Unit 5: DECIMAL NUMBERS

5.1.- DECIMAL NUMBERS. PLACE VALUE

Decimal numbers are widely used in everyday life. We see them frequently in money and in measurements of length, area, weight and so on.

Exercise: Use decimal numbers to express the value of the following coins.

The value of a digit in a decimal depends on its position, or **place**, in the decimal. Each place is 10 times the value of the next place to its right.

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
		4.	0		
		5.	4		
		5.	0	4	
		5.	0	0	4

The 4 is in the *ones* place.
Its value is 4 *ones*, or 4.

The 4 in 5.4 is in the *tenths* place.
Its value is 4 *tenths*, or 0.4.

The 4 in 5.04 is in the *hundredths* place.
Its value is 4 *hundredths*, or 0.04.

The 4 in 5.004 is in the *thousandths place.*
Its value is 4 *thousandths*, or 0.004.

To read a decimal less than 1:

- Start at the decimal point.
- Read the number as a whole number. Then say the name of the place.

0.92 — hundredths

Read: ninety-two hundredths

To read a decimal greater than 1:

37.19 — whole number part | decimal part

Read: thirty-seven *and* nineteen hundredths

Exercise 1:

Write in words the following numbers.

a) 0.8 b) 35.004 c) 0.00001 d) 152.43 e) 0.0075

Exercise 2:

Write the place of the underlined digit. Then write its value.

a) 2̲6.9 b) 7.0045̲1 c) 9.61̲3 d) 14.01̲6 e) 3.7̲4

Exercise 3:

Write in figures the following numbers.

a) Fifty-nine and four hundredths.
b) Twelve ten-thousandths.
c) Four hundred and sixty-five thousandths.
d) Eight millionths.

Exercise 4:

Look at the table below and answer the questions.

Ones .	Tenths	Hundredths	Thousandths	Ten-thousandths
	3	2	0	
1	8	0		
		5	0	0
	6	0	0	0

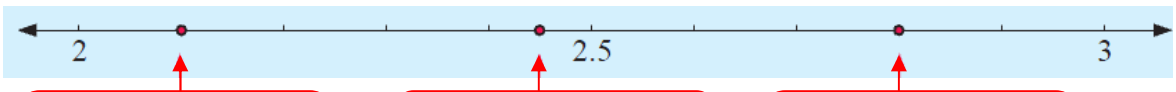
a) How many hundredths are there in 320 thousandths?
b) How many hundredths are there in 18 tenths?
c) How many hundredths are there in 500 ten-thousandths?
d) How many ten-thousandths are there in 6 tenths?

5.2.- ORDERING DECIMAL NUMBERS

Just like whole numbers, decimal numbers can be shown in a **number line**. To do this we generally divide each segment of the number line in **ten equal parts**.

Example 1: Place the values 2.1, 2.45 and 2.8 on a number line.

Divide a number line from 2 to 3 into ten equal parts.

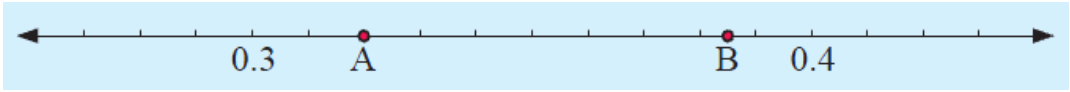


2 is placed $\frac{1}{10}$ of the way from 2 to 3.

2.45 lies half-way between 2.4 and 2.5.

2 is placed $\frac{8}{10}$ of the way from 2 to 3.

Example 2: Write down the values of A and B on the number line.



The segment between 0.3 and 0.4 is divided into 10 equal parts, so the number line shows 0.30, 0.31, 0.32, …, 0.39, 040.

A lies at 0.32 B lies half-way between 0.38 and 0.39, so B is 0.385.

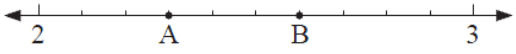
Exercise 5:

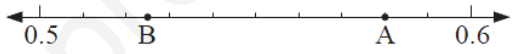
Place the following decimal numbers on separate number lines.

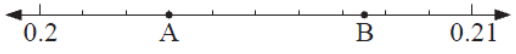
a) 4.3, 4.75, 4.8 b) 68.7, 68.2, 69.1 c) 0.22, 0.26, 0.29

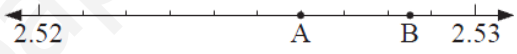
Exercise 6:

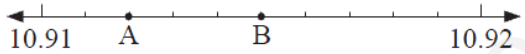
Write down the values of A and B on the following number lines.

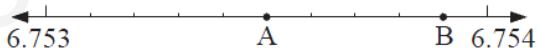
a) 

b) 

c) 

e) 

d) 

f) 

Example 3:

Insert <, > or = between the numbers 5.301 and 5.31.

Both numbers have 5 wholes and three tenths.
5.302 has zero hundredths whereas 5.31 has one hundredth.
So, 5.302 < 5.31

Remenber that:
> means
"is greater than"
< means
"is less than"

Example 4:

Write the following decimal numbers in ascending order (from smallest to largest): 7.35, 7.28, 7.095

To help compare the numbers we write them with the same number of places after the decimal point:

We can write zeros at the end of decimal numbers without changing the place value of the other digits.

7.350, 7.280, 7.095

The numbers each have the same whole number part: 7.350, 7.280, 7.095 but different values in the tenths place:

7.350, 7.280, 7.095

So, 7.095, 7.28, 7.35 are in ascending order.

Exercise 7:

Insert $<$, $>$ or $=$ between these pairs of numbers.

a) 3.63, 3.6	b) 7.07, 7.7	c) 0.00876, 0.0786
d) 0.229, 0.292	e) 0.7, 7/100	f) 21.101, 21.011
g) 7.5, 7.500	h) 0.47, 0.5	i) 0.746, 0.467

Exercise 8:

Place these numbers in order from lowest to highest to spell a team going backwards.

A	0.01857	N	-0.036	E	0.02
T	-0.037	D	-0.041	O	0.29
R	0.3	M	0.017	E	-0.0375
H	0.17	T	0.172	R	0.019
H	0.0186	U	-0.03	I	-0.0364

Exercise 9:

Matthew's best four times for and 80 m sprint are 9.9 seconds, 9.09 seconds, 9.99 seconds and 9.89 seconds. Place these times in order from fastest to slowest.

5.3.- DECIMAL APPROXIMATIONS

We are often given measurements as decimal numbers. In such cases we approximate the decimal by **rounding off** to the required accuracy.

We have previously seen how to round off whole numbers. For example:

3628	$\approx$3600	(to the nearest hundred)
	$\approx$4000	(to the nearest thousand)

We round off decimal numbers in the same way. For example:

0.3872	$\approx$0.387	(to 3 decimal places)
	$\approx$0.39	(to 2 decimal places)
	$\approx$0.4	(to 1 decimal place)

Rules for rounding off decimal numbers

- If the digit after the one being rounded is **less than 5**, then we round **down**.
- If the digit after the one being rounded is **5 or more**, then we round **up**.

Example: Find $\frac{36}{17}$ correct to 3 decimal places.

Rather than do long division, we can use a calculator.
Pressing 36 ÷ 17 = , the result is 2.117647059

So, $\frac{36}{17} \approx 2.118$ (to 3 decimal places)

Exercise 10:

Write 0.07149 correct to:

a) 1 decimal place b) 2 decimal places c) 3 decimal places

Exercise 11:

Tye calculates the interest due on her savings account to be $78.1983. Round this to the nearest cent.

Exercise 12:

Use your calculator to round to the number of decimal places in the square brackets.

a) $39 \div 17$ [4] b) $56.9 \div 11.7$ [3] c) $(0.367)^2$ [3]

d) $(8.391)^3$ [3] e) $0.637 \times (0.21)^2$ [4] f) $\frac{21.62}{(8.37)^2}$ [3]

5.4.- ADDING AND SUBTRACTING DECIMALS

When **adding** or **subtracting** decimal numbers, we write the numbers under one another so the decimal points are directly underneath each other.

When this is done, the digits in each place value will also lie under one another. We then add or subtract as for whole numbers.

Examples: a) $152.97 + 387.25$ b) $49.6 - 44.574$

```
  1  1  1  1
  1  5  2 . 9  7
+ 3  8  7 . 2  5
-----------------
  5  4  0 . 2  2
```

```
                9
          5  10 10
  4  9 . 6  Ø  Ø
- 4  4 . 5  7  4
-----------------
     5 . 0  2  6
```

Write two zeros in the minuend since the subtrahend has three decimal places.

Exercise 13:

Compute with pencil and paper:

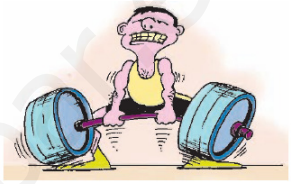
a) $12.66 + 1.302$ b) $0.021 + 0.979$ c) $0.16 + 2.09 + 0.895$
d) $0.083 - 0.0091$ e) $3 - 0.72$ f) $0.16 + 0.093 - 0.131$

Exercise 14:

A 20 m length of rope is cut into 4 pieces. Three of the pieces have lengths 5.62 m, 8.05 m, and 2.6 m. Find the length of the fourth piece.

Exercise 15:

A weightlifter snatches 135.8 kg, 142.9 kg, and 153.7 kg in consecutive lifts. Find the total mass lifted.



Exercise 16:

Taxation	€507.90
Private Health Cover	€119.20
Superannuation	€95.62
Union Fees	€14.82

Each fortnight Alex is paid €1700 less the deductions given in the table alongside. What is Alex's actual take home pay each fortnight?

Exercise 17:

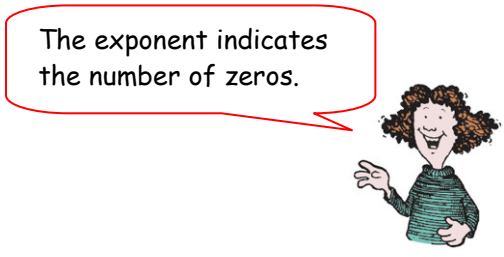
Continue the number sequences by writing the next three terms:

a) 3.25, 4, 4.75, 5.5, … b) 8.65, 8.5, 8.35, 8.2, … c) 1.5, 1.62, 1.74, 1.86, …

5.5.- MULTIPLYING AND DIVIDING BY POWERS OF 10

Remember:

$10^1 = 10$
$10^2 = 100$
$10^3 = 1000$
$10^4 = 10\,000$
$\vdots$



When multiplying by 10^n we shift the decimal point n places to the **right**. The number becomes 10^n times **larger** than it was originally.

Examples:

a) $9.8 \times 10 = 98$ ($10 = 10^1$, so shift the decimal point 1 place right)
b) $0.0751 \times 100 = 7.51$ ($100 = 10^2$, so shift the decimal point 2 places right)
c) $13.26 \times 1000 = 13260$ ($1000 = 10^3$, so shift the decimal point 3 places right)

When dividing by 10^n we shift the decimal point n places to the **left**.
The number becomes 10^n times **smaller** than it was originally.

Examples:

a) $0.4 \div 10 = 0.04$ ($10 = 10^1$, so shift the decimal point 1 place left)
b) $8721 \div 100 = 87.21$ ($100 = 10^2$, so shift the decimal point 2 places left)
c) $85 \div 1000 = 0.085$ ($1000 = 10^3$, so shift the decimal point 3 places left)

Exercise 18:

Compute:

a) 0.0583×100 b) 25×1000 c) 1.89×10^4
d) $463 \div 10000$ e) $375.6 \div 100$ f) $0.02 \div 1000$

5.6.- MULTIPLYING DECIMAL NUMBERS

To **multiply** decimals:

- Multiply as you would multiply whole numbers.
- Write the product.
- Then count the number of decimal places in both factors.
- Mark off that number of decimal places in the product.

Examples:

19 × 0.843 = ?

0.843 ← 3 decimal places
× 19. ← no decimal places
7587
843
16.017 ← 3 decimal places

1.78 × 0.0079 = ?

0.0079 ← 4 places
× 1.78 ← 2 places
632
553
79
0.014062 ← 6 decimal places (Insert a zero.)

Exercise 19:

Multiply.

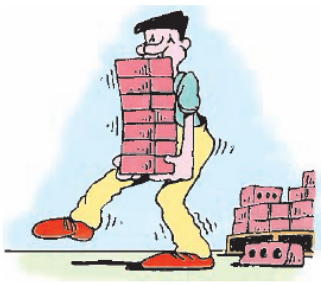
a) 0.42×0.08 b) 3.70×1.20 c) 27.5×10.4 d) 66.3×0.82

Exercise 20:

In order to bake cakes for the school fair, I buy 180 kg of flour at $0.84 per kg and 25 kg of sugar at $1.17 per kg. How much money have I spent?

Exercise 21:

House bricks have a mass of 4.3 kg each and I buy 2500 of them to build a wall around my courtyard.



a) Find the total mass of the bricks.
b) If my truck can carry only 2 tonnes at a time, how many truck loads are necessary to transport the bricks?

5.7.- DIVIDING DECIMAL NUMBERS

Investigation:

What to do:

1. Copy and complete the following divisions. Look for patterns to use when the divisions involve decimals.

   a) $800 \div 200 = \square$, $80 \div 20 = \square$, $8 \div 2 = \square$, $0.8 \div 0.2 = \square$

   b) $800 \div 20 = \square$, $80 \div 2 = \square$, $8 \div 0.2 = \square$, $0.8 \div 0.02 = \square$

   c) $80 \div 200 = \square$, $8 \div 20 = \square$, $0.8 \div 2 = \square$, $0.08 \div 0.2 = \square$

2. In each set of divisions, what did you notice about the answers?

3. Did you find that in each set the division by the smallest *whole* number was the easiest?

From the investigation you should have observed that:

> Multiplying or dividing both numbers in a division by the same factor **does not change** the result.

Dividing a decimal by a whole number

To divide a decimal by a whole number:

- Divide as you would with whole numbers.
- Place the decimal point in the quotient in the same place as the dividend.

Example: $815.424 \div 24$

```
8 1 5 . 4 2 4 | 2 4
0 9 5         | 3 3 . 9 7 6
  2 3 4
    1 8 2
      1 4 4
        0 0
```

Dividing by a decimal

To divide by a decimal:

- Change the divisor to a whole number by moving the decimal point the necessary number of places *to the right*.
- Move the decimal point in the dividend the same number of places *to the right*.
  (Remember: Multiplying or dividing both numbers in a division by the same factor **does not change** the result).
- Divide as you would with a decimal by a whole number.

Example: $64.452 \div 2.46$

$64.452 \div 2.46 \Rightarrow 6445.2 \div 246$ (We have multiplied the dividend and the divisor by 100)

$$\begin{array}{r|l} 6445.2 & 246 \\ \cline{2-2} 1525 & 26.2 \\ 0492 & \\ 000 & \end{array}$$

Exercise 22:

One digit in each answer below is wrong. Correct it.

a) $72.24 \div 8 = 9.01$ b) $74.58 \div 6 = 12.33$ c) $31.92 \div 7 = 4.66$

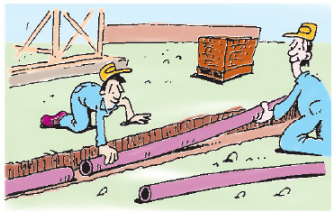
Exercise 23:

Round the quotient to the nearest hundredth.

a) $5.485 \div 1.13$ b) $5 \div 0.03$ c) $0.1252 \div 24.6$

Exercise 24:

Determine the number of 2.4 lengths of piping required to construct a 720 drain.



Exercise 25:

A car averages 6.3 kilometres per litre of petrol. How much petrol will be needed for a 110.25-km trip?

Exercise 26:

A car transporter weighs 12.6 tonnes when empty and 23.4 tonnes when loaded with 12 cars. What is the weight of one car?

Exercise 27:

In the triple-jump competition, Michael jumped 14.23 m, 14.78 m, 14.43 m, 14.37 m, 14.62 m and 13.97 m.

a) What was his mean jump?
b) How many centimetres below 15 m was his mean jump?

Exercise 28:

Work out these expressions and put the answers in order from highest to lowest. Do you agree with what you spell?

S	$34 \div 1000$	N	490×0.01
M	$651 \div 0.01$	I	5×100
T	72×10	G	6×0.1
Y	$-63 \div 0.1$	I	12×0.01
S	-9×10	I	$112 \div 10$
A	$-350 \div 100$	L	$3 \div 0.01$
L	10×93	U	5.4×1000
E	0.3×0.1	P	35×10
Y	$10640 \div 100$		

Exercise 29:

Steve is 30 cm shorter than Peter. Bernard is 86 mm shorter than Carl. Peter is 1.2 cm taller than Bernard. Peter is 1.84 m tall. What are the heights of Steve, Carl and Bernard?

Exercise 30:

Fill in the boxes with the digits 1 to 9, using each digit only once.

$\square.\square\square + \square.\square + \square.\square\square = \square$

Exercise 31:

Copy and complete this multiplication grid:

x		0.7		11
3	0.3			
				55
	1.6		128	

Use your grid to work out 19.8×24 .