

OPERACIONES CON POLINOMIOS

Sean los polinomios:

$$p(x) = \frac{1}{3}x^4 + \frac{1}{2}x^2 - x + 2$$

$$q(x) = 2x^4 + 5x^3 - 3x^2 + 2x$$

$$c(x) = (3x^2 + 2)$$

Calcular

$$\begin{aligned} \text{a) } & 5(p(x) - 2q(x)) - (q(x) - p(x)) = \\ & = 5p(x) - 10q(x) - q(x) + p(x) = \\ & = 6p(x) - 11q(x) = \\ & = 6\left(\frac{1}{3}x^4 + \frac{1}{2}x^2 - x + 2\right) - 11(2x^4 + 5x^3 - 3x^2 + 2x) = \\ & = 2x^4 + 3x^2 - 6x + 12 - 22x^4 - 55x^3 + 33x^2 - 22x = \\ & = -20x^4 + 36x^2 - 28x + 12 - 55x^3 \end{aligned}$$

$$\begin{aligned} \text{b) } & c(x)^2 + (2p(x) - q(x)) - (c(x) + p(x)) + 3c(x) = \\ & = c(x)^2 + 2p(x) - q(x) - c(x) - p(x) + 3c(x) = \\ & = c(x)^2 + p(x) - q(x) + 2c(x) = \\ & = (3x^2 + 2)^2 + \frac{1}{3}x^4 + \frac{1}{2}x^2 - x + 2 - (2x^4 + 5x^3 - 3x^2 + 2x) + \\ & \quad 2 \cdot (3x^2 + 2) = \end{aligned}$$

$$\begin{aligned} \text{c) } & \text{Calcular: } q(x) \cdot c(x) = (2x^4 + 5x^3 - 3x^2 + 2x) \cdot (3x^2 + 2) = \\ & = 6x^6 + 4x^4 + 15x^5 + 10x^3 - 9x^4 - 6x^2 + 6x^3 + 4x = \\ & = 6x^6 - 5x^4 + 15x^5 + 16x^3 - 6x^2 + 4x. \end{aligned}$$

$$\begin{aligned} & = 9x^4 + 4 + 12x^2 + \frac{1}{3}x^4 + \frac{1}{2}x^2 - x + 2 - 2x^4 - 5x^3 + 3x^2 - 2x + \\ & \quad + 6x^2 + 4 = \\ & = \frac{22}{3}x^4 + 10 + \frac{43}{2}x^2 - 3x - 5x^3 \end{aligned}$$

$$\begin{aligned}
& \left(6p(x) + \frac{1}{3}q(x)\right) - \left(\frac{1}{2}p(x) + q(x)\right) + c(x)^3 = \\
& = 6p(x) + \frac{1}{3}q(x) - \frac{1}{2}p(x) - q(x) + c(x)^3 = \\
& = \frac{11}{2}p(x) - \frac{2}{3}q(x) + c(x)^3 = \\
& = \frac{11}{2} \left(\frac{1}{3}x^4 + \frac{1}{2}x^2 - x + 2\right) - \frac{2}{3}(2x^4 + 5x^3 - 3x^2 + 2x) + \\
& \quad (3x^2 + 2)^3 = \\
& = \left(\frac{11}{6}x^4 + \frac{11}{4}x^2 - \frac{11}{2}x + \frac{22}{2} - \frac{4}{3}x^4 - \frac{10}{3}x^3 + \frac{6}{3}x^2 - \frac{4}{3}x + \right. \\
& \quad \left.(3x^2)^3 + 2^3 + 3(3x^2)^2 \cdot 2 + 3 \cdot 3x^2 \cdot 2^2 = \right. \\
& = \frac{1}{2}x^4 + \frac{19}{4}x^2 - \frac{41}{6}x + 11 - \frac{10}{3}x^3 + 27x^6 + 8 + 54x^4 + 36x^2 = \\
& = \frac{109}{2}x^4 + \frac{163}{4}x^2 - \frac{41}{6}x + 19 - \frac{10}{3}x^3 + 27x^6
\end{aligned}$$

¿Cuánto han de valer a y b para que la siguiente división sea exacta?

$$x^4 - 5x^3 + 3x^2 + \overbrace{ax}^{d?} + \overbrace{b}^{d?} : x^2 - 5x + 1$$

Efectuemos primero la división de los dos polinomios.

$$\begin{array}{r}
x^4 - 5x^3 + 3x^2 + ax + b \quad | \quad x^2 - 5x + 1 \\
\underline{-x^4 + 5x^3 - x^2} \\
 2x^2 + ax + b \\
\underline{-2x^2 + 10x - 2} \\
 (10+a)x + (b-2)
\end{array}$$

Al efectuar la división nos ha dado como polinomio cociente $x^2 + 2$.

$$C(x) = x^2 + 2$$

$$R(x) = (10+a)x + (b-2)$$

Calcular a y b.

$$R(x) = 0$$

$$(10+a)x + (b-2) = 0x + 0$$

$$10+a=0 \Rightarrow a = -10$$

$$b-2=0 \Rightarrow b = 2$$

2 polinomios del mismo grado son iguales cuando sean iguales sus respectivos coeficientes.

Calcula el cociente y el resto de las divisiones.

$$\begin{array}{r} x^4 \quad -4x^2 + 12x - 9 \\ -x^4 + 2x^3 - 3x^2 \\ \hline \end{array} \quad \left| \begin{array}{l} x^2 - 2x + 3 \\ x^2 + 2x - 3 \end{array} \right.$$

$$\begin{array}{r} + 2x^3 - 7x^2 + 12x - 9 \\ - 2x^3 + 4x^2 - 6x \\ \hline \end{array}$$

$$\begin{array}{r} -3x^2 + 6x - 9 \\ + 3x^2 - 6x + 9 \\ \hline \end{array}$$

0

$$R(x) = 0$$

$$C(x) = x^2 + 2x - 3$$

$$\begin{array}{r} 3x^3 - 5x^2 + 7x - 3 \\ -3x^3 \quad \quad + 3x \\ \hline \end{array} \quad \left| \begin{array}{l} x^2 - 1 \\ 3x - 5 \end{array} \right.$$

$$\begin{array}{r} -5x^2 + 10x - 3 \\ + 5x^2 \quad \quad - 5 \\ \hline \end{array}$$

$$10x - 8$$

$$C(x) = 3x - 5$$

$$R(x) = 10x - 8$$

$$\begin{array}{r} 3x^4 - x^2 - 1 \\ -3x^4 + 3x^3 + 4x^2 \\ \hline \end{array} \quad \left| \begin{array}{l} 3x^2 - 3x - 4 \\ x^2 + x + 2 \end{array} \right.$$

$$\begin{array}{r} 3x^3 + 3x^2 - 1 \\ 3x^3 + 3x^2 + 4x \\ \hline \end{array}$$

$$\begin{array}{r} 6x^2 + 4x - 1 \\ -6x^2 + 6x + 8 \\ \hline \end{array}$$

$$10x + 7$$

$$C(x) = x^2 + x + 2$$

$$R(x) = 10x + 7$$

$$\begin{array}{r}
 6x^5 + 9x^4 - 7x^3 + 7x^2 - 8x + 5 \quad | \quad 3x^2 - 3x - 1 \\
 - 6x^5 + 6x^4 + 2x^3 \\
 \hline
 + 15x^4 - 5x^3 + 7x^2 - 8x + 5 \\
 - 15x^4 + 15x^3 + 5x^2 \\
 \hline
 + 10x^3 + 12x^2 - 8x + 5 \\
 - 10x^3 + 10x^2 + \frac{10}{3}x \\
 \hline
 + 22x^2 - \frac{14}{3}x + 5 \\
 - 22x^2 + 22x + \frac{22}{3} \\
 \hline
 \frac{52}{3}x + \frac{37}{3}
 \end{array}$$

$$-8 + \frac{10}{3} = \frac{-24 + 10}{3} = \frac{-14}{3}$$

$$-\frac{14}{3} + 22 = \frac{-14 + 66}{3} = \frac{52}{3}$$

$$5 + \frac{22}{3} = \frac{15 + 22}{3} = \frac{37}{3}$$

$$C(x) = 2x^3 + 5x^2 + \frac{10}{3}x + \frac{22}{3}$$

$$R(x) = \frac{52}{3}x + \frac{37}{3}$$