1.- (3 puntos)

a) Desarrollar la expresión de $tg\ 3a$ dejándola en función de $tg\ a$

b) Sea a un ángulo del II cuadrante y $sen\ a = 3/5$, calcular las razones de $\frac{a}{2}$ (seno, coseno y tangente)

2.- (2 puntos)

a) Resolver la ecuación: $2\ sen^2 x + \cos 2x = 4 \cos^2 x$

b) Calcular en función de h la sec 203°, siendo $cotg\ 67° = h$

3.- (3 puntos)

a) Resuelve el triángulo y calcula su área si se conocen $A = 80°$, $a = 10$ m y $b = 5$ m

b) Calcular $\sqrt[5]{\frac{-1-\sqrt{3}i}{-3+\sqrt{3}i}}$

4.- (2 puntos)

a) Dado el vector $u = (-10, 12)$, hallar sus coordenadas respecto de la base $B = \{v, w\}$ donde $v = (3, -4)$ y $w = (1, -1)$

b) Dados los complejos $z_1 = 2_{60°}$, $z_2 = -1 + i$ y $z_3 = 2(\cos 210° + i\ sen\ 210°)$ calcular

$$\frac{z_1 \cdot \overline{z_2}}{z_3}$$ en forma binómica

① a) $tg\,3a = tg\,(2a+a) = \dfrac{tg\,2a + tg\,a}{1 + tg\,2a \cdot tg\,a} = \dfrac{\dfrac{2tg\,a}{1+tg^2a} + tg\,a}{1 + \dfrac{2tg\,a}{1+tg^2a} \cdot tg\,a} =$

$= \dfrac{\dfrac{2tg\,a + tg\,a + tg^3a}{\cancel{1+tg^2a}}}{\dfrac{1+tg^2a+2tg^2a}{\cancel{1+tg^2a}}} = \boxed{\dfrac{3tg\,a + tg^3a}{1+3tg^2a}}$

b) $\alpha \in II$, $sen\,\alpha = \dfrac{3}{5} \Rightarrow sen^2\alpha + cos^2\alpha = 1 \Rightarrow \dfrac{9}{25} + cos^2\alpha = 1 \Rightarrow$

$cos^2\alpha = 1 - \dfrac{9}{25} \Rightarrow cos^2\alpha = \dfrac{16}{25} \Rightarrow \left.\begin{array}{l} cos\,\alpha = \pm\sqrt{\dfrac{16}{25}} \\ \alpha \in II \end{array}\right\} cos\,\alpha = -\dfrac{4}{5}$.

$\dfrac{\alpha}{2} \in I$:

$sen\,\dfrac{\alpha}{2} = +\sqrt{\dfrac{1-cos\alpha}{2}} = \sqrt{\dfrac{1+\frac{4}{5}}{2}} = \sqrt{\dfrac{\frac{9}{5}}{2}} = \sqrt{\dfrac{9}{10}} = \boxed{\dfrac{3}{\sqrt{10}}}$

$cos\,\dfrac{\alpha}{2} = +\sqrt{\dfrac{1+cos\alpha}{2}} = \sqrt{\dfrac{1-\frac{4}{5}}{2}} = \sqrt{\dfrac{\frac{1}{5}}{2}} = \sqrt{\dfrac{1}{10}} = \boxed{\dfrac{1}{\sqrt{10}}}$

$tg\,\dfrac{\alpha}{2} = \dfrac{\frac{3}{\cancel{\sqrt{10}}}}{\frac{1}{\cancel{\sqrt{10}}}} = \boxed{3}$

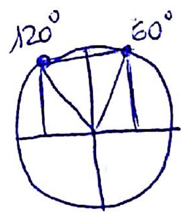
② a) $2\,sen^2x + cos\,2x = 4\,cos^2x$

$2\,sen^2x + cos^2x - sen^2x = 4cos^2x$

$sen^2x - 3cos^2x = 0 \Rightarrow sen^2x - 3(1-sen^2x) = 0 \Rightarrow$

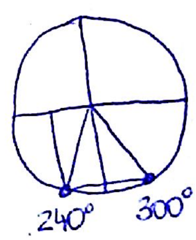
$\Rightarrow sen^2x - 3 + 3\,sen^2x = 0 \Rightarrow 4\,sen^2x - 3 = 0 \Rightarrow 4\,sen^2x = 3 \Rightarrow$

$\Rightarrow sen^2x = \dfrac{3}{4} \Rightarrow sen\,x = \pm\sqrt{\dfrac{3}{4}} = \pm\dfrac{\sqrt{3}}{2}$



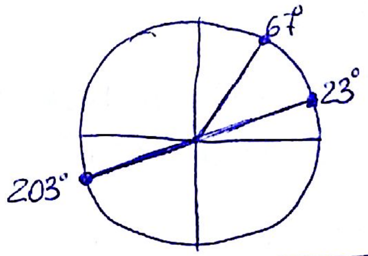
$sen\,x = \dfrac{\sqrt{3}}{2} \Rightarrow$ $x = \begin{cases} 60° + 2\pi K \\ 120° + 2\pi K \end{cases}$

$sen\,x = -\dfrac{\sqrt{3}}{2} \Rightarrow$ $x = \begin{cases} 240° + 2\pi K \\ 300° + 2\pi K \end{cases}$



② b) $\sec 203^\circ$, si $\cotg 67^\circ = h$

$h = \cotg 67^\circ = \tg 23^\circ$ por ser complementarios los ángulos.



$$\sec^2 203^\circ = 1 + \tg^2 203^\circ \Longrightarrow \underset{203^\circ \in \text{III}}{\sec^2 203^\circ = 1 + h^2} \Rightarrow \boxed{\sec 203^\circ = -\sqrt{1+h^2}}$$

Como $\tg 23^\circ = \tg 203^\circ = h$

③ a) $A = 80^\circ$, $a = 10\,m$ y $b = 5\,m$

$$\frac{a}{\operatorname{sen} A} = \frac{b}{\operatorname{sen} B} \Rightarrow \operatorname{sen} B = \frac{b \cdot \operatorname{sen} A}{a} = \frac{5 \cdot \operatorname{sen} 80^\circ}{10} = \frac{\operatorname{sen} 80^\circ}{2} = 0{,}4924$$

$$B = \operatorname{arc\,sen} 0{,}4924 = \begin{cases} \boxed{29{,}49^\circ} \\ \cancel{150{,}5^\circ} \text{ NO VALE } (B = 80^\circ) \end{cases}$$

$$\boxed{C = 180^\circ - 80^\circ - 29{,}49^\circ = 70{,}51^\circ}$$

$$\frac{a}{\operatorname{sen} A} = \frac{c}{\operatorname{sen} C} \Rightarrow \frac{10}{\operatorname{sen} 80^\circ} = \frac{c}{\operatorname{sen} 70{,}51^\circ} \Rightarrow \boxed{c = \frac{10 \cdot \operatorname{sen} 70{,}51^\circ}{\operatorname{sen} 80^\circ} = 9{,}57\,m}$$

$$A = \frac{1}{2}\, a \cdot b \cdot \operatorname{sen} C = \frac{1}{2} \cdot 10 \cdot 5 \cdot \operatorname{sen} 70{,}51^\circ = 25 \cdot \operatorname{sen} 70{,}51^\circ = \boxed{23{,}57\,m^2}$$

b) $\sqrt[5]{\dfrac{-1-\sqrt{3}i}{-3+\sqrt{3}i}}$

$$-1-\sqrt{3}i \Rightarrow |-1-\sqrt{3}i| = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\underset{A \in \text{III}}{\alpha = \operatorname{arc\,tg} \frac{-\sqrt{3}}{-1} = \operatorname{arc\,tg} \sqrt{3}} = \begin{cases} \cancel{60^\circ} \text{ Afijo } (-1, -\sqrt{3}) \in \text{III} \\ 240^\circ \end{cases} \quad 2_{240^\circ}$$

$$-3+\sqrt{3}i \Rightarrow |-3+\sqrt{3}i| = \sqrt{(-3)^2 + (\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$$

$$\underset{B \in \text{II}}{\beta = \operatorname{arc\,tg} \frac{\sqrt{3}}{-3}} = \begin{cases} -30^\circ \rightarrow \cancel{330^\circ} \text{ Afijo } (-3, \sqrt{3}) \in \text{II} \\ 150^\circ \end{cases} \quad 2\sqrt{3}_{150^\circ}$$

$$\sqrt[5]{\frac{2_{240^\circ}}{2\sqrt{3}_{150^\circ}}} = \sqrt[5]{\left(\frac{\cancel{2}}{\cancel{2}\sqrt{3}}\right)_{90^\circ}} = \sqrt[5]{\left(\frac{1}{\sqrt{3}}\right)_{90^\circ}} = \left(\sqrt[5]{\frac{1}{\sqrt{3}}}\right)_{\underbrace{\frac{90^\circ + 2\pi K}{5}}_{18^\circ + 72K\ \forall K = 0,1,2,3,4}} = \begin{cases} \text{Si } K=0 \Rightarrow \left(\sqrt[5]{1/\sqrt{3}}\right)_{18^\circ} \\ \text{Si } K=1 \Rightarrow \left(\sqrt[5]{1/\sqrt{3}}\right)_{90^\circ} \\ \text{Si } K=2 \Rightarrow \left(\sqrt[5]{1/\sqrt{3}}\right)_{162^\circ} \\ \text{Si } K=3 \Rightarrow \left(\sqrt[5]{1/\sqrt{3}}\right)_{234^\circ} \\ \text{Si } K=4 \Rightarrow \left(\sqrt[5]{1/\sqrt{3}}\right)_{306^\circ} \end{cases}$$

(4)

a) $\vec{u} = (-10, 12) \qquad B = \{(3,-4), (1,-1)\}$

$(-10, 12) = \alpha(3,-4) + \beta(1,-1) = (3\alpha+\beta, -4\alpha-\beta)$

$\begin{cases} 3\alpha+\beta = -10 \\ -4\alpha-\beta = 12 \end{cases} \longrightarrow 3\cdot(-2)+\beta = -10 \Rightarrow -6+\beta = -10 \Rightarrow$

$\Rightarrow \beta = -10+6 \Rightarrow \beta = -4$

$\oplus \quad -\alpha = 2 \Rightarrow \alpha = -2$

$\boxed{\vec{u} = (-2,-4)}$ respecto de B

b) $z_1 = 2_{60^\circ} \qquad z_2 = -1+i \Rightarrow \overline{z_2} = -1-i \quad , \quad z_3 = 2(\cos 210^\circ + i \operatorname{sen} 210^\circ) = 2_{210^\circ}$

$\overline{z_2} = \sqrt{2}_{225^\circ} \begin{cases} |\overline{z_2}| = \sqrt{(-1)^2+(-1)^2} = \sqrt{1+1} = \sqrt{2} \\ \alpha = \operatorname{arctg} \frac{-1}{-1} = \operatorname{arctg} 1 = \begin{cases} \cancel{45^\circ} \\ 225^\circ \end{cases} \quad A \in \text{III} \\ \qquad A(-1,-1) \in \text{III} \end{cases}$

$$\frac{z_1 \cdot \overline{z_2}}{z_3} = \frac{2_{60^\circ} \cdot \sqrt{2}_{225^\circ}}{2_{210^\circ}} = \left(\frac{\cancel{2}\sqrt{2}}{\cancel{2}}\right)_{60^\circ+225^\circ-210^\circ} = \sqrt{2}_{75^\circ} =$$

$$= \sqrt{2}(\cos 75^\circ + i \operatorname{sen} 75^\circ) = \boxed{\sqrt{2}\cos 75^\circ + \sqrt{2}\operatorname{sen} 75^\circ \cdot i}$$