Resuelve las siguientes inecuaciones racionales:

a) $\dfrac{5x-2}{2x+1} \geq 0$

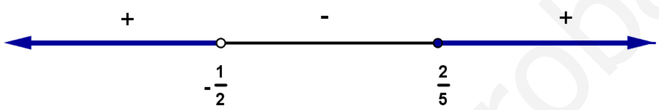
- Ceros

  $5x-2=0 \Rightarrow \underline{x=\dfrac{2}{5}}$

- Polos

  $2x+1=0 \Rightarrow \underline{x=-\dfrac{1}{2}}$

$x=-1 \Rightarrow \dfrac{(-)}{(-)}=+$ $x=0 \Rightarrow \dfrac{(-)}{(+)}=-$ $x=1 \Rightarrow \dfrac{(+)}{(+)}=+$



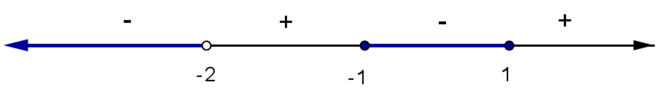
Solución: $x \in \left(-\infty, -\dfrac{1}{2}\right) \cup \left[\dfrac{2}{5}, +\infty\right)$

b) $\dfrac{x^2-1}{x+2} \leq 0$

- Ceros: $x^2-1=0 \Rightarrow x^2=1 \Rightarrow \begin{cases} \underline{x=-1} \\ \underline{x=1} \end{cases}$
- Polos: $x+2=0 \Rightarrow \underline{x=-2}$
- Luego, factorizando, tenemos: $\underline{\dfrac{x^2-1}{x+2}<0 \Leftrightarrow \dfrac{(x-1)(x+1)}{(x+2)} \leq 0}$

$x=-3 \Rightarrow \dfrac{(-)(-)}{(-)}=-$ $x=0 \Rightarrow \dfrac{(-)(+)}{(+)}=-$

$x=-1{,}5 \Rightarrow \dfrac{(-)(-)}{(+)}=+$ $x=2 \Rightarrow \dfrac{(+)(+)}{(+)}=+$



Solución: $x \in (-\infty, -2) \cup [-1, 1]$

c) $\dfrac{x^2-5x+4}{x^2-5x+6} \geq 0$

- Ceros: $x^2-5x+4=0 \Rightarrow x=\dfrac{5\pm\sqrt{25-16}}{2}=\dfrac{5\pm 3}{2}=\begin{cases}\underline{x=4}\\ \underline{x=1}\end{cases}$
- Polos: $x^2-5x+6=0 \Rightarrow x=\dfrac{5\pm\sqrt{25-24}}{2}=\dfrac{5\pm 1}{2}=\begin{cases}\underline{x=3}\\ \underline{x=2}\end{cases}$

- Luego, factorizando, tenemos: $\underline{\dfrac{x^2-5x+4}{x^2-5x+6} \geq 0 \Leftrightarrow \dfrac{(x-4)(x-1)}{(x-3)(x-2)} \geq 0}$

$x=0 \Rightarrow \dfrac{(-)(-)}{(-)(-)}=+$

$x=1{,}5 \Rightarrow \dfrac{(-)(+)}{(-)(-)}=-$

$x=2{,}5 \Rightarrow \dfrac{(-)(+)}{(-)(+)}=+$

$x=3{,}5 \Rightarrow \dfrac{(-)(+)}{(+)(+)}=-$

$x=4 \Rightarrow \dfrac{(+)(+)}{(+)(+)}=+$

Solución: $x \in (-\infty,1] \cup (2,3) \cup [4,+\infty)$

d) $\dfrac{x^3+x^2-5x+3}{x^3+5x^2+3x-9} \leq 0$

- Ceros: $x^3+x^2-5x+3=0$

Posibles raíces enteras = {divisores de 3} = $\{\pm 1, \pm 3\}$

	1	+1	−5	+3
1		+1	+2	−3
	1	+2	−3	0

$\Rightarrow x^3+x^2-5x+3=(x-1)(x^2+2x-3)$

Luego $x^3+x^2-5x+3=0 \Leftrightarrow (x-1)(x^2+2x-3)=0 \Leftrightarrow \begin{cases}x-1=0 \Rightarrow \underline{x=1}\\ x^2+2x-3=0 \ \ (*)\end{cases}$

$(*)\ \ x^2+2x-3=0 \Rightarrow x=\dfrac{-2\pm\sqrt{4+12}}{2}=\dfrac{-2\pm 4}{2}=\begin{cases}\underline{x=1}\\ \underline{x=-3}\end{cases}$

Por tanto, $\underline{Ceros \rightarrow x = 1 (\text{doble}) \quad x = -3}$

- $\underline{\text{Polos}}$: $x^3 + 5x^2 + 3x - 9 = 0$

Posibles raíces enteras = {divisores de -9 } = $\{\pm 1, \pm 3, \pm 9\}$

	1	+5	+3	−9
1		+1	+6	+9
	1	+6	+9	0

$\Rightarrow x^3 + 5x^2 + 3x - 9 = (x-1)(x^2 + 6x + 9)$

Luego $x^3 + 5x^2 + 3x - 9 = 0 \Leftrightarrow (x-1)(x^2+6x+9) = 0 \Leftrightarrow \begin{cases} x - 1 = 0 \Rightarrow \underline{x = 1} \\ x^2 + 6x + 9 = 0 \Rightarrow (x+3)^2 = 0 \Rightarrow x = -3\,(\text{doble}) \end{cases}$

Por tanto, $\underline{Polos \rightarrow x = -3(\text{doble}) \quad x = 1}$

- Luego, factorizando, tenemos: $\underline{\dfrac{x^3 + x^2 - 5x + 3}{x^3 + 5x^2 + 3x - 9} \le 0 \Leftrightarrow \dfrac{(x-1)^2(x+3)}{(x+3)^2(x-1)} \le 0}$

$x = -4 \Rightarrow \dfrac{(+)(-)}{(+)(-)} = +$ $\qquad$ $x = 0 \Rightarrow \dfrac{(+)(+)}{(+)(-)} = -$ $\qquad$ $x = 2 \Rightarrow \dfrac{(+)(+)}{(+)(+)} = +$

<u>Solución</u>: $x \in (-3, -1)$

e) $\dfrac{x^2 - 2x - 3}{x^2 + x - 2} < 0$

- $\underline{\text{Ceros}}$: $x^2 - 2x - 3 = 0 \Rightarrow x = \dfrac{2 \pm \sqrt{4+12}}{2} = \dfrac{2 \pm 4}{2} = \begin{cases} \underline{x = 3} \\ \underline{x = -1} \end{cases}$

- $\underline{\text{Polos}}$: $x^2 + x - 2 = 0 \Rightarrow x = \dfrac{-1 \pm \sqrt{1+8}}{2} = \dfrac{-1 \pm 3}{2} = \begin{cases} \underline{x = 1} \\ \underline{x = -2} \end{cases}$

- Luego, factorizando, tenemos: $\underline{\dfrac{x^2 - 2x - 3}{x^2 + x - 2} < 0 \Leftrightarrow \dfrac{(x-3)(x+1)}{(x-1)(x+2)} < 0}$

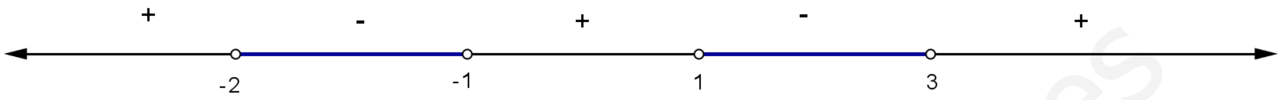
$x=-3 \Rightarrow \frac{(-)(-)}{(-)(-)}=+$

$x=-1{,}5 \Rightarrow \frac{(-)(-)}{(-)(+)}=-$

$x=0 \Rightarrow \frac{(-)(+)}{(-)(+)}=+$

$x=2 \Rightarrow \frac{(-)(+)}{(+)(+)}=-$

$x=4 \Rightarrow \frac{(+)(+)}{(+)(+)}=+$



<u>Solución</u>: $x \in (-2,-1) \cup (1,3)$

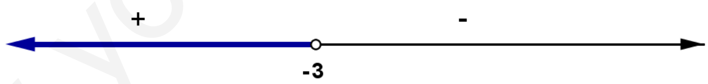
f) $\frac{x-1}{x+3}-1>0 \Rightarrow \frac{x-1-1\cdot(x+3)}{x+3}>0 \Rightarrow \frac{x-1-x-3}{x+3}>0 \Rightarrow \underline{\frac{-4}{x+3}>0}$

- <u>Ceros</u>: No tiene

- <u>Polos</u>: $x+3=0 \Rightarrow x=-3$

$x=-4 \Rightarrow \frac{(-)}{(-)}=+$

$x=-2 \Rightarrow \frac{(-)}{(+)}=-$



<u>Solución</u>: $x \in (-\infty,-3)$

g) $\frac{x^2}{x^2-2} \le 2 \Rightarrow \frac{x^2}{x^2-2}-2 \le 0 \Rightarrow \frac{x^2-2(x^2-2)}{x^2-2} \le 0 \Rightarrow \frac{x^2-2x^2+4}{x^2-2} \le 0 \Rightarrow \underline{\frac{-x^2+4}{x^2-2} \le 0}$

- <u>Ceros</u>: $-x^2+4=0 \Rightarrow x^2=4 \Rightarrow \begin{cases} \underline{x=-2} \\ \underline{x=2} \end{cases}$

- <u>Polos</u>: $x^2-2=0 \Rightarrow x^2=2 \Rightarrow \begin{cases} \underline{x=-\sqrt{2}} \\ \underline{x=\sqrt{2}} \end{cases}$

- Luego, factorizando, tenemos: $\dfrac{-x^2+4}{x^2-2}\leq 0 \Leftrightarrow \dfrac{-1(x-2)(x+2)}{(x-\sqrt{2})(x+\sqrt{2})}\leq 0 \xrightarrow{:(-1)} \underline{\dfrac{(x-2)(x+2)}{(x-\sqrt{2})(x+\sqrt{2})}\geq 0}$

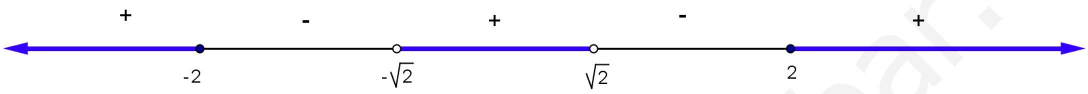
$x=-3 \Rightarrow \dfrac{(-)(-)}{(-)(-)}=+$

$x=-1{,}5 \Rightarrow \dfrac{(-)(+)}{(-)(-)}=-$

$x=0 \Rightarrow \dfrac{(-)(+)}{(-)(+)}=+$

$x=1{,}5 \Rightarrow \dfrac{(-)(+)}{(+)(+)}=-$

$x=3 \Rightarrow \dfrac{(+)(+)}{(+)(+)}=+$



Solución: $x\in(-\infty,-2]\cup\left(-\sqrt{2},\sqrt{2}\right)\cup[2,+\infty)$

h) $\dfrac{x^2-5}{x^2+3}<x \Rightarrow \dfrac{x^2-5}{x^2+3}-x<0 \Rightarrow \dfrac{x^2-5-x(x^2+3)}{x^2+3}<0 \Rightarrow \dfrac{x^2-5-x^3-3x}{x^2+3}<0 \Rightarrow \underline{\dfrac{-x^3+x^2-3x-5}{x^2+3}<0}$

- Ceros: $-x^3+x^2-3x-5=0$

Posibles raíces enteras = {divisores de -5 } = $\{\pm 1, \pm 5\}$

	-1	$+1$	-3	-5
-1		$+1$	-2	$+5$
	-1	$+2$	-5	0

$\Rightarrow -x^3+x^2-3x-5=(x+1)(-x^2+2x-5)$

Luego $-x^3+x^2-3x-5=0 \Leftrightarrow (x+1)(-x^2+2x-5)=0 \Leftrightarrow \begin{cases} x+1=0 \Rightarrow \underline{x=-1} \\ -x^2+2x-5=0 \ (*) \end{cases}$

(*) $-x^2+2x-5=0 \Rightarrow x=\dfrac{-2\pm\sqrt{4-20}}{-2} \Rightarrow \nexists$ solución real

Por tanto, $\underline{\textit{Ceros} \rightarrow x=-1}$

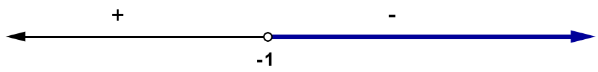
- Polos: $x^2+3=0 \Rightarrow x^2=-3 \Rightarrow \nexists$ solución real

Por tanto, <u>*Polos* → No tiene</u>

- Luego, factorizando, tenemos: $\underline{\dfrac{-x^3+x^2-3x-5}{x^2+3}<0 \Leftrightarrow \dfrac{(x+1)(-x^2+2x-5)}{x^2+3}<0}$

$$x=-2 \Rightarrow \frac{(-)(-)}{(+)}=+$$

$$x=0 \Rightarrow \frac{(+)(-)}{(+)}=-$$



<u>Solución</u>: $x \in (-1,+\infty)$

i) $\dfrac{1}{x-1}<\dfrac{2}{x+4} \Rightarrow \dfrac{1}{x-1}-\dfrac{2}{x+4}<0 \Rightarrow \dfrac{1(x+4)-2(x-1)}{(x-1)(x+4)}<0 \Rightarrow \dfrac{x+4-2x+2}{(x-1)(x+4)}<0 \Rightarrow \dfrac{6-x}{(x-1)(x+4)}<0$

<u>Por tanto, hay que resolver la inecuación</u>: $\dfrac{6-x}{(x-1)(x+4)}<0$

- <u>Ceros</u>: $6-x=0 \Rightarrow \underline{x=6}$

- <u>Polos</u>: $(x-1)(x+4)=0 \Rightarrow \begin{cases} \underline{x=1} \\ \underline{x=-4} \end{cases}$

- $\dfrac{6-x}{(x-1)(x+4)}<0$

$x=-5 \Rightarrow \dfrac{(+)}{(-)(-)}=+$

$x=0 \Rightarrow \dfrac{(+)}{(-)(+)}=-$

$x=2 \Rightarrow \dfrac{(+)}{(+)(+)}=+$

$x=7 \Rightarrow \dfrac{(-)}{(+)(+)}=-$

<u>Solución</u>: $x \in (-4,1) \cup (6,+\infty)$

j) $\dfrac{3x-2}{x-1}-1\geq\dfrac{2x-1}{x+1}\Rightarrow\dfrac{3x-2}{x-1}-1-\dfrac{2x-1}{x+1}\geq 0\Rightarrow\dfrac{(3x-2)(x+1)-(x-1)(x+1)-(2x-1)(x-1)}{(x-1)(x+1)}\geq 0\Rightarrow$

$\Rightarrow\dfrac{(3x^2+3x-2x-2)-(x^2-1)-(2x^2-2x-x+1)}{(x-1)(x+1)}\geq 0\Rightarrow$

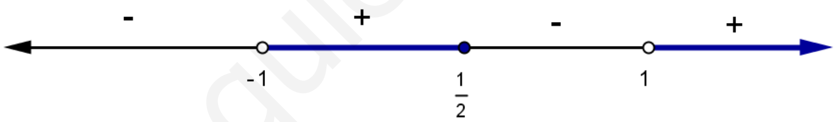
$\Rightarrow\dfrac{3x^2+3x-2x-2-x^2+1-2x^2+2x+x-1}{(x-1)(x+1)}\geq 0\Rightarrow\dfrac{4x-2}{(x-1)(x+1)}\geq 0$

<u>Por tanto, hay que resolver la inecuación</u>: $\dfrac{4x-2}{(x-1)(x+1)}\geq 0$

- <u>Ceros</u>: $4x-2=0\Rightarrow \underline{x=\dfrac{1}{2}}$
- <u>Polos</u>: $(x-1)(x+1)=0\Rightarrow\begin{cases}\underline{x=1}\\ \underline{x=-1}\end{cases}$
- $\dfrac{4x-2}{(x-1)(x+1)}\geq 0$

$x=-2\Rightarrow\dfrac{(-)}{(-)(-)}=-$ $\qquad$ $x=0{,}75\Rightarrow\dfrac{(+)}{(-)(+)}=-$

$x=0\Rightarrow\dfrac{(-)}{(-)(+)}=+$ $\qquad$ $x=2\Rightarrow\dfrac{(+)}{(+)(+)}=+$



<u>Solución</u>: $x\in\left(-1,\dfrac{1}{2}\right]\cup(1,+\infty)$

k) $\dfrac{1}{x+3}-\dfrac{1}{x^2-9}\leq\dfrac{-1}{x-3}\Rightarrow\dfrac{1}{x+3}-\dfrac{1}{(x-3)(x+3)}+\dfrac{1}{x-3}\leq 0\Rightarrow\dfrac{x-3-1+x+3}{(x-3)(x+3)}\leq 0\Rightarrow\dfrac{2x-1}{(x-3)(x+3)}\leq 0$

<u>Por tanto, hay que resolver la inecuación</u>: $\dfrac{2x-1}{(x-3)(x+3)}\leq 0$

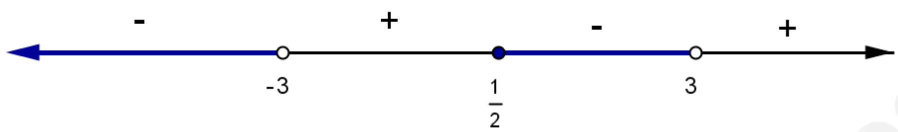
- <u>Ceros</u>: $2x-1=0\Rightarrow \underline{x=\dfrac{1}{2}}$
- <u>Polos</u>: $(x-3)(x+3)=0\Rightarrow\begin{cases}\underline{x=3}\\ \underline{x=-3}\end{cases}$
- $\dfrac{2x-1}{(x-3)(x+3)}\leq 0$

$x=-4 \Rightarrow \dfrac{(-)}{(-)(-)}=-$

$x=0 \Rightarrow \dfrac{(-)}{(-)(+)}=+$

$x=1 \Rightarrow \dfrac{(+)}{(-)(+)}=-$

$x=4 \Rightarrow \dfrac{(+)}{(+)(+)}=+$



<u>Solución</u>: $x \in (-\infty,-3) \cup \left[\dfrac{1}{2},3\right)$

l) $\dfrac{-x}{x-2} \leq \dfrac{x+1}{x+2} - \dfrac{6x+1}{x^2-4} \Rightarrow \dfrac{6x+1}{(x-2)(x+2)} - \dfrac{x+1}{x+2} - \dfrac{x}{x-2} \leq 0 \Rightarrow$

$\Rightarrow \dfrac{6x+1-(x-2)(x+1)-x(x+2)}{(x-2)(x+2)} \leq 0 \Rightarrow \dfrac{6x+1-x^2+2x-x+2-x^2-2x}{(x-2)(x+2)} \leq 0 \Rightarrow \dfrac{-2x^2+5x+3}{(x-2)(x+2)} \leq 0$

<u>Por tanto, hay que resolver la inecuación</u>: $\dfrac{-2x^2+5x+3}{(x-2)(x+2)} \leq 0$

- <u>Ceros</u>: $-2x^2+5x+3=0$

$$x=\frac{-5 \pm \sqrt{(5)^2-4\cdot(-2)\cdot 3}}{2\cdot(-2)}=\frac{-5 \pm \sqrt{25+24}}{-4}=\frac{-5 \pm 7}{-4}=\begin{cases} x=-\dfrac{1}{2} \\ \underline{x=3} \end{cases}$$

- <u>Polos</u>: $(x-2)(x+2)=0 \Rightarrow \begin{cases} \underline{x=2} \\ \underline{x=-2} \end{cases}$

- Luego, factorizando, tenemos: $\underline{\dfrac{-2x^2+5x+3}{(x-2)(x+2)} \leq 0 \Leftrightarrow \dfrac{-2\left(x+\frac{1}{2}\right)(x-3)}{(x-2)(x+2)} \leq 0}$

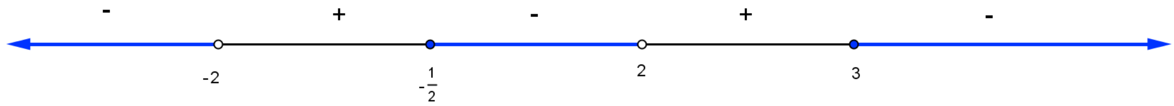
$x=-3 \Rightarrow \dfrac{(-)(-)(-)}{(-)(-)}=-$

$x=-1 \Rightarrow \dfrac{(-)(-)(-)}{(-)(+)}=+$

$x=0 \Rightarrow \dfrac{(-)(+)(-)}{(-)(+)}=-$

$x=2{,}5 \Rightarrow \dfrac{(-)(+)(-)}{(+)(+)}=+$

$x=4 \Rightarrow \dfrac{(-)(+)(+)}{(+)(+)}=-$



Solución: $x \in (-\infty,-2) \cup \left[-\frac{1}{2},2\right) \cup [3,+\infty)$

m) $x^2 - \frac{64}{x^2} \geq -12 \Rightarrow x^2 - \frac{64}{x^2} + 12 \geq 0 \Rightarrow \frac{x^4 - 64 + 12x^2}{x^2} \geq 0 \Rightarrow \frac{x^4 + 12x^2 - 64}{x^2} \geq 0$

Por tanto, hay que resolver la inecuación: $\underline{\frac{x^4 + 12x^2 - 64}{x^2} \geq 0}$

➢ Ceros: $x^4 + 12x^2 - 64 = 0$

1) Hacemos el cambio de variable $\underline{x^2 = t}$ y la ecuación se convierte en la ecuación de 2º grado:

$$t^2 + 12t - 64 = 0$$

2) Resolvemos la ecuación de segundo grado:

$$t^2 + 12t - 64 = 0 \Leftrightarrow t = \frac{-12 \pm \sqrt{144 + 256}}{2} = \frac{-12 \pm 20}{2} = \begin{cases} t = 4 \\ t = -16 \end{cases}$$

3) Deshacemos el cambio de variable

- $t = 4 \Rightarrow x^2 = 4 \Rightarrow x = \sqrt{4} \Rightarrow \underline{x = \pm 2}$
- $t = -16 \Rightarrow x^2 = -16 \Rightarrow x = \sqrt{-16} \Rightarrow$ no tiene solución real

➢ Polos: $x^2 = 0 \Rightarrow \underline{x = 0}$

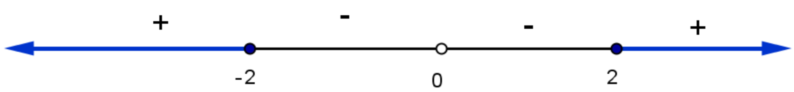
➢ $\underline{\frac{x^4 + 12x^2 - 64}{x^2} \geq 0}$

$x = -3 \Rightarrow \frac{(+)}{(+)} = +$

$x = -1 \Rightarrow \frac{(-)}{(+)} = -$

$x = 1 \Rightarrow \frac{(-)}{(+)} = -$

$x = 3 \Rightarrow \frac{(+)}{(+)} = +$



Solución: $x \in (-\infty,-2] \cup [2,+\infty)$

n) $\frac{3x-3}{x-1} \leq \frac{7x+1}{x^2-1} - \frac{x^2+2}{x+1} \Rightarrow$

$$\Rightarrow \frac{3x-3}{x-1} - \frac{7x+1}{(x-1)(x+1)} + \frac{x^2+2}{x+1} \leq 0 \Rightarrow \frac{(3x-3)(x+1)-(7x+1)+(x^2+2)(x-1)}{(x-1)(x+1)} \leq 0$$

$$\Rightarrow \frac{3x^2+3x-3x-3-7x-1+x^3-x^2+2x-2}{(x-1)(x+1)} \leq 0 \Rightarrow \frac{x^3+2x^2-5x-6}{(x-1)(x+1)} \leq 0$$

<u>Por tanto, hay que resolver la inecuación</u>: $\frac{x^3+2x^2-5x-6}{(x-1)(x+1)} \leq 0$

- <u>Ceros</u>: $x^3+2x^2-5x-6=0$

$$\begin{array}{r|rrrr} & 1 & +2 & -5 & -6 \\ -1 & & -1 & -1 & +6 \\ \hline & 1 & +1 & -6 & \boxed{0} \end{array} \Rightarrow x^3+2x^2-5x-6=(x+1)(x^2+x-6)$$

$$x^3+2x^2-5x-6=0 \Leftrightarrow (x+1)(x^2+x-6)=0 \Leftrightarrow \begin{cases} x+1=0 \Rightarrow \underline{x=-1} \\ x^2+x-6=0 \ \ (*) \end{cases}$$

$$(*) \ \ x^2+x-6=0 \Rightarrow x=\frac{-1\pm\sqrt{1+24}}{2}=\frac{-1\pm 5}{2}=\begin{cases} \underline{x=2} \\ \underline{x=-3} \end{cases}$$

Por tanto, los ceros del polinomio son**:** $x=-1$, $x=2$ y $x=-3$

- <u>Polos</u>: $(x-1)(x+1)=0 \Rightarrow \begin{cases} \underline{x=1} \\ \underline{x=-1} \end{cases}$

- Luego, factorizando, tenemos: $\underline{\frac{x^3+2x^2-5x-6}{(x-1)(x+1)} \leq 0 \Leftrightarrow \frac{(x+1)(x-2)(x+3) \leq 0}{(x-1)(x+1)}}$

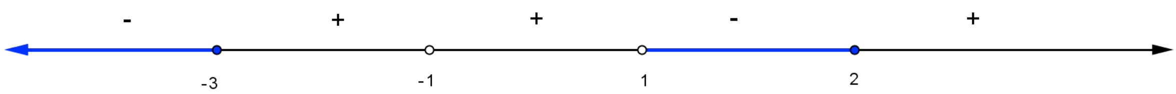
$x=-4 \Rightarrow \frac{(-)(-)(-)}{(-)(-)}=-$

$x=-2 \Rightarrow \frac{(-)(-)(+)}{(-)(-)}=+$

$x=0 \Rightarrow \frac{(+)(-)(+)}{(-)(+)}=+$

$x=1{,}5 \Rightarrow \frac{(+)(-)(+)}{(+)(+)}=-$

$$x = 3 \Rightarrow \frac{(+)(+)(+)}{(+)(+)} = +$$



Solución: $x \in (-\infty, -3] \cup (1, 2]$