Nombre:_____

1. Opera con radicales dejando el resultado de la forma más reducida posible:

   a) $\frac{\sqrt[4]{a^3b}\sqrt[6]{a^4b^3}}{\frac{1}{\sqrt{b}}} \div \sqrt[6]{a}$ (1 punto)
   b) $\sqrt[3]{81a^3} + 2a\sqrt[3]{24}$ (0,5 punto)

   c) $\sqrt{x}\sqrt[3]{x}\sqrt[4]{x^3}$ (0,5 punto)

   d) $\sqrt[3]{\sqrt{2}\sqrt[3]{4}}$ (0,5 punto)

2. Opera y simplifica: (1 punto)

$$\frac{3}{\sqrt{3}-\sqrt{2}} - \frac{2}{\sqrt{3}+\sqrt{2}}$$

3. Expresa mediante intervalo los siguientes conjuntos de números reales y represéntalos en la recta. (1 punto)

   a) $|2x+6| < 1$

   b) $\left|x - \frac{1}{2}\right| > \frac{1}{4}$

4. Reduce a un solo logaritmo, aplicando las propiedades de los logaritmos. (1 punto)

$$3\log(x+10) - log\frac{2x+20}{3} + log\frac{3}{2}$$

5. Resuelve las siguientes ecuaciones. (3 puntos)

   a) $4^{x^2+1} = 2^{5x+5}$
   b) $\sqrt{x+1} + \sqrt{2x+3} = 5$
   c) $6x^3 - 7x^2 - 14x + 15 = 0$

6. Resuelve el siguiente sistema. (1,5 puntos)

$$\begin{cases} 2x + 4y = 10 \\ x^2 + 3xy = -8 \end{cases}$$

① a) $\dfrac{\sqrt[12]{(a^3b)^3}\cdot\sqrt[12]{(a^4b^3)^2}}{\sqrt[12]{\frac{1}{b^6}}} : \sqrt[12]{a^2} = \left(\sqrt[12]{\dfrac{a^9b^3\cdot a^8\cdot b^6}{1} : \dfrac{1}{b^6}}\right) : \sqrt[12]{a^2} =$

$\sqrt[12]{a^9b^3a^8b^6\cdot b^6 : a^2} = \sqrt[12]{a^{9+8-2}\cdot b^{3+6+6}} = \sqrt[12]{a^{15}\cdot b^{15}} = \sqrt[4]{a^5\cdot b^5} = ab\sqrt[4]{ab}.$

b) $\sqrt[3]{81a^3} + 2a\sqrt[3]{24} = \sqrt[3]{3^4a^3} + 2a\sqrt[3]{2^3\cdot 3} = 3a\sqrt[3]{3} + 2a\cdot 2\sqrt[3]{3} =$

$= \boxed{7a\sqrt[3]{3}}$

c) $\sqrt{x}\sqrt[3]{x}\sqrt[4]{x^3} = \sqrt[12]{x^6\cdot x^4\cdot (x^3)^3} = \sqrt[12]{x^{19}} = \boxed{x\sqrt[12]{x^7}}$

d) $\sqrt[3]{\sqrt{2}\sqrt[3]{4}} = \sqrt[3]{\sqrt[6]{2^3(2^2)^2}} = \sqrt[18]{2^3\cdot 2^4} = \boxed{\sqrt[18]{2^7}}$

② $\dfrac{3}{\sqrt{3}-\sqrt{2}} - \dfrac{2}{\sqrt{3}+\sqrt{2}} = 3\sqrt{3}+3\sqrt{2}-2\sqrt{3}+2\sqrt{2} = \boxed{\sqrt{3}+5\sqrt{2}}$

$\left(\dfrac{3}{\sqrt{3}-\sqrt{2}}\right)\cdot\left(\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right) = \dfrac{3\sqrt{3}+3\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \dfrac{3\sqrt{3}+3\sqrt{2}}{3-2} = 3\sqrt{3}+3\sqrt{2}$

$\left(\dfrac{2}{\sqrt{3}+\sqrt{2}}\right)\cdot\left(\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right) = \dfrac{2\sqrt{3}-2\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \dfrac{2\sqrt{3}-2\sqrt{2}}{3-2} = 2\sqrt{3}-2\sqrt{2}$

③ a) $|2x+6| < 1$ — $2x+6 < 1 \rightarrow x < \dfrac{1-6}{2}$; $x < -\dfrac{5}{2}$

$2x+6 > -1 \rightarrow x > -7$; $x > -\dfrac{7}{2}$

$-\dfrac{7}{2} < x < -\dfrac{5}{2} \rightarrow \boxed{\left(-\dfrac{7}{2}, -\dfrac{5}{2}\right)}$

b) $\left|x-\dfrac{1}{2}\right| > \dfrac{1}{4} \Rightarrow x-\dfrac{1}{2} > \dfrac{1}{4}$; $x > \dfrac{1}{4}+\dfrac{1}{2}$; $\boxed{x > \dfrac{3}{4}}$

$x-\dfrac{1}{2} < -\dfrac{1}{4}$; $x < -\dfrac{1}{4}+\dfrac{1}{2}$; $x < \dfrac{-1+2}{4}$; $\boxed{x < \dfrac{1}{4}}$

$\boxed{\left(-\infty, \dfrac{1}{4}\right) \cup \left(\dfrac{3}{4}, \infty\right)}$

④ $\log(x+10)^3 - \log\dfrac{2x+20}{3} + \log\dfrac{3}{2} = \log\left(\dfrac{(x+10)^3}{\frac{2x+20}{3}}\cdot\dfrac{3}{2}\right)$

$= \log\left[\dfrac{3\cdot(x+10)^{\cancel{3}}}{2\cancel{(x+10)}}\cdot\dfrac{3}{2}\right] = \boxed{\log\left(\dfrac{9(x+10)^2}{4}\right)}$

a) $4^{x^2+1} = 2^{5x+5}$; $(2^2)^{x^2+1} = 2^{5x+5}$; $2^{2(x^2+1)} = 2^{5x+5}$

$2x^2+2 = 5x+5$; $2x^2-5x-3=0$; $x = \frac{5 \pm \sqrt{25-4\cdot 2(-3)}}{2\cdot 2} = \frac{5 \pm 7}{4}$

$$\boxed{\begin{array}{l} x = \frac{5+7}{4} = \frac{12}{4} = 3 \\ x = \frac{5-7}{4} = \frac{-2}{4} = -\frac{1}{2} \end{array}}$$ Soluciones

b) $\sqrt{x+1} + \sqrt{2x+3} = 5$

$(\sqrt{x+1})^2 = (5-\sqrt{2x+3})^2$; $x+1 = 25 + (\sqrt{2x+3})^2 - 2\cdot 5\cdot\sqrt{2x+3}$;

$x+1 = 25+2x+3-10\sqrt{2x+3}$; $10\sqrt{2x+3} = 25+2x+3-x-1$;

$(10\sqrt{2x+3})^2 = (27+x)^2$; $\frac{10^2}{100}\cdot(2x+3) = 27^2+x^2+2\cdot 27x$;

$200x+300 = 729+x^2+54x$; $x^2-146x+429=0$

$x = \frac{146 \pm \sqrt{146^2-4\cdot 1\cdot 429}}{2} = \frac{146 \pm \sqrt{19600}}{2} = \frac{146 \pm 140}{2}$

$x = \frac{146+140}{2} = 143$ No $\quad \sqrt{143+1} + \sqrt{2\cdot 143+3} = 12+\dots \neq 5$

Comprobamos

$x = \frac{146-140}{2} = 3$ Sí $\quad \sqrt{3+1} + \sqrt{2\cdot 3+3} = \sqrt{4}+\sqrt{9} = 2+3 = 5$

c) $6x^3-7x^2-14x+15=0$

	6	-7	-14	15
1		6	-1	-15
	6	-1	-15	0

$6x^2-x-15=0$

$x = \frac{1 \pm \sqrt{1-4\cdot 6(-15)}}{2\cdot 6} = \frac{1 \pm \sqrt{361}}{12}$

$x = \frac{1 \pm 19}{12}$; $\frac{20}{12} = \frac{5}{3} = x$; $\frac{-18}{12} = -\frac{3}{2} = x$

$x = 1$

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$$\begin{cases} 2x+4y=10 \\ x^2+3xy=-8 \end{cases} \longrightarrow x+2y=5 \; ; \; x=5-2y$$

$$(5-2y)^2+3(5-2y)y=-8 ; \; 25+4y^2-20y+15y-6y^2=-8 ;$$

$$-2y^2-5y+33=0$$

$$y=\frac{5\pm\sqrt{5^2-4\cdot(-2)\cdot 33}}{2(-2)}=\frac{5\pm\sqrt{289}}{-4}=\frac{5\pm 17}{-4} \begin{cases} \frac{22}{-4}=-\frac{11}{2} \\ \frac{-12}{-4}=3 \end{cases}$$