$$\int xe^{x}dx$$ Sol: $e^{x}(x-1)+k$

Por el método de <u>*integración por partes:*</u>

$$\int xe^{x}dx=\left\{\begin{array}{l} u=x\Rightarrow du=dx \\ dv=e^{x}dx\Rightarrow v=e^{x}\end{array}\right\}=xe^{x}-\int e^{x}dx=xe^{x}-e^{x}+k=(x-1)e^{x}+k$$

$$I=\int(x^{2}-3x+5)e^{x}dx$$ Sol: $e^{x}(x^{2}-5x+10)+k$

Por el método de integración por partes:

$$I=\int(x^{2}-3x+5)e^{x}dx=\left\{\begin{array}{l} u=x^{2}-3x+5\Rightarrow du=(2x-3)dx \\ dv=e^{x}dx\Rightarrow v=e^{x}\end{array}\right\}=$$

$$=(x^{2}-3x+5)\ \ e\ \ -\int e^{x}(2x-3)dx=$$

La integral que nos ha quedado es del mismo tipo que la que pretendemos calcular, por lo que nuevamente aplicaremos el método de integración de partes:

Hacemos $\left\{\begin{array}{l} u=2x-3\Rightarrow du=2dx \\ dv=e^{x}dx\Rightarrow v=e^{x}\end{array}\right\}$ y sustituimos:

$$I=(x^{2}-3x+5)e^{x}-\int e^{x}(2x-3)dx=(x^{2}-3x+5)e^{x}-\left[(2x-3e^{x}-\int 2e^{x}dx\right]=$$

$$=(x^{2}-3x+5)e^{x}-(2x-3)e^{x}+2\int e^{x}dx=(x^{2}-3x+5)e^{x}-(2x-3)e^{x}+2e^{x}+k=$$

$$=\left[(x^{2}-3x+5)-(2x-3)+2\right]e^{x}+k=e^{x}(x^{2}-5x+10)+k$$

$$\int x\,\mathrm{Ln}(x)dx$$ Sol: $\frac{1}{2}x^{2}\left(\mathrm{Ln}(x)-\frac{1}{2}\right)+k$

Por el método de integración por partes:

$$\int x\,\mathrm{Ln}(x)dx=\left\{\begin{array}{l} u=\mathrm{Ln}(x)\Rightarrow du=\frac{1}{x}dx \\ dv=xdx\Rightarrow v=\frac{1}{2}x^{2}\end{array}\right\}=\frac{1}{2}x^{2}\,\mathrm{Ln}(x)-\int\frac{1}{2}x^{2}\cdot\frac{1}{x}dx=$$

$$=\frac{1}{2}x^{2}\,\mathrm{Ln}(x)-\frac{1}{2}\int x\,dx=\frac{1}{2}x^{2}\,\mathrm{Ln}(x)-\frac{1}{2}\cdot\frac{1}{2}x^{2}+k=\frac{1}{2}x^{2}\left(\mathrm{Ln}(x)-\frac{1}{2}\right)+k$$

$$\int e^{-x}\cos x dx \qquad \text{Sol: } \frac{1}{2}e^{-x}(\text{sen}\, x-\cos x)+k$$

$$I=\int e^{-x}\cos x dx=\left\{\begin{matrix} u=e^{-x} \Rightarrow du=-e^{-x}dx \\ dv=\cos x dx \Rightarrow v=\text{sen}\, x\end{matrix}\right\}=e^{-x}\text{sen}\, x-\int -e^{-x}\,\text{sen}\, x dx=$$

$$=e^{-x}\text{sen}\, x+\int e^{-x}\,\text{sen}\, x dx$$

Al aplicar el método de partes nos ha quedado una integral del mismo tipo que la que pretendemos calcular, por lo que volvemos a aplicar el mismo método. En ella hacemos:

$$\begin{matrix} u=e^{-x} & \Rightarrow & du=-e \quad dx \\ dv=\text{sen}\, x\, dx & \Rightarrow & v=-\cos x\end{matrix}$$

Sustituyendo en la expresión anterior nos queda:

$$I=e^{-x}\text{sen}\, x+\int e^{-x}\,\text{sen}\, x dx=e^{-x}\text{sen}\, x+\left[-\cos x\cdot e^{-x}-\int -\cos x\cdot(-e^{-x})dx\right]=$$

$$=e^{-x}\text{sen}\, x-\cos x\cdot e^{-x}-\int \cos x\cdot e^{-x}dx$$

es decir, volvemos a la misma integral que pretendemos calcular. Entonces:

$$I=e^{-x}\text{sen}\, x-\cos x\cdot e^{-x}-I \quad\Rightarrow\quad 2I=e^{-x}\text{sen}\, x-\cos x\cdot e^{-x} \quad\Rightarrow\quad I=\frac{e^{-x}(\text{sen}\, x-\cos x)}{2}$$

En consecuencia:

$$I=\int e^{-x}\cos x dx=\frac{e^{-x}(\text{sen}\, x-\cos x)}{2}+k$$

$$\int \text{Ln}(1-x)dx \qquad \text{Sol: } -x-(1-x)\text{Ln}(1-x)+k$$

$$\int \text{Ln}(1-x)dx=\left\{\begin{matrix} u=\text{Ln}(1-x)\Rightarrow du=\frac{-1}{1-x}dx \\ dv=dx\Rightarrow v=x\end{matrix}\right\}=x\text{Ln}(1-x)-\int x\cdot\frac{-1}{1-x}dx=$$

$$=x\text{Ln}(1-x)-\int\frac{-x}{1-x}dx=x\text{Ln}(1-x)-\int\frac{1-x-1}{1-x}dx=x\text{Ln}(1-x)-\int\left(1+\frac{-1}{1-x}\right)dx=$$

$$=x\text{Ln}(1-x)-\left(x+\text{Ln}(1-x)\right)+k=x\text{Ln}(1-x)-x-\text{Ln}(1-x)+k=$$

$$=-x-(1-x)\text{Ln}(1-x)+k$$

$$\int x^n \,\mathrm{Ln}(x)\,dx \qquad \text{Sol: } \frac{x^{n+1}}{n+1}\left(\mathrm{Ln}(x)-\frac{1}{n+1}\right)+k$$

Por el método de integración por partes:

$$\int x^n \,\mathrm{Ln}(x)\,dx = \left\{ \begin{array}{l} u=\mathrm{Ln}(x) \Rightarrow du = \frac{1}{x}dx \\ dv = x^n dx \Rightarrow v = \frac{1}{n+1}x^{n+1} \end{array} \right\} = \frac{1}{n+1}x^{n+1}\,\mathrm{Ln}(x) - \int \frac{1}{n+1}x^{n+1}\cdot\frac{1}{x}dx =$$

$$= \frac{1}{n+1}x^{n+1}\,\mathrm{Ln}(x) - \frac{1}{n+1}\int x^n\,dx = \frac{1}{n+1}x^{n+1}\,\mathrm{Ln}(x) - \frac{1}{n+1}\cdot\frac{1}{n+1}x^{n+1}+k =$$

$$= \frac{x^{n+1}}{n+1}\left(\mathrm{Ln}(x)-\frac{1}{n+1}\right)+k$$

$$\int \mathrm{arc\,sen}\,x\,dx \qquad \text{Sol: } x\,\mathrm{arcsen}\,x + \sqrt{1-x^2}+k$$

Hacemos el siguiente cambio: $\left\{ \begin{array}{l} u = \mathrm{arc\,sen}\,x \\ dv = dx \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} du = \frac{1}{\sqrt{1-x^2}}\cdot dx \\ v = x \end{array} \right.$

Sustituyendo en la fórmula de integración por partes obtenemos:

$$\int \mathrm{arc\,sen}\,x.dx = x.\mathrm{arc\,sen}\,x - \int x\cdot\frac{1}{\sqrt{1-x^2}}\cdot dx = x.\mathrm{arc\,sen}\,x - \int x.(1-x^2)^{-\frac{1}{2}}.dx =$$

$$= x.\mathrm{arc\,sen}\,x + \frac{1}{2}\int -2x.(1-x^2)^{-\frac{1}{2}}.dx = x.\mathrm{arc\,sen}\,x + \frac{1}{2}\cdot\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}}+k =$$

$$= x.\mathrm{arc\,sen}\,x + \sqrt{1-x^2}+k$$

$$\int \sqrt{1-x^2}\,dx \qquad \text{Sol: } \frac{1}{2}\left(\mathrm{arcsen}\,x + x\sqrt{1-x^2}\right)+k$$

$$\int \sqrt{1-x^2}\,dx = \int \frac{1-x^2}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}}dx + \int \frac{-x^2}{\sqrt{1-x^2}}dx =$$

$$= \mathrm{arc\,sen}\,x + \int \frac{-x^2}{\sqrt{1-x^2}}\cdot dx =$$

La integral que nos queda la realizaremos por partes:

$$\int \frac{-x^2}{\sqrt{1-x^2}}\cdot dx = \int x\cdot\frac{-x}{\sqrt{1-x^2}}\cdot dx = \left\{ \begin{array}{l} u = x \Rightarrow dx \\ dv = \frac{-x}{\sqrt{1-x^2}}dx \Rightarrow v = \sqrt{1-x^2} \end{array} \right\} = \quad x\sqrt{1-x^2} - \int \sqrt{1-x^2}\,dx$$

Sustituyendo nos queda:

$$\int \sqrt{1-x^2}dx = \text{arc sen}\,x + \int \frac{-x^2}{\sqrt{1-x^2}} \cdot dx = \text{arc sen}\,x + x\sqrt{1-x^2} - \int \sqrt{1-x^2}dx$$

y se nos repite la misma integral. Entonces:

$$\int \sqrt{1-x^2}dx = \text{arc sen}\,x + x\sqrt{1-x^2} - \int \sqrt{1-x^2}dx \Rightarrow$$

$$\Rightarrow 2\int \sqrt{1-x^2}dx = \text{arc sen}\,x + x\sqrt{1-x^2} \Rightarrow \int \sqrt{1-x^2}dx = \frac{1}{2}\left(\text{arc sen}\,x + x\sqrt{1-x^2}\right) + k$$

$\int x\,\text{arc sen}\,x\,dx$ Sol: $\frac{1}{4}\left[(2x^2-1)\,\text{arcsen}\,x + x\sqrt{1-x^2}\right] + k$

Hacemos el siguiente cambio: $\left\{\begin{matrix} u = \text{arc sen}\,x \\ dv = xdx \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} du = \frac{1}{\sqrt{1-x^2}} \cdot dx \\ v = \frac{x^2}{2} \end{matrix}\right.$

Sustituyendo en la fórmula de integración por partes obtenemos:

$$\int x\text{arc sen}\,x \cdot dx = \frac{x^2}{2} \cdot \text{arc sen}\,x - \int \frac{x^2}{2} \cdot \frac{1}{\sqrt{1-x^2}} \cdot dx = \frac{x^2}{2} \cdot \text{arc sen}\,x + \frac{1}{2}\int \frac{-x^2}{\sqrt{1-x^2}}dx =$$

Por el ejercicio anterior tenemos que :

$$= x\sqrt{1-x^2} - \int \sqrt{1-x^2}dx = x\sqrt{1-x^2} - \frac{1-x^2}{\sqrt{1-x^2}}dx \Rightarrow$$

$$\int \frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}}dx - \int \frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \text{arc sen}\,x - \int \frac{-x^2}{\sqrt{1-x^2}}dx$$

En consecuencia:

$$\int \frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \text{arc sen}\,x - \int \frac{-x^2}{\sqrt{1-x^2}}dx$$

Por tanto:

$$2\int \frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \text{arc sen}\,x \Rightarrow \int \frac{-x^2}{\sqrt{1-x^2}}dx = \frac{1}{2}\left(x\sqrt{1-x^2} - \text{arc sen}\,x\right)$$

Sustituyendo obtenemos:

$$\int x\text{arc sen}\,x \cdot dx = \frac{x^2}{2} \cdot \text{arc sen}\,x + \frac{1}{2}\int \frac{-x^2}{\sqrt{1-x^2}}dx =$$

$$= \frac{x^2}{2} \cdot \text{arc sen}\, x + \frac{1}{2} \cdot \frac{1}{2}\left(x\sqrt{1-x^2} - \text{arc sen}\, x\right) + k =$$

$$= \frac{x^2}{2} \cdot \text{arc sen}\, x + \frac{1}{4} x\sqrt{1-x^2} - \frac{1}{4}\text{arc sen}\, x + k =$$

$$= \frac{1}{4}\left[(2x^2 - 1)\,\text{arcsen}\, x + x\sqrt{1-x^2}\right] + k$$

$\int \text{arc tg}\, x dx$ Sol: $x\, \text{arc tg}\, x - \frac{1}{2}\text{Ln}\,(1+x^2) + k$

$$\int \text{arc tg}\, x dx = \left\{ \begin{array}{c} u = \text{arc tg}\, x \Rightarrow du = \frac{1}{1+x^2} dx \\ dv = dx \Rightarrow v = x \end{array} \right\} = x \cdot \text{arc tg}\, x - \int x \cdot \frac{1}{1+x^2} dx =$$

$$= x \cdot \text{arc tg}\, x - \int \frac{x}{1+x^2} dx = x \cdot \text{arc tg}\, x - \frac{1}{2}\int \frac{2x}{1+x^2} dx = x \cdot \text{arc tg}\, x - \frac{1}{2}\text{Ln}\,(1+x^2) + k$$

$\int \text{arc tg}\, \sqrt{x}\, dx$ Sol: $(x+1)\,\text{arc tg}\, \sqrt{x} - \sqrt{x} + k$

$$\int \text{arc tg}\, \sqrt{x}\, dx = \left\{ \begin{array}{c} u = \text{arc tg}\, \sqrt{x} \Rightarrow du = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} dx \\ dv = dx \Rightarrow v = x \end{array} \right\} = x\text{arc tg}\, \sqrt{x} - \int x \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} dx =$$

$$= x\text{arc tg}\, \sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{1+x} \cdot dx = \left\{ \begin{array}{c} x = t^2 \Rightarrow \\ \Rightarrow dx = 2tdt \end{array} \right\} = x\text{arc tg}\, \sqrt{x} - \frac{1}{2}\int \frac{t}{1+t^2} \cdot 2tdt =$$

$$= x\text{arc tg}\, \sqrt{x} - \int \frac{t^2}{1+t^2} dt = x\text{arc tg}\, \sqrt{x} - \int \frac{1+t^2-1}{1+t^2} dt =$$

$$= x\text{arc tg}\, \sqrt{x} - \int \frac{1+t^2-1}{1+t^2} dt = x\text{arc tg}\, \sqrt{x} - \int \left(1 - \frac{1}{1+t^2}\right) dt =$$

$$= x\text{arc tg}\, \sqrt{x} - \int dt + \int \frac{1}{1+t^2} dt = x\text{arc tg}\, \sqrt{x} - t + \text{arc tg}\, t + k =$$

$$= x\text{arc tg}\, \sqrt{x} - \sqrt{x} + \text{arc tg}\, \sqrt{x} + k = (x+1)\,\text{arc tg}\, \sqrt{x} - \sqrt{x} + k$$

$$\int \mathbf{Ln}\,(x+\sqrt{1+x^2})dx \qquad \text{Sol: } \; x\,\mathbf{Ln}\,(x+\sqrt{1+x^2})-\sqrt{1+x^2}+k$$

Hacemos: $u=\mathbf{Ln}\,(x+\sqrt{1+x^2})$ y $dv=dx$ con lo cual

$$du=\frac{1}{x+\sqrt{1+x^2}}\cdot\left(1+\frac{2x}{2\sqrt{1+x^2}}\right)dx=\frac{1}{x+\sqrt{1+x^2}}\cdot\left(1+\frac{x}{\sqrt{1+x^2}}\right)dx\Rightarrow$$

$$\Rightarrow du=\frac{1}{x+\sqrt{1+x^2}}\cdot\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right)dx=\frac{1}{\sqrt{1+x^2}}dx \quad \text{y} \quad v=x$$

Sustituyendo en la fórmula de integración por partes, obtenemos:

$$\int \mathbf{Ln}\,(x+\sqrt{1+x^2})dx=x\cdot\mathbf{Ln}\,(x+\sqrt{1+x^2})-\int x\cdot\frac{1}{\sqrt{1+x^2}}dx=$$

$$=x\cdot\mathbf{Ln}\,(x+\sqrt{1+x^2})-\int\frac{2x}{2\sqrt{1+x^2}}dx=x\cdot\mathbf{Ln}\,(x+\sqrt{1+x^2})-\sqrt{1+x^2}+k$$

$$\int\frac{x\,\mathbf{arc\,sen}\,x}{\sqrt{1-x^2}}dx \qquad \text{Sol: } \; x-\sqrt{1-x^2}\,\mathbf{arc\,sen}\,x+k$$

$$\int\frac{x\,\mathbf{arc\,sen}\,x}{\sqrt{1-x^2}}dx=\left\{\begin{array}{c} u=\mathbf{arc\,sen}\,x\Rightarrow du=\frac{1}{\sqrt{1-x^2}}dx \\ dv=\frac{x}{\sqrt{1-x^2}}dx\Rightarrow v=-\int\frac{-2x}{2\sqrt{1-x^2}}dx=-\sqrt{1-x^2}\end{array}\right\}=$$

$$=-\sqrt{1-x^2}\cdot\mathbf{arc\,sen}\,x-\int-\sqrt{1-x^2}\cdot\frac{1}{\sqrt{1-x^2}}dx=-\sqrt{1-x^2}\cdot\mathbf{arc\,sen}\,x+\int dx=$$

$$=-\sqrt{1-x^2}\cdot\mathbf{arc\,sen}\,x+x+k$$