CALCULA LOS SIGUIENTES LÍMITES:

1) $\lim\limits_{x\to 1}\dfrac{x^2-1}{x-1}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to 1}\dfrac{(x-1)(x+1)}{(x-1)}=\lim\limits_{x\to 1}(x+1)=2$

2) $\lim\limits_{x\to 1}\dfrac{x^3-1}{x^2-1}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to 1}\dfrac{(x-1)(x^2+x+1)}{(x-1)(x+1)}=\lim\limits_{x\to 1}\dfrac{x^2+x+1}{x+1}=\dfrac{(1)^2+(1)+1}{(1)+1}=\dfrac{3}{2}$

$$
\begin{array}{c|ccc}
 & 1 & 0 & 0 & -1 \\
1 & & +1 & +1 & +1 \\
\hline
 & 1 & +1 & +1 & \boxed{0}
\end{array}
\;\Rightarrow\; x^3-1=(x-1)(x^2+x+1)
$$

3) $\lim\limits_{x\to -1}\dfrac{2x^2+5x+3}{x^3+2x^2+2x+1}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to -1}\dfrac{(x+1)(2x+3)}{(x+1)(x^2+x+1)}=\lim\limits_{x\to -1}\dfrac{(2x+3)}{(x^2+x+1)}=\dfrac{1}{1}=1$

$$
\begin{array}{c|ccc}
 & 2 & 5 & 3 \\
-1 & & -2 & -3 \\
\hline
 & 2 & +3 & \boxed{0}
\end{array}
\;\Rightarrow\; 2x^2+5x+3=(x+1)(2x+3)
$$

$$
\begin{array}{c|cccc}
 & 1 & +2 & +2 & +1 \\
-1 & & -1 & -1 & -1 \\
\hline
 & 1 & +1 & +1 & \boxed{0}
\end{array}
\;\Rightarrow\; x^3+2x^2+2x+1=(x+1)(x^2+x+1)
$$

4) $\lim\limits_{x\to 3}\dfrac{x^2-6x+9}{x-3}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to 3}\dfrac{(x-3)^2}{(x-3)}=\lim\limits_{x\to 3}(x-3)=0$

5) $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{x^2-4x+4}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to 2}\dfrac{(x-2)(x-3)}{(x-2)^2}=\lim\limits_{x\to 2}\dfrac{x-3}{x-2}=\dfrac{-1}{0}=\begin{cases}\lim\limits_{x\to 2^-}\dfrac{x-3}{x-2}=\dfrac{-1}{0^-}=+\infty\\[2mm]\lim\limits_{x\to 2^+}\dfrac{x-3}{x-2}=\dfrac{-1}{0^+}=-\infty\end{cases}\Rightarrow x=2 \text{ es A.V.}$

6) $\lim\limits_{x\to 0}\dfrac{2x^3+6x^2-3x}{2x^2+5x}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to 0}\dfrac{x(2x^2+6x-3)}{x(2x+5)}=\lim\limits_{x\to 0}\dfrac{2x^2+6x-3}{2x+5}=\dfrac{2\cdot(0)^2+6\cdot(0)-3}{2\cdot(0)+5}=-\dfrac{3}{5}$

$$\lim\limits_{x\to +\infty}\dfrac{2x^3+6x^2-3x}{2x^2+5x}=\dfrac{+\infty}{+\infty}=(\text{I})=\lim\limits_{x\to +\infty}\dfrac{\dfrac{2x^3}{x^3}+\dfrac{6x^2}{x^3}-\dfrac{3x}{x^3}}{\dfrac{2x^2}{x^3}+\dfrac{5x}{x^3}}=\lim\limits_{x\to +\infty}\dfrac{2+\dfrac{6}{x}-\dfrac{3}{x^2}}{\dfrac{2}{x}+\dfrac{5x}{x^2}}=\dfrac{2+0-0}{0+0}=+\infty$$

$$\lim\limits_{x\to -\infty}\dfrac{2x^3+6x^2-3x}{2x^2+5x}=\dfrac{-\infty}{+\infty}=(\text{I})=\lim\limits_{x\to +\infty}\dfrac{\dfrac{2x^3}{x^3}+\dfrac{6x^2}{x^3}-\dfrac{3x}{x^3}}{\dfrac{2x^2}{x^3}+\dfrac{5x}{x^3}}=\lim\limits_{x\to +\infty}\dfrac{2+\dfrac{6}{x}-\dfrac{3}{x^2}}{\dfrac{2}{x}+\dfrac{5x}{x^2}}=\dfrac{2+0-0}{0+0}=-\infty$$

7) $\lim\limits_{x\to 3^+}\left(\dfrac{4x-2}{x-3}\right)^{\frac{1}{x}} = \left(\dfrac{12}{0^+}\right)^{\frac{1}{3}} = (+\infty)^{\frac{1}{3}} = +\infty$

8) $\lim\limits_{x\to +\infty}\dfrac{1+2x-3x^3}{x^2-x^3-4} = \dfrac{-\infty}{-\infty} = (I) = \lim\limits_{x\to +\infty}\dfrac{\dfrac{1}{x^3}+\dfrac{2x}{x^3}-\dfrac{3x^3}{x^3}}{\dfrac{x^2}{x^3}-\dfrac{x^3}{x^3}-\dfrac{4}{x^3}} = \lim\limits_{x\to +\infty}\dfrac{\dfrac{1}{x^3}+\dfrac{2}{x^2}-3}{\dfrac{1}{x}-1-\dfrac{4}{x^3}} = \dfrac{0+0-3}{0-1-0} = \dfrac{-3}{-1} = 3$

9) $\lim\limits_{x\to -\infty}\dfrac{5x^2-3x+1}{x^2-x^3-4} = \dfrac{+\infty}{+\infty} = (I) = \lim\limits_{x\to -\infty}\dfrac{\dfrac{5x^2}{x^3}-\dfrac{3x}{x^3}+\dfrac{1}{x^3}}{\dfrac{x^2}{x^3}-\dfrac{x^3}{x^3}-\dfrac{4}{x^3}} = \lim\limits_{x\to -\infty}\dfrac{\dfrac{5}{x}-\dfrac{3}{x^2}+\dfrac{1}{x^3}}{\dfrac{1}{x}-1-\dfrac{4}{x^3}} = \dfrac{0+0+0}{0-1-0} = \dfrac{0}{-1} = 0$

10) $\lim\limits_{x\to -\infty}\dfrac{x^4-3x+1}{2x-3x^3-4} = \dfrac{+\infty}{+\infty} = (I) = \lim\limits_{x\to -\infty}\dfrac{\dfrac{x^4}{x^4}-\dfrac{3x}{x^4}+\dfrac{1}{x^4}}{\dfrac{2x}{x^4}-\dfrac{3x^3}{x^4}-\dfrac{4}{x^4}} = \lim\limits_{x\to -\infty}\dfrac{1-\dfrac{3}{x^3}+\dfrac{1}{x^4}}{\dfrac{2}{x^3}-\dfrac{3}{x}-\dfrac{4}{x^4}} = \dfrac{1-0+0}{0-0-0} = \dfrac{1}{0} = +\infty$

11) $\lim\limits_{x\to -\infty}\dfrac{-2x^4-3x+1}{x^3-3x^4-4} = \dfrac{-\infty}{-\infty} = (I) = \lim\limits_{x\to -\infty}\dfrac{\dfrac{-2x^4}{x^4}-\dfrac{3x}{x^4}+\dfrac{1}{x^4}}{\dfrac{x^3}{x^4}-\dfrac{3x^4}{x^4}-\dfrac{4}{x^4}} = \lim\limits_{x\to -\infty}\dfrac{-2-\dfrac{3}{x^3}+\dfrac{1}{x^4}}{\dfrac{1}{x}-3-\dfrac{4}{x^4}} = \dfrac{-2-0+0}{0-3-0} = \dfrac{2}{3}$

12) $\lim\limits_{x\to 1}\dfrac{2x^3-4x^2+2x}{x^3-x^2-x+1} = \dfrac{0}{0}(I) = \lim\limits_{x\to 1}\dfrac{(x-1)^2\cdot 2x}{(x-1)^2\cdot(x+1)} = \lim\limits_{x\to 1}\dfrac{2x}{(x+1)} = \dfrac{2}{2} = 1$

	2	−4	+2	0
+1		+2	−2	0
	2	−2	0	0
+1		+2	0	
	2	0	0	

$\Rightarrow 2x^3-4x^2+2x = (x-1)^2\cdot 2x$

	1	−1	−1	+1
+1		+1	0	0
	1	0	−1	0
+1		+1	+1	
	1	+1	0	

$\Rightarrow x^3-x^2-x+1 = (x-1)^2\cdot(x+1)$

13) $\displaystyle \lim_{x \to -2} \frac{2x^2 + 7x + 6}{x^3 + 3x^2 + 3x + 2} = \frac{0}{0}(\text{I}) = \lim_{x \to -2} \frac{(x+2)(2x+3)}{(x+2)(x^2+x+1)} = \lim_{x \to -2} \frac{2x+3}{x^2+x+1} = \frac{2 \cdot (-2) + 3}{(-2)^2 + (-2) + 1} = -\frac{1}{3}$

$$
\begin{array}{r|rrr}
 & 2 & +7 & +6 \\
-2 & & -3 & -6 \\
\hline
 & 2 & +3 & \boxed{0}
\end{array}
\quad \Rightarrow 2x^2 + 7x + 6 = (x+2)(2x+3)
$$

$$
\begin{array}{r|rrrr}
 & 1 & +3 & +3 & +2 \\
-2 & & -2 & -2 & -2 \\
\hline
 & 1 & +1 & +1 & \boxed{0}
\end{array}
\quad \Rightarrow x^3 + 3x^2 + 3x + 2 = (x+2)(x^2+x+1)
$$

14) $\displaystyle \lim_{x \to 1} \frac{x^4 - x^3 + x^2 - 2x + 1}{x^3 - x^2 + x - 1} = \frac{0}{0}(\text{I}) = \lim_{x \to 1} \frac{(x-1)(x^3+x-1)}{(x-1)(x^2+1)} = \lim_{x \to 1} \frac{x^3 + x - 1}{x^2 + 1} = \frac{1}{2}$

$$
\begin{array}{r|rrrrr}
 & 1 & -1 & +1 & -2 & +1 \\
1 & & +1 & 0 & +1 & -1 \\
\hline
 & 1 & 0 & +1 & -1 & \boxed{0}
\end{array}
\quad \Rightarrow x^4 - x^3 + x^2 - 2x + 1 = (x-1)(x^3+x-1)
$$

$$
\begin{array}{r|rrrr}
 & 1 & -1 & +1 & -1 \\
1 & & +1 & 0 & +1 \\
\hline
 & 1 & 0 & +1 & \boxed{0}
\end{array}
\quad \Rightarrow x^3 - x^2 + x - 1 = (x-1)(x^2+1)
$$

15) $\displaystyle \lim_{x \to \frac{1}{2}} \frac{8x^3 + 4x^2 - 10x + 3}{4x^4 - 15x^2 + 13x - 3} = \frac{0}{0}(\text{I}) = \lim_{x \to \frac{1}{2}} \frac{\left(x - \frac{1}{2}\right)^2 (8x + 12)}{\left(x - \frac{1}{2}\right)^2 (4x^2 + 4x - 12)} = \lim_{x \to \frac{1}{2}} \frac{(8x + 12)}{(4x^2 + 4x - 12)} = \frac{4 + 12}{1 + 2 - 12} = -\frac{16}{9}$

$$
\begin{array}{r|rrrr}
 & 8 & +4 & -10 & +3 \\
\frac{1}{2} & & +4 & +4 & -3 \\
\hline
 & 8 & +8 & -6 & \boxed{0} \\
\frac{1}{2} & & +4 & +6 & \\
\hline
 & 8 & +12 & \boxed{0} &
\end{array}
\quad \Rightarrow \; 8x^3 + 4x^2 - 10x + 3 = \left(x - \frac{1}{2}\right)^2 \cdot (8x + 12)
$$

$$
\begin{array}{r|rrrrr}
 & 4 & 0 & -15 & +13 & -3 \\
\frac{1}{2} & & +2 & +1 & -7 & +3 \\
\hline
 & 4 & +2 & -14 & +6 & \boxed{0} \\
\frac{1}{2} & & +2 & +2 & -6 & \\
\hline
 & 4 & +4 & -12 & \boxed{0} &
\end{array}
\quad \Rightarrow \; 4x^4 - 15x^2 + 13x - 3 = \left(x - \frac{1}{2}\right)^2 \cdot (4x^2 + 4x - 12)
$$

16) $\lim\limits_{x\to a}\dfrac{x^6-a^6}{x^3-a^3}=\dfrac{0}{0}(I)=\lim\limits_{x\to a}\dfrac{(x^3-a^3)(x^3+a^3)}{(x^3-a^3)}=\lim\limits_{x\to a}(x^3+a^3)=a^3+a^3=2a^3$

17) $\lim\limits_{x\to a}\dfrac{x^2-(a+1)x+a}{x^2-a^2}=\lim\limits_{x\to a}\dfrac{(x-a)(x-1)}{(x-a)(x+a)}=\lim\limits_{x\to a}\dfrac{x-1}{x+a}=\dfrac{a-1}{2a}$

$$\begin{array}{c|ccc}
 & 1 & -(a+1) & +a \\
a & & +a & -a \\
\hline
 & 1 & -1 & \boxed{0}
\end{array}\Rightarrow x^2-(a+1)x+a=(x-a)(x-1)$$

18) $\lim\limits_{x\to a}\dfrac{x^3-ax^2-a^2x+a^3}{x^3-3ax^2+3a^2x-a^3}=\dfrac{0}{0}(I)=\lim\limits_{x\to a}\dfrac{(x-a)^2(x+a)}{(x-a)^3}=\lim\limits_{x\to a}\dfrac{x+a}{x-a}=$

$\begin{cases}
\bullet\ \text{Si}\ a>0\Rightarrow\begin{cases}\lim\limits_{x\to a^-}\dfrac{x+a}{x-a}=\dfrac{2a}{0^-}=-\infty \\[2mm] \lim\limits_{x\to a^+}\dfrac{x+a}{x-a}=\dfrac{2a}{0^+}=+\infty\end{cases}\\[8mm]
\bullet\ \text{Si}\ a<0\Rightarrow\begin{cases}\lim\limits_{x\to a^-}\dfrac{x+a}{x-a}=\dfrac{2a}{0^-}=+\infty \\[2mm] \lim\limits_{x\to a^+}\dfrac{x+a}{x-a}=\dfrac{2a}{0^+}=-\infty\end{cases}\\[8mm]
\bullet\ \text{Si}\ a=0\Rightarrow\lim\limits_{x\to a}\dfrac{x}{x}=\lim\limits_{x\to a}1=1
\end{cases}$

$$\begin{array}{c|cccc}
 & 1 & -a & -a^2 & +a^3 \\
a & & +a & 0 & -a^3 \\
\hline
 & 1 & 0 & -a^2 & \boxed{0} \\
a & & +a & +a^2 & \\
\hline
 & 1 & +a & \boxed{0} & \Rightarrow\ x^3-ax^2-a^2x+a^3=(x-a)^2\cdot(x+a)
\end{array}$$

$$\begin{array}{c|cccc}
 & 1 & -3a & +3a^2 & -a^3 \\
a & & +a & -2a^2 & +a^3 \\
\hline
 & 1 & -2a & +a^2 & \boxed{0} \\
a & & +a & -a^2 & \\
\hline
 & 1 & -a & \boxed{0} & \Rightarrow\ x^3-3ax^2+3a^2x-a^3=(x-a)^2\cdot(x-a)=(x-a)^3
\end{array}$$

19) $\lim\limits_{x\to 2}\left(\dfrac{3}{x^2-5x+6}-\dfrac{4}{x-2}\right)=\infty-\infty\ (I)=\lim\limits_{x\to 2}\left(\dfrac{3}{(x-2)(x-3)}-\dfrac{4}{x-2}\right)=\lim\limits_{x\to 2}\left(\dfrac{3-4(x-3)}{(x-2)(x-3)}\right)=$

$$=\lim\limits_{x\to 2}\left(\dfrac{3-4x+12)}{(x-2)(x-3)}\right)=\lim\limits_{x\to 2}\dfrac{15-4x}{(x-2)(x-3)}=\dfrac{7}{0}=\begin{cases}\lim\limits_{x\to 2^-}\dfrac{15-4x}{(x-2)(x-3)}=\dfrac{7}{(0^-)(-1)}=\dfrac{7}{0^+}=+\infty\\[2mm]\lim\limits_{x\to 2^+}\dfrac{15-4x}{(x-2)(x-3)}=\dfrac{7}{(0^+)(-1)}=\dfrac{7}{0^-}=-\infty\end{cases}$$

20) $\lim\limits_{x\to+\infty}\left(\dfrac{4x^2-x+3}{3x^2+x-3}\right)^{\frac{x}{1-x}}=\left[\lim\limits_{x\to+\infty}\left(\dfrac{4x^2-x+3}{3x^2+x-3}\right)\right]^{\lim\limits_{x\to+\infty}\left(\frac{x}{1-x}\right)}=\left(\dfrac{4}{3}\right)^{-1}=\dfrac{3}{4}$

- $\lim\limits_{x\to+\infty}\left(\dfrac{4x^2-x+3}{3x^2+x-3}\right)=\dfrac{+\infty}{+\infty}(I)=\lim\limits_{x\to+\infty}\dfrac{\dfrac{4x^2}{x^2}-\dfrac{x}{x^2}+\dfrac{3}{x^2}}{\dfrac{3x^2}{x^2}+\dfrac{x}{x^2}-\dfrac{3}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{4-\dfrac{1}{x}+\dfrac{3}{x^2}}{3+\dfrac{1}{x}-\dfrac{3}{x^2}}=\dfrac{4-0+0}{3+0-0}=\dfrac{4}{3}$

- $\lim\limits_{x\to+\infty}\left(\dfrac{x}{1-x}\right)=\dfrac{+\infty}{-\infty}(I)=\lim\limits_{x\to+\infty}\dfrac{\dfrac{x}{x}}{\dfrac{1}{x}-\dfrac{x}{x}}=\lim\limits_{x\to+\infty}\dfrac{1}{\dfrac{1}{x}-1}=\dfrac{1}{0-1}=-1$

21) $\lim\limits_{x\to+\infty}\left(\dfrac{x}{3x^2+2}\right)^{\frac{x^2}{1+x}}=\left[\lim\limits_{x\to+\infty}\left(\dfrac{x}{3x^2+2}\right)\right]^{\lim\limits_{x\to+\infty}\left(\frac{x^2}{1+x}\right)}=0^{+\infty}=0$

- $\lim\limits_{x\to+\infty}\left(\dfrac{x}{3x^2+2}\right)=\dfrac{+\infty}{+\infty}(I)=\lim\limits_{x\to+\infty}\dfrac{\dfrac{x}{x^2}}{\dfrac{3x^2}{x^2}+\dfrac{2}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{\dfrac{1}{x}}{3+\dfrac{2}{x^2}}=\dfrac{0}{3+0}=\dfrac{0}{3}=0$

- $\lim\limits_{x\to+\infty}\left(\dfrac{x^2}{1+x}\right)=\dfrac{+\infty}{+\infty}(I)=\lim\limits_{x\to+\infty}\dfrac{\dfrac{x^2}{x^2}}{\dfrac{1}{x^2}+\dfrac{x}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{1}{\dfrac{1}{x^2}+\dfrac{1}{x}}=\dfrac{1}{0}=+\infty$

22) $\lim\limits_{x\to+\infty}\left(\dfrac{x^2+3}{3x^2-5}\right)^{\frac{3x^2}{x-2}}=\left[\lim\limits_{x\to+\infty}\left(\dfrac{x^2+3}{3x^2-5}\right)\right]^{\lim\limits_{x\to+\infty}\left(\frac{3x^2}{x-2}\right)}=\left(\dfrac{1}{3}\right)^{+\infty}=0$

- $\lim\limits_{x\to+\infty}\left(\dfrac{x^2+3}{3x^2-5}\right)=\dfrac{+\infty}{+\infty}(I)=\lim\limits_{x\to+\infty}\dfrac{\dfrac{x^2}{x^2}+\dfrac{3}{x^2}}{\dfrac{3x^2}{x^2}-\dfrac{5}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{1+\dfrac{3}{x^2}}{3-\dfrac{5}{x^2}}=\dfrac{1+0}{3-0}=\dfrac{1}{3}$

- $\lim\limits_{x\to+\infty}\left(\dfrac{3x^2}{x-2}\right)=\dfrac{+\infty}{+\infty}(I)=\lim\limits_{x\to+\infty}\dfrac{\dfrac{3x^2}{x^2}}{\dfrac{x}{x^2}-\dfrac{2}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{3}{\dfrac{1}{x}-\dfrac{2}{x^2}}=\dfrac{3}{0}=+\infty$

23) $\lim\limits_{x\to+\infty}\left(\dfrac{x^2+3}{3x^2-5}\right)^{\frac{x^2}{2-x}}=\left[\lim\limits_{x\to+\infty}\left(\dfrac{x^2+3}{3x^2-5}\right)\right]^{\lim\limits_{x\to+\infty}\left(\frac{x^2}{2-x}\right)}=\left(\dfrac{1}{3}\right)^{-\infty}=3^{+\infty}=+\infty$

- $\lim\limits_{x\to+\infty}\left(\dfrac{x^2+3}{3x^2-5}\right)=\dfrac{+\infty}{+\infty}(I)=\lim\limits_{x\to+\infty}\dfrac{\frac{x^2}{x^2}+\frac{3}{x^2}}{\frac{3x^2}{x^2}-\frac{5}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{1+\frac{3}{x^2}}{3-\frac{5}{x^2}}=\dfrac{1+0}{3-0}=\dfrac{1}{3}$

- $\lim\limits_{x\to+\infty}\left(\dfrac{3x^2}{2-x}\right)=\dfrac{+\infty}{-\infty}(I)=\lim\limits_{x\to+\infty}\dfrac{\frac{3x^2}{x^2}}{\frac{2}{x^2}-\frac{x}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{3}{\frac{2}{x^2}-\frac{1}{x}}=\dfrac{3}{0}=-\infty$

24) $\lim\limits_{x\to+\infty}\left(\dfrac{x^2+3}{3x^2-5}\right)^{\frac{x}{x-2}}=\left[\lim\limits_{x\to+\infty}\left(\dfrac{x^2+3}{3x^2-5}\right)\right]^{\lim\limits_{x\to+\infty}\left(\frac{x}{x-2}\right)}=\left(\dfrac{1}{3}\right)^{1}=\dfrac{1}{3}$

- $\lim\limits_{x\to+\infty}\left(\dfrac{x^2+3}{3x^2-5}\right)=\dfrac{+\infty}{+\infty}(I)=\lim\limits_{x\to+\infty}\dfrac{\frac{x^2}{x^2}+\frac{3}{x^2}}{\frac{3x^2}{x^2}-\frac{5}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{1+\frac{3}{x^2}}{3-\frac{5}{x^2}}=\dfrac{1+0}{3-0}=\dfrac{1}{3}$

- $\lim\limits_{x\to+\infty}\left(\dfrac{x}{x-2}\right)=\dfrac{+\infty}{+\infty}(I)=\lim\limits_{x\to+\infty}\dfrac{\frac{x}{x}}{\frac{x}{x}-\frac{2}{x}}=\lim\limits_{x\to+\infty}\dfrac{1}{1-\frac{2}{x}}=\dfrac{1}{1-0}=1$

25) $\lim\limits_{x\to1}\dfrac{x-1}{\sqrt{x}-1}=\dfrac{0}{0}(I)=\lim\limits_{x\to1}\dfrac{(x-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)}=\lim\limits_{x\to1}\dfrac{(x-1)(\sqrt{x}+1)}{(\sqrt{x})^2-1^2}=\lim\limits_{x\to1}\dfrac{(x-1)(\sqrt{x}+1)}{(x-1)}=\lim\limits_{x\to1}(\sqrt{x}+1)=1+1=2$

26) $\lim\limits_{x\to0}\dfrac{2-\sqrt{4-x}}{x}=\dfrac{0}{0}(I)=\lim\limits_{x\to0}\dfrac{(2-\sqrt{4-x})(2+\sqrt{4-x})}{x(2+\sqrt{4-x})}=\lim\limits_{x\to0}\dfrac{2^2-(\sqrt{4-x})^2}{x(2+\sqrt{4-x})}=\lim\limits_{x\to0}\dfrac{4-(4-x)}{x(2+\sqrt{4-x})}=$

$=\lim\limits_{x\to0}\dfrac{4-4+x}{x(2+\sqrt{4-x})}=\lim\limits_{x\to0}\dfrac{x}{x(2+\sqrt{4-x})}\underset{\text{simplificar}}{=}\lim\limits_{x\to0}\dfrac{1}{(2+\sqrt{4-x})}=\dfrac{1}{2+\sqrt{4}}=\dfrac{1}{2+2}=\dfrac{1}{4}$

27) $\lim\limits_{x\to+\infty}(\sqrt{x^3+x+1}-x)=\infty-\infty(I)=\lim\limits_{x\to+\infty}(\sqrt{x^3}-x)=\lim\limits_{x\to+\infty}(x^{\frac{3}{2}}-x^1)=+\infty$

28) $\lim\limits_{x\to+\infty}(x-\sqrt{4x^3+x+1})=\infty-\infty(I)=\lim\limits_{x\to+\infty}(x-\sqrt{4x^3})=\lim\limits_{x\to+\infty}(x-2x^{\frac{3}{2}})=-\infty$

29) $\lim\limits_{x\to+\infty}(\sqrt{4x^2+x+1}-5x)=\infty-\infty(I)=\lim\limits_{x\to+\infty}(\sqrt{4x^2}-5x)=\lim\limits_{x\to+\infty}(2x-5x)=-\infty$

30) $\lim\limits_{x\to+\infty}(\sqrt{x^2+x+1}-x)=\infty-\infty\text{(I)}=\lim\limits_{x\to+\infty}\dfrac{(\sqrt{x^2+x+1}-x)(\sqrt{x^2+x+1}+x)}{(\sqrt{x^2+x+1}+x)}=$

$=\lim\limits_{x\to+\infty}\dfrac{(\sqrt{x^2+x+1})^2-x^2}{(\sqrt{x^2+x+1}+x)}=\lim\limits_{x\to+\infty}\dfrac{x^2+x+1-x^2}{(\sqrt{x^2+x+1}+x)}=\lim\limits_{x\to+\infty}\dfrac{x+1}{\sqrt{x^2+x+1}+x}=\dfrac{+\infty}{+\infty}\text{(I)}=\lim\limits_{x\to+\infty}\dfrac{\dfrac{x}{x}+\dfrac{1}{x}}{\sqrt{\dfrac{x^2}{x^2}+\dfrac{x}{x^2}+\dfrac{1}{x^2}}+\dfrac{x}{x}}=$

$=\lim\limits_{x\to\infty}\dfrac{1+\dfrac{1}{x}}{\sqrt{1+\dfrac{1}{x}+\dfrac{1}{x^2}}+1}=\dfrac{1}{\sqrt{1+0+0}+1}=\dfrac{1}{\sqrt{1}+1}=\dfrac{1}{2}$

31) $\lim\limits_{x\to+\infty}(\sqrt{x+2}-\sqrt{x-2})=\infty-\infty\text{(I)}=\lim\limits_{x\to+\infty}\dfrac{(\sqrt{x+2}-\sqrt{x-2})(\sqrt{x+2}+\sqrt{x-2})}{(\sqrt{x+2}+\sqrt{x-2})}=$

$=\lim\limits_{x\to+\infty}\dfrac{(\sqrt{x+2})^2-(\sqrt{x-2})^2}{(\sqrt{x+2}+\sqrt{x-2})}=\lim\limits_{x\to+\infty}\dfrac{x+2-(x-2)}{(\sqrt{x+2}+\sqrt{x-2})}=\lim\limits_{x\to+\infty}\dfrac{x+2-x+2}{(\sqrt{x+2}+\sqrt{x-2})}=\lim\limits_{x\to+\infty}\dfrac{4}{(\sqrt{x+2}+\sqrt{x-2})}=$

$=\dfrac{4}{\infty+\infty}=\dfrac{4}{+\infty}=0$

32) $\lim\limits_{x\to+\infty}(x-\sqrt{x^2+x+1})=\infty-\infty\text{(I)}=\lim\limits_{x\to+\infty}\dfrac{(x-\sqrt{x^2+x+1})(x+\sqrt{x^2+x+1})}{(x+\sqrt{x^2+x+1})}=$

$=\lim\limits_{x\to+\infty}\dfrac{(x)^2-(\sqrt{x^2+x+1})^2}{(x+\sqrt{x^2+x+1})}=\lim\limits_{x\to+\infty}\dfrac{x^2-(x^2+x+1)^2}{(x+\sqrt{x^2+x+1})}=\lim\limits_{x\to+\infty}\dfrac{x^2-x^2-x-1}{(x+\sqrt{x^2+x+1})}=\lim\limits_{x\to+\infty}\dfrac{-x-1}{(x+\sqrt{x^2+x+1})}=$

$=\dfrac{-\infty}{+\infty}\text{(I)}=\lim\limits_{x\to+\infty}\dfrac{-\dfrac{x}{x}-\dfrac{1}{x}}{\dfrac{x}{x}+\sqrt{\dfrac{x^2}{x^2}+\dfrac{x}{x^2}+\dfrac{1}{x}}}=\lim\limits_{x\to+\infty}\dfrac{-1-\dfrac{1}{x}}{1+\sqrt{1+\dfrac{1}{x}+\dfrac{1}{x^2}}}=\dfrac{-1+0}{1+\sqrt{1+0+0}}=\dfrac{-1}{1+\sqrt{1}}=-\dfrac{1}{2}$

33) $\lim\limits_{x\to0}\dfrac{x}{\sqrt{1+x}-\sqrt{1-x}}=\dfrac{0}{0}\text{(I)}=\lim\limits_{x\to0}\dfrac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}=\lim\limits_{x\to0}\dfrac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2}=$

$=\lim\limits_{x\to0}\dfrac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)}=\lim\limits_{x\to0}\dfrac{x(\sqrt{1+x}+\sqrt{1-x})}{1+x-1+x}=\lim\limits_{x\to0}\dfrac{x(\sqrt{1+x}+\sqrt{1-x})}{2x}\underset{\text{simplificar}}{=}\lim\limits_{x\to0}\dfrac{\sqrt{1+x}+\sqrt{1-x}}{2}=$

$=\dfrac{\sqrt{1+0}+\sqrt{1-0}}{2}=\dfrac{2}{2}=1$

34) $\lim\limits_{x\to 3}\dfrac{\sqrt[3]{x^3-27}}{\sqrt[3]{x^2+6x-27}}=\lim\limits_{x\to 3}\sqrt[3]{\dfrac{x^3-27}{x^2+6x-27}}=\sqrt[3]{\lim\limits_{x\to 3}\dfrac{x^3-27}{x^2+6x-27}}=\sqrt[3]{\dfrac{9}{4}}$

$\lim\limits_{x\to 3}\dfrac{x^3-27}{x^2+6x-27}=\dfrac{0}{0}\,(I)=\lim\limits_{x\to 3}\dfrac{(x-3)(x^2+3x+9)}{(x-3)(x+9)}=\lim\limits_{x\to 3}\dfrac{x^2+3x+9}{x+9}=\dfrac{(3)^2+3(3)+9}{3+9}=\dfrac{9+9+9}{12}=\dfrac{27}{12}=\dfrac{9}{4}$

$$
\begin{array}{r|rrrr}
 & 1 & 0 & 0 & -27\\
3 & & +3 & +9 & +27\\
\hline
 & 1 & +3 & +9 & \boxed{0}
\end{array}\quad\Rightarrow x^3-27=(x-3)(x^2+3x+9)
$$

$$
\begin{array}{r|rrr}
 & 1 & +6 & -27\\
3 & & +3 & +27\\
\hline
 & 1 & +9 & \boxed{0}
\end{array}\quad\Rightarrow x^2+6x-27=(x-3)(x+9)
$$

35) $\lim\limits_{x\to 5}\dfrac{3-\sqrt{4+x}}{x-5}=\dfrac{0}{0}\,(I)=\lim\limits_{x\to 5}\dfrac{(3-\sqrt{4+x})(3+\sqrt{4+x})}{(x-5)(3+\sqrt{4+x})}=\lim\limits_{x\to 5}\dfrac{(3)^2-(\sqrt{4+x})^2}{(x-5)(3+\sqrt{4+x})}=\lim\limits_{x\to 5}\dfrac{9-(4+x)}{(x-5)(3+\sqrt{4+x})}=$

$=\lim\limits_{x\to 5}\dfrac{9-4-x}{(x-5)(3+\sqrt{4+x})}=\lim\limits_{x\to 5}\dfrac{5-x}{(x-5)(3+\sqrt{4+x})}=\lim\limits_{x\to 5}\dfrac{-(x-5)}{(x-5)(3+\sqrt{4+x})}\underset{\text{simplificar}}{=}$

$=\lim\limits_{x\to 5}\dfrac{-1}{(3+\sqrt{4+x})}=\dfrac{-1}{3+\sqrt{4+5}}=\dfrac{-1}{3+\sqrt{9}}=-\dfrac{1}{6}$

36) $\lim\limits_{x\to 4}\dfrac{3-\sqrt{5+x}}{1-\sqrt{5-x}}=\dfrac{0}{0}\,(I)=\lim\limits_{x\to 4}\dfrac{[(3-\sqrt{5+x})(3+\sqrt{5+x})](1+\sqrt{5-x})}{[(1-\sqrt{5-x})(1+\sqrt{5-x})](3+\sqrt{5+x})}=$

$=\lim\limits_{x\to 4}\dfrac{[(3)^2-(\sqrt{5+x})^2](1+\sqrt{5-x})}{[(1)^2-(\sqrt{5-x})^2](3+\sqrt{5+x})}=\lim\limits_{x\to 4}\dfrac{[9-(5+x)](1+\sqrt{5-x})}{[1-(5-x)](3+\sqrt{5+x})}=\lim\limits_{x\to 4}\dfrac{(9-5-x)(1+\sqrt{5-x})}{(1-5+x)(3+\sqrt{5+x})}=$

$=\lim\limits_{x\to 4}\dfrac{(4-x)(1+\sqrt{5-x})}{(x-4)(3+\sqrt{5+x})}=\lim\limits_{x\to 4}\dfrac{-(x-4)(1+\sqrt{5-x})}{(x-4)(3+\sqrt{5+x})}\underset{\text{simplificar}}{=}\lim\limits_{x\to 4}\dfrac{-(1+\sqrt{5-x})}{(3+\sqrt{5+x})}=\dfrac{-(1+\sqrt{5-4})}{(3+\sqrt{5+4})}=-\dfrac{2}{6}=-\dfrac{1}{3}$

37) $\lim\limits_{x\to 2}\dfrac{x-\sqrt{x+2}}{\sqrt{4x+1}-3}=\dfrac{0}{0}\,(I)==\lim\limits_{x\to 2}\dfrac{[(x-\sqrt{x+2})(x+\sqrt{x+2})](\sqrt{4x+1}+3)}{[(\sqrt{4x+1}-3)(\sqrt{4x+1}+3)](x+\sqrt{x+2})}=$

$=\lim\limits_{x\to 2}\dfrac{[(x)^2-(\sqrt{x+2})^2](\sqrt{4x+1}+3)}{[(\sqrt{4x+1})^2-(3)^2](x+\sqrt{x+2})}=\lim\limits_{x\to 2}\dfrac{[x^2-(x+2)](\sqrt{4x+1}+3)}{[4x+1-9](x+\sqrt{x+2})}=$

$=\lim\limits_{x\to 2}\dfrac{(x^2-x-2)(\sqrt{4x+1}+3)}{(4x-8)(x+\sqrt{x+2})}\underset{\substack{\text{factorizar}\\\text{polinomios}}}{=}=\lim\limits_{x\to 2}\dfrac{(x-2)(x+1)((\sqrt{4x+1}+3)}{4(x-2)(x\sqrt{x+2})}\underset{\text{simplificar}}{=}$

$=\lim\limits_{x\to 2}\dfrac{(x+1)(\sqrt{4x+1}+3)}{4(x+\sqrt{x+2})}=\dfrac{(2+1)(\sqrt{4\cdot 2+1}+3)}{4(2+\sqrt{2+2})}=\dfrac{3\cdot(\sqrt{9}+3)}{4\cdot(2+\sqrt{4})}=\dfrac{3\cdot 6}{4\cdot 4}=\dfrac{3\cdot 3\cdot 2}{4\cdot 2\cdot 2}=\dfrac{9}{8}$

38) $\displaystyle\lim_{\substack{x\to 4 \\ x\to 2}}\frac{\sqrt{3x+4}-x}{x-\sqrt{5x-4}}=\frac{0}{0}\text{(I)}=\lim_{x\to 4}\frac{[(\sqrt{3x+4}-x)(\sqrt{3x+4}+x)](x+\sqrt{5x-4})}{[(x-\sqrt{5x-4})(x+\sqrt{5x-4})](\sqrt{3x+4}+x)}=$

$\displaystyle=\lim_{x\to 4}\frac{[(\sqrt{3x+4})^2-(x)^2](x+\sqrt{5x-4})}{[(x)^2-(\sqrt{5x-4})^2](\sqrt{3x+4}+x)}=\lim_{x\to 4}\frac{[(3x+4)-x^2](x+\sqrt{5x-4})}{[x^2-(5x-4)](\sqrt{3x+4}+x)}=$

$\displaystyle=\lim_{x\to 4}\frac{(-x^2+3x+4)(x+\sqrt{5x-4})}{(x^2-5x+4)(\sqrt{3x+4}+x)}\underset{\substack{\text{factorizar}\\\text{polinomios}}}{=}\lim_{x\to 4}\frac{-(x-4)(x+1)(x+\sqrt{5x-4})}{(x-4)(x-1)(\sqrt{3x+4}+x)}\underset{\text{simplificar}}{=}$

$\displaystyle=\lim_{x\to 4}\frac{-(x+1)(x+\sqrt{5x-4})}{(x-1)(\sqrt{3x+4}+x)}=\frac{-(4+1)(4+\sqrt{5(4)-4})}{(4-1)(\sqrt{3(4)+4}+4)}=\frac{-5\cdot 8}{3\cdot 8}=-\frac{5}{3}$

39) $\displaystyle\lim_{x\to +\infty}\frac{\sqrt{3x^2+1}}{2+5x}=\frac{+\infty}{+\infty}=\lim_{x\to +\infty}\frac{\sqrt{\dfrac{3x^2}{x^2}+\dfrac{1}{x^2}}}{\dfrac{2}{x}+\dfrac{5x}{x}}=\lim_{x\to +\infty}\frac{\sqrt{3+\dfrac{1}{x^2}}}{\dfrac{2}{x}+5}=\frac{\sqrt{3}+0}{0+5}=\frac{\sqrt{3}}{5}$

40) $\displaystyle\lim_{x\to -\infty}\frac{\sqrt{x^2-3x+1}}{2+5x}=\lim_{x\to +\infty}\frac{\sqrt{(-x)^2-3(-x)+1}}{2+5(-x)}=\lim_{x\to +\infty}\frac{\sqrt{x^2+3x+1}}{2-5x}=\frac{+\infty}{-\infty}\text{(I)}=\lim_{x\to +\infty}\frac{\sqrt{\dfrac{x^2}{x^2}+\dfrac{3x}{x^2}+\dfrac{1}{x^2}}}{\dfrac{2}{x}-\dfrac{5x}{x}}=$

$\displaystyle=\lim_{x\to +\infty}\frac{\sqrt{1+\dfrac{3}{x}+\dfrac{1}{x^2}}}{\dfrac{2}{x}-5}=\frac{\sqrt{1+0+0}}{0-5}=-\frac{1}{5}$

41) $\displaystyle\lim_{x\to +\infty}\frac{\sqrt[3]{x^7-2x+4}}{3x^2-1}=\frac{+\infty}{+\infty}\text{(I)}=\lim_{x\to +\infty}\frac{\sqrt[3]{x^7}}{3x^2}=\lim_{x\to +\infty}\frac{x^{\frac{7}{3}}}{x^2}\underset{\text{grado N>grado D}}{=}+\infty$

42) $\displaystyle\lim_{x\to -\infty}\frac{\sqrt[3]{x^7-2x+4}}{3x^2-1}=\lim_{x\to +\infty}\frac{\sqrt[3]{(-x)^7-2(-x)+4}}{3(-x)^2-1}=\lim_{x\to +\infty}\frac{\sqrt[3]{-x^7+2x+4}}{3x^2-1}=\frac{-\infty}{+\infty}\text{(I)}=$

$\displaystyle=\lim_{x\to +\infty}\frac{\sqrt[3]{-x^7}}{3x^2}=\lim_{x\to +\infty}\frac{-x^{\frac{7}{3}}}{x^2}\underset{\text{grado N>grado D}}{=}-\infty$

43) $\displaystyle\lim_{x\to +\infty}\left(\frac{\sqrt{x+9}-3}{x^2}\right)=\frac{+\infty}{+\infty}\text{(I)}=\lim_{x\to +\infty}\left(\frac{\sqrt{x}}{x^2}\right)=\lim_{x\to +\infty}\frac{x^{\frac{1}{2}}}{x^2}\underset{\text{grado N<grado D}}{=}0$

44) $\displaystyle\lim_{x\to +\infty}2^{\left(\frac{x^3-1}{x^3+x}\right)^{x^2+1}}=2^{\lim\limits_{x\to +\infty}\left(\frac{x^3-1}{x^3+x}\right)^{x^2+1}}\underset{(*)}{=}2^{\frac{1}{e}}=\sqrt[e]{2}$

$\displaystyle(*)\lim_{x\to +\infty}\left(\frac{x^3-1}{x^3+x}\right)^{x^2+1}=1^{+\infty}\text{(I)}=e^{\lim\limits_{x\to +\infty}\left(\frac{x^3-1}{x^3+x}-1\right)\cdot(x^2+1)}\underset{(**)}{=}e^{-1}=\frac{1}{e}$

$\displaystyle(**)\ \lim_{x\to +\infty}\left(\frac{x^3-1}{x^3+x}-1\right)\cdot(x^2+1)=\lim_{x\to +\infty}\left(\frac{x^3-1-x^3-x}{x(x^2+1)}\right)\cdot(x^2+1)=\lim_{x\to +\infty}\left(\frac{-1-x}{x(x^2+1)}\right)\cdot(x^2+1)=\lim_{x\to +\infty}\frac{-1-x}{x}=-1$

45) $\lim\limits_{x\to 1}\left(\dfrac{x^3+1}{x^2+1}\right)^{\frac{3}{x-1}} = 1^\infty\,(I) = e^{\lim\limits_{x\to 1}\left(\frac{x^3+1}{x^2+1}-1\right)\cdot\frac{3}{x-1}} \underset{(*)}{=} e^{\frac{3}{2}} = \sqrt{e^3} = e\sqrt{e}$

$(*)\lim\limits_{x\to 1}\left(\dfrac{x^3+1}{x^2+1}-1\right)\cdot\dfrac{3}{x-1} = \lim\limits_{x\to 1}\left(\dfrac{x^3+1-x^2-1}{x^2+1}\right)\cdot\dfrac{3}{x-1} = \lim\limits_{x\to 1}\left(\dfrac{x^3-x^2}{x^2+1}\right)\cdot\dfrac{3}{x-1} = \lim\limits_{x\to 1}\dfrac{x^2(x-1)}{x^2+1}\cdot\dfrac{3}{(x-1)} =$

$= \lim\limits_{x\to 1}\dfrac{3x^2}{x^2+1} = \dfrac{3}{2}$

46) $\lim\limits_{x\to +\infty}\left(\sqrt{\dfrac{x-2}{x+5}}\right)^{x} = \lim\limits_{x\to +\infty}\left(\dfrac{x-2}{x+5}\right)^{\frac{x}{2}} = 1^\infty\,(I) = e^{\lim\limits_{x\to +\infty}\left(\frac{x-2}{x+5}-1\right)\cdot\frac{x}{2}} \underset{(*)}{=} e^{-\frac{7}{2}} = \dfrac{1}{e^{\frac{7}{2}}} = \dfrac{1}{\sqrt{e^7}} = \dfrac{1}{e^3\sqrt{e}}$

$(*)\lim\limits_{x\to +\infty}\left(\dfrac{x-2}{x+5}-1\right)\cdot\dfrac{x}{2} = \lim\limits_{x\to +\infty}\left(\dfrac{x-2-x-5}{x+5}\right)\cdot\dfrac{x}{2} = \lim\limits_{x\to +\infty}\left(\dfrac{-7}{x+5}\right)\cdot\dfrac{x}{2} = \lim\limits_{x\to +\infty}\dfrac{-7x}{2x+10} = \lim\limits_{x\to +\infty}\dfrac{-\dfrac{7x}{x}}{\dfrac{2x}{x}+\dfrac{10}{x}} =$

$= \lim\limits_{x\to +\infty}\dfrac{-7}{2+\dfrac{10}{x}} = -\dfrac{7}{2+0} = -\dfrac{7}{2}$

47) $\lim\limits_{x\to 2}(x-1)^{\frac{1}{x-2}} = 1^\infty\,(I) = e^{\lim\limits_{x\to 2}(x-1-1)\cdot\frac{1}{x-2}} = e^{\lim\limits_{x\to 2}(x-2)\frac{1}{x-2}} = e^{\lim\limits_{x\to 2}1} = e^1 = e$

48) $\lim\limits_{x\to 1}x^{\frac{2}{1-x}} = 1^\infty\,(I) = e^{\lim\limits_{x\to 1}(x-1)\cdot\frac{2}{1-x}} = e^{\lim\limits_{x\to 1}\frac{2(x-1)}{(1-x)}} = e^{\lim\limits_{x\to 1}\frac{2(x-1)}{-(x-1)}} = e^{\lim\limits_{x\to 1}(-2)} = e^{-2} = \dfrac{1}{e^2}$

49) $\lim\limits_{x\to 0}\sqrt[2x]{1-4x} = \lim\limits_{x\to 0}1-4x(1-4x)^{\frac{1}{2x}} = 1^\infty\,(I) = e^{\lim\limits_{x\to 0}(1-4x-1)\cdot\frac{1}{2x}} = e^{\lim\limits_{x\to 0}\frac{-4x}{2x}} = e^{\lim\limits_{x\to 0}(-2)} = e^{-2} = \dfrac{1}{e^2}$

50) $\lim\limits_{x\to 2}\left(\dfrac{2x-1}{x+1}\right)^{\frac{x}{x-2}} = 1^\infty\,(I) = e^{\lim\limits_{x\to 2}\left(\frac{2x-1}{x+1}-1\right)\cdot\frac{x}{x-2}} \underset{(*)}{=} e^{\frac{2}{3}} = \sqrt[3]{e^2}$

$(*)\lim\limits_{x\to 2}\left(\dfrac{2x-1}{x+1}-1\right)\cdot\dfrac{x}{x-2} = \lim\limits_{x\to 2}\left(\dfrac{2x-1-x-1}{x+1}\right)\cdot\dfrac{x}{x-2} = \lim\limits_{x\to 2}\left(\dfrac{x-2}{x+1}\right)\cdot\dfrac{x}{x-2} = \lim\limits_{x\to 2}\dfrac{x}{x+1} = \dfrac{2}{3}$

51) $\lim\dfrac{x-1}{x^2+x-2}$ cuando $x\to +\infty,\ x\to 0,\ x\to 1,\ x\to 3$

- $\lim\limits_{x\to +\infty}\dfrac{x-1}{x^2+x-2} = \dfrac{+\infty}{+\infty}(I) = \lim\limits_{x\to +\infty}\dfrac{\dfrac{x}{x^2}-\dfrac{1}{x^2}}{\dfrac{x^2}{x^2}+\dfrac{x}{x^2}-\dfrac{2}{x^2}} == \lim\limits_{x\to +\infty}\dfrac{\dfrac{1}{x}-\dfrac{1}{x^2}}{1+\dfrac{1}{x}-\dfrac{2}{x^2}} = \dfrac{0-0}{1+0-0} = \dfrac{0}{1} = 0$

- $\lim\limits_{x\to 0}\dfrac{x-1}{x^2+x-2} = \dfrac{-1}{-2} = \dfrac{1}{2}$

- $\lim\limits_{x\to 1}\dfrac{x-1}{x^2+x-2} = \dfrac{0}{0}(I) = \lim\limits_{x\to 1}\dfrac{(x-1)}{(x-1)(x+2)} = \lim\limits_{x\to 1}\dfrac{1}{(x+2)} = \dfrac{1}{3}$

- $\lim\limits_{x\to 3}\dfrac{x-1}{x^2+x-2} = \dfrac{2}{10} = \dfrac{1}{5}$

52) $\lim \dfrac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}$ cuando $x\to+\infty,\ x\to 0,\ x\to -1,\ x\to -3$

- $\displaystyle\lim_{x\to+\infty}\frac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}=\frac{+\infty}{+\infty}(I)=\lim_{x\to+\infty}\frac{\dfrac{x^5}{x^4}+\dfrac{2x^4}{x^5}+\dfrac{4x^3}{x^5}+\dfrac{8x^2}{x^5}}{\dfrac{4x^4}{x^5}+\dfrac{12x^3}{x^5}+\dfrac{13x^2}{x^5}+\dfrac{6x}{x^5}+\dfrac{1}{x^5}}=$

$$=\lim_{x\to+\infty}\frac{x+\dfrac{2}{x}+\dfrac{4}{x^2}+\dfrac{8}{x^3}}{\dfrac{4}{x}+\dfrac{12}{x^2}+\dfrac{13}{x^3}+\dfrac{6}{x^4}+\dfrac{1}{x^5}}=\frac{+\infty}{0}=+\infty$$

- $\displaystyle\lim_{x\to0}\frac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}=\frac{0}{1}=0$

- $\displaystyle\lim_{x\to-1}\frac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}=\frac{5}{0}=\begin{cases}\displaystyle\lim_{x\to-1^-}\frac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}=\dfrac{5}{0^+}=+\infty\\[4mm]\displaystyle\lim_{x\to-1^+}\frac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}=\dfrac{5}{0^+}=+\infty\end{cases}$

- $\displaystyle\lim_{x\to-3}\frac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}=\frac{(-3)^5+2(-3)^4+4(-3)^3+8(-3)^2}{4(-3)^4+12(-3)^3+13(-3)^2+6(-3)+1}=\frac{-243+162-108+72}{324-324+117-18+1}=-\frac{117}{100}$