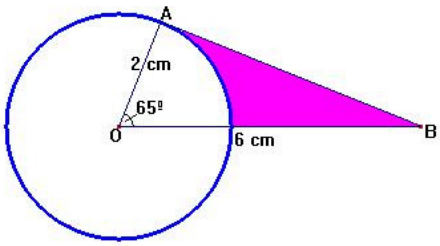




Similar triangles (congruent angles)

$\frac{8}{12} = \frac{x}{15} \rightarrow x = \frac{8 \times 15}{12}$

The length of the shadow of the cardboard box is 10 ft



7. Calculate:
   a) The length of AB
   b) The area of the shaded part.

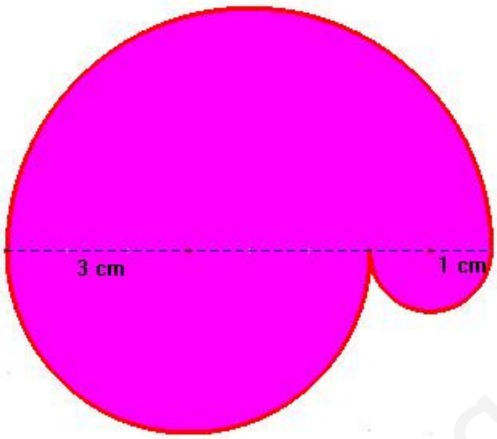
a)It is a right triangle, Pythagorean Theorem:

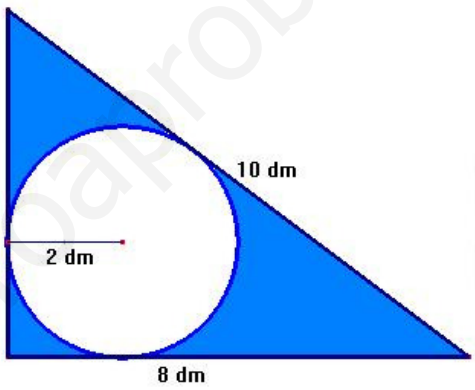
$$2^2 + x^2 = 6^2 \rightarrow x^2 = 32 \rightarrow x = \sqrt{32}\text{cm} \rightarrow \overline{AB} = \sqrt{32}\text{cm}$$

b)The area of the shaded part is: Area of the triangle – Area of the circle sector

$$A_t = \frac{2\sqrt{32}}{2} = \sqrt{32};\ A_{CS} = \frac{\pi \cdot 2^2 \cdot 65}{360} = 2.269 \rightarrow A = \sqrt{32} - 2.269 \approx 3.388\text{cm}^2$$

8. Find the area and perimeter of the shaded part in each diagram:





There are three semi-circles:

$$r=1 \rightarrow A_1 = \frac{\pi \cdot 1^2}{2}$$

$$r=3 \rightarrow A_2 = \frac{\pi \cdot 3^2}{2}$$

$$r=4 \rightarrow A_1 = \frac{\pi \cdot 4^2}{2}$$

$$A = A_1 + A_2 + A_3 = 40.84\text{cm}^2$$

Perimeter: $P = P_1 + P_2 + P_3$ (semicircles)

$$P = \pi \cdot 1 + \pi \cdot 3 + \pi \cdot 4 = 8\pi$$

$$P = 8\pi = 25.13\text{cm}$$

The shaded area is:

$$A = A_{Triangle} - A_{circle}$$

Area triangle, we need the height:

$$10^2 = 8^2 + x^2 \rightarrow x^2 = 100 - 64 \rightarrow x = 6$$

$$A_{Triangle} = \frac{8 \times 6}{2} = 24\text{dm}^2$$

$$A_{Circle} = \pi \times 2^2 = 12.566\text{dm}^2$$

$$A = A_{Triangle} - A_{circle} = 11.43\text{dm}^2$$

Perimeter: $P = P_{Triangle} + P_{circle}$

$$P = (6 + 8 + 10) + 2\pi \cdot 2 = 36.57\text{dm}$$