

EXAM UNIT 5 (EQUATIONS)

Name:.....

1) Solve the following equations: (5 points)

a. $2x - 3(x + 7) + 2x - 1 = 2(x - 3) - x$

b. $\frac{x-5}{2} + \frac{x+1}{3} - 1 = \frac{x+4}{3} - \frac{x-1}{2}$

c. $(2x-1)^2 - 3x + 1 = (2x-3)(2x+3) - 3$

d. $\frac{3x+2}{5} - \frac{4x-1}{10} + \frac{5x-2}{8} = \frac{x+1}{4}$

e. $\frac{3x+1}{2} = 2x - \frac{x-1}{2}$

2) Solve the following equations: (4 points)

a. $9x^2 - 12x + 4 = 0$

b. $4x^2 - 9 = 0$

c. $(x+4)^2 - (2x-1)^2 = 8x$

d. $\frac{(5x-4)(5x+4)}{4} = \frac{(3x-1)^2 - 9}{2}$

3) Translate into an equation: (1 point)

a. Ten subtracted from five times a number is equal to twenty.

b. One third of a number minus four is equal to seventeen.

c. A number plus three, all multiplied by eight is equal to forty-eight.

SOLUTION

1) Solve the following equations:

a. $2x - 3(x + 7) + 2x - 1 = 2(x - 3) - x$

$$2x - 3x - 21 + 2x - 1 = 2x - 6 - x \rightarrow 2x - 3x + 2x - 2x + x = -6 + 21 + 1$$

$$0x = 16 \rightarrow \text{No solution}$$

b. $\frac{x-5}{2} + \frac{x+1}{3} - 1 = \frac{x+4}{3} - \frac{x-1}{2} \rightarrow \frac{3(x-5)}{6} + \frac{2(x+1)}{6} - \frac{6}{6} = \frac{2(x+4)}{6} - \frac{3(x-1)}{6}$

$$3x - 15 + 2x + 2 - 6 = 2x + 8 - 3x + 3 \rightarrow 3x + 2x - 2x + 3x = 8 + 3 + 6 - 2 + 15$$

$$6x = 8 + 3 + 6 - 2 + 15 \rightarrow 6x = 30 \rightarrow x = \frac{30}{6} = 5 \quad \text{Solution: } x = 5$$

c. $(2x-1)^2 - 3x + 1 = (2x-3)(2x+3) - 3 \rightarrow 4x^2 - 4x + 1 - 3x + 1 = 4x^2 - 9 - 3$

$$-4x - 3x = -9 - 3 - 2 \rightarrow -7x = -14 \rightarrow x = \frac{14}{7} = 2 \quad \text{Solution: } x = 2$$

d. $\frac{3x+2}{5} - \frac{4x-1}{10} + \frac{5x-2}{8} = \frac{x+1}{4} \rightarrow \frac{8(3x+2)}{40} - \frac{4(4x-1)}{40} + \frac{5(5x-2)}{40} = \frac{10(x+1)}{40}$

$$24x + 16 - 16x + 4 + 25x - 10 = 10x + 10 \rightarrow 24x - 16x + 25x - 10x = 10 + 10 - 4 - 16$$

$$23x = 0 \rightarrow x = \frac{0}{23} = 0 \quad \text{Solution: } x = 0$$

e. $\frac{3x+1}{2} = 2x - \frac{x-1}{2} \rightarrow \frac{3x+1}{2} = \frac{4x}{2} - \frac{x-1}{2} \rightarrow 3x + 1 = 4x - x + 1$

$$3x - 4x + x = 1 - 1 \rightarrow 0x = 0 \quad \text{It is an identity} \quad \text{Solution: all numbers}$$

2) Solve the following equations:

a. $9x^2 - 12x + 4 = 0$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4 \cdot 9 \cdot 4}}{2 \cdot 9} = \frac{12 \pm \sqrt{144 - 144}}{18} = \frac{12}{18} = \frac{2}{3} \quad \text{Solution: } \frac{2}{3} \text{ (just one)}$$

b. $4x^2 - 9 = 0 \rightarrow 4x^2 = 9 \rightarrow x^2 = \frac{9}{4} \rightarrow x = \pm \sqrt{\frac{9}{4}} \rightarrow x = \pm \frac{3}{2} \quad \text{Solution: } \pm \frac{3}{2}$

c. $(x+4)^2 - (2x-1)^2 = 8x \rightarrow (x^2 + 8x + 16) - (4x^2 - 4x + 1) = 8x$

$$x^2 + 8x + 16 - 4x^2 + 4x - 1 = 8x \rightarrow -3x^2 + 4x + 15 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-3) \cdot 15}}{2 \cdot (-3)} = \frac{-4 \pm \sqrt{16 + 180}}{-6} = \frac{-4 \pm \sqrt{196}}{-6} = \frac{-4 \pm 14}{-6} = \begin{cases} \frac{3}{-6} \\ \frac{10}{-6} \end{cases} = \begin{cases} -\frac{5}{3} \\ -\frac{5}{3} \end{cases}$$

$$\text{Solution: } 3 \text{ and } -\frac{5}{3}$$

d. $\frac{(5x-4)(5x+4)}{4} = \frac{(3x-1)^2 - 9}{2}$

$$\frac{25x^2 - 16}{4} = \frac{9x^2 - 6x + 1 - 9}{2} \rightarrow 25x^2 - 16 = 18x^2 - 12x - 16 \rightarrow 7x^2 + 12x = 0$$

$$x(7x+12) = 0 \begin{cases} x=0 \\ 7x+12=0 \rightarrow x=-\frac{12}{7} \end{cases}$$

Solution: 0 and $-\frac{12}{7}$

3) Translate into an equation:

d. Ten subtracted from five times a number is equal to twenty.

$$5x - 10 = 20$$

e. One third of a number minus four is equal to seventeen.

$$\frac{x}{3} - 4 = 17$$

f. A number plus three, all multiplied by eight is equal to forty-eight.

$$8(x+3) = 48$$