EXAM 2_2 (PROGRESSIONS /ALGEBRA)

Remember: in each question, write the steps you have taken to reach the solution.

1) For each of the following sequences, determine whether it is arithmetic or geometric. Find the tenth term, and the sum of the first 20 terms. (3.75 points)

a) 3, -1, -5, -9,…

b) 5, 10, 20, …

c) 1, 10, 19,…

2) Find the sum to 10 terms of a geometric progression whose first term is 3 and the common ratio 1/3. Find the sum of all the terms. (1.25 p)

3) Work out and simplify: (1.25 p)

$(2x-3)(2x+3)-4(x+2)^2 =$

4) Solve the following equations: (2.5 p)

a) $\frac{3-x}{4}-\frac{x+1}{6}=2-\frac{2-x}{3}$

b) $(x-1)^2-\frac{3(x+2)(x-2)}{2}=-(2x+1)$

5) Solve the following simultaneous equation: (1.25 p)

$$\left.\begin{array}{l}\frac{x+y}{4}-\frac{x-y}{2}=1\\ 3x-\frac{2y}{3}=13\end{array}\right\}$$

SOLUTION

1) For each of the following sequences, determine whether it is arithmetic or geometric. Find the tenth term, and the sum of the first 20 terms.

(3.75 points)

a) 3, -1, -5, -9,... AP $d=-4,\quad a_1=3 \qquad a_n=3+(n-1)(-4)$

$a_{10}=3+9(-4)=3-36=-33\,;\quad a_{20}=3+19(-4)=3-76=-73$

$$S_{20}=\frac{(a_1+a_{20})\cdot 20}{2}=(3-73)\cdot 10=-700$$

b) 5, 10, 20, ... GP $r=2, a_1=5\rightarrow a_n=5\cdot 2^{n-1}$

$a_{10}=5\cdot 2^9=5\cdot 512=2560\,;\ a_{20}=5\cdot 2^{19}$

$$S_{20}=\frac{5\cdot 2^{19}\cdot 2-5}{2-1}=\frac{5\cdot 2^{20}-5}{1}=5\left(2^{20}-1\right)=5242875$$

c) 1, 10, 19,... AP $d=9,\quad a_1=1 \quad a_n=1+(n-1)\cdot 9=1+9n-9=9n-8$

$a_{10}=9\cdot 10-8=82\,,\ a_{20}=9\cdot 20-8=172$

$$S_{20}=\frac{(a_1+a_{20})20}{2}=(1+172)\cdot 10=1730$$

2) Find the sum to 10 terms of a geometric progression whose first term is 3 and the common ratio 1/3. Find the sum of all the terms.

GP $a_1=3, r=\frac{1}{3}\rightarrow a_{10}=a_1\cdot r^9\Rightarrow a_{10}=3\cdot\left(\frac{1}{3}\right)^9\Rightarrow a_{10}=\frac{3}{3^9}=\frac{1}{3^8}$

$$S_{10}=\frac{a_{10}\cdot r-a_1}{r-1}=\frac{\frac{1}{3^8}\cdot\frac{1}{3}-3}{\frac{1}{3}-1}=\frac{\frac{1}{3^9}-3}{-\frac{2}{3}}$$

$S=\frac{a_1}{1-r}=\frac{3}{1-\frac{1}{3}}=\frac{3}{\frac{2}{3}}=\frac{9}{2}$ The sum of all its terms is $\frac{9}{2}$

3) Work out and simplify:

$(2x-3)(2x+3)-4(x+2)^2=(2x)^2-3^2-4\left(x^2+4x+4\right)=4x^2-9-4x^2-16x-16=-16x-25$

4) Solve the following equations:

a) $\frac{3-x}{4}-\frac{x+1}{6}=2-\frac{2-x}{3}\rightarrow\frac{3(3-x)}{12}-\frac{2(x+1)}{12}=\frac{24}{12}-\frac{4(2-x)}{12}$

$9-3x-2x-2=24-8+4x\rightarrow -5x-4x=24-8+2-9\rightarrow -9x=9\rightarrow x=\frac{9}{-9}=-1$

b) $(x-1)^2-\frac{3(x+2)(x-2)}{2}=-(2x+1)\rightarrow 2(x-1)^2-3(x+2)(x-2)=-2(2x+1)$

$2(x^2-2x+1)-3(x^2-4)=-2(2x+1)\rightarrow 2x^2-4x+2-3x^2+12=-4x-2$

$-x^2-4x+4x+2+2+12=0\rightarrow -x^2=-16\rightarrow x^2=16\rightarrow x=\sqrt{16}=\pm 4$

5) Solve the following simultaneous equation:

$$\left.\begin{array}{l}\frac{x+y}{4}-\frac{x-y}{2}=1\\ 3x-\frac{2y}{3}=13\end{array}\right\}\rightarrow\left.\begin{array}{l}\frac{x+y}{4}-\frac{2(x-y)}{4}=\frac{4}{4}\\ \frac{9x}{3}-\frac{2y}{3}=\frac{39}{3}\end{array}\right\}\rightarrow\left.\begin{array}{l}x+y-2x+2y=4\\ 9x-2y=39\end{array}\right\}\rightarrow\left.\begin{array}{l}-x+3y=4\\ 9x-2y=39\end{array}\right\}$$

$$\left.\begin{array}{l}-x+3y=4\\ 9x-2y=39\end{array}\right\}\rightarrow x=3y-4\rightarrow 9(3y-4)-2y=39\rightarrow 27y-36-2y=39\rightarrow 25y=75$$

$25y=75\rightarrow y=3\Rightarrow x=3y-4\rightarrow x=9-4=5\rightarrow$ Solution : x = 5, y = 3