

Resuelve las siguientes integrales indefinidas de funciones racionales:

$$a) \int \frac{x^2+1}{x^3+x^2-2x} dx$$

$$b) \int \frac{x^2+1}{x^3-6x^2+12x-8} dx$$

$$c) \int \frac{4}{x^3-5x^2+7x-3} dx$$

$$d) \int \frac{x^2-3x+2}{x^2-9} dx$$

$$e) \int \frac{x^2+x-8}{x^3-4x^2} dx$$

$$f) \int \frac{2x^2-x+1}{x^2+8x+16} dx$$

$$g) \int \frac{x^4+3x^3-2x^2+1}{x+5} dx$$

Solución

$$a) \int \frac{x^2+1}{x^3+x^2-2x} dx \Rightarrow x^3+x^2-2x = x(x^2+x-2) = x(x-1)(x+2)$$

$$\frac{x^2+1}{x^3+x^2-2x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2}$$

$$x^2+1 = A(x-1)(x+2) + Bx(x+2) + Cx(x-1)$$

$$\left. \begin{array}{l} x=0: 1 = -2A \\ x=1: 2 = 3B \\ x=-2: 5 = 6C \end{array} \right\} \Rightarrow \begin{array}{l} A = -1/2 \\ B = 2/3 \\ C = 5/6 \end{array}$$

$$\begin{aligned} \int \frac{x^2+1}{x^3+x^2-2x} dx &= -\frac{1}{2} \int \frac{dx}{x} + \frac{2}{3} \int \frac{dx}{x-1} + \frac{5}{6} \int \frac{dx}{x+2} = \\ &= -\frac{1}{2} \ln|x| + \frac{2}{3} \ln|x-1| + \frac{5}{6} \ln|x+2| + C \end{aligned}$$

$$b) \int \frac{x^2+1}{x^3-6x^2+12x-8} dx \Rightarrow x^3-6x^2+12x-8 = (x-2)^3$$

$$\frac{x^2+1}{(x-2)^3} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$$

$$x^2+1 = A(x-2)^2 + B(x-2) + C = Ax^2 - 4Ax + 4A + Bx - 2B + C$$

$$\left. \begin{array}{l} x^2: 1 = A \\ x^1: 0 = -4A + B \\ x^0: 1 = 4A - 2B + C \end{array} \right\} \Rightarrow \begin{array}{l} A = 1 \\ B = 4 \\ C = 5 \end{array}$$

$$\begin{aligned} \int \frac{x^2 + 1}{x^3 - 6x^2 + 12x - 8} dx &= \int \frac{dx}{x-2} + 4 \int \frac{dx}{(x-2)^2} + 5 \int \frac{dx}{(x-2)^3} = \\ &= \ln|x-2| - \frac{4}{x-2} - \frac{5}{2(x-2)^2} + C \end{aligned}$$

$$c) \int \frac{4}{x^3 - 5x^2 + 7x - 3} dx \Rightarrow x^3 - 5x^2 + 7x - 3 = (x-1)^2(x-3)$$

$$\frac{4}{(x-1)^2(x-3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-3}$$

$$4 = A(x-1)(x-3) + B(x-3) + C(x-1)^2$$

$$\left. \begin{array}{l} x=1: 4 = -2B \\ x=3: 4 = 4C \\ x=0: 4 = 3A - 3B + C \end{array} \right\} \Rightarrow \begin{array}{l} A = -1 \\ B = -2 \\ C = 1 \end{array}$$

$$\begin{aligned} \int \frac{4}{x^3 - 5x^2 + 7x - 3} dx &= - \int \frac{dx}{x-1} - 2 \int \frac{dx}{(x-1)^2} + \int \frac{dx}{x-3} = \\ &= -\ln|x-1| + \frac{2}{x-1} + \ln|x-3| + C \end{aligned}$$

$$d) \int \frac{x^2 - 3x + 2}{x^2 - 9} dx \Rightarrow x^2 - 3x + 2 = 1 \cdot (x^2 - 9) + (-3x + 11)$$

$$\int \frac{x^2 - 3x + 2}{x^2 - 9} dx = \int dx + \int \frac{-3x + 11}{x^2 - 9} dx \Rightarrow x^2 - 9 = (x-3)(x+3)$$

$$\frac{-3x + 11}{x^2 - 9} = \frac{A}{x-3} + \frac{B}{x+3} \Rightarrow -3x + 11 = A(x+3) + B(x-3)$$

$$\left. \begin{array}{l} x=3: 2 = 6A \\ x=-3: 20 = -6B \end{array} \right\} \Rightarrow \begin{array}{l} A = 1/3 \\ B = -10/3 \end{array}$$

$$\begin{aligned} \int \frac{x^2 - 3x + 2}{x^2 - 9} dx &= \int dx + \int \frac{-3x + 11}{x^2 - 9} dx = x + \frac{1}{3} \int \frac{dx}{x-3} - \frac{10}{3} \int \frac{dx}{x+3} = \\ &= x + \frac{1}{3} \ln|x-3| - \frac{10}{3} \ln|x+3| + C \end{aligned}$$

$$e) \int \frac{x^2 + x - 8}{x^3 - 4x^2} dx \Rightarrow x^3 - 4x^2 = x^2(x - 4)$$

$$\frac{x^2 + x - 8}{x^3 - 4x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 4}$$

$$x^2 + x - 8 = Ax(x - 4) + B(x - 4) + Cx^2 = Ax^2 - 4Ax + Bx - 4B + Cx^2$$

$$\left. \begin{array}{l} x^2: 1 = A + C \\ x^1: 1 = -4A + B \\ x^0: -8 = -4B \end{array} \right\} \Rightarrow \begin{array}{l} A = 1/4 \\ B = 2 \\ C = 3/4 \end{array}$$

$$\begin{aligned} \int \frac{x^2 + x - 8}{x^3 - 4x^2} dx &= \frac{1}{4} \int \frac{1}{x} dx + 2 \int \frac{1}{x^2} dx + \frac{3}{4} \int \frac{1}{(x - 4)} dx = \\ &= \frac{1}{4} \ln|x| - \frac{2}{x} + \frac{3}{4} \ln|x - 4| + C \end{aligned}$$

$$f) \int \frac{2x^2 - x + 1}{x^2 + 8x + 16} dx \Rightarrow 2x^2 - x + 1 = 2 \cdot (x^2 + 8x + 16) + (-17x - 31)$$

$$\int \frac{2x^2 - x + 1}{x^2 + 8x + 16} dx = 2 \int dx + \int \frac{-17x - 31}{x^2 + 8x + 16} dx \Rightarrow x^2 + 8x + 16 = (x + 4)^2$$

$$\frac{-17x - 31}{(x + 4)^2} = \frac{A}{x + 4} + \frac{B}{(x + 4)^2} \Rightarrow -17x - 31 = A(x + 4) + B$$

$$\left. \begin{array}{l} x^1: -17 = A \\ x^0: -31 = 4A + B \end{array} \right\} \Rightarrow \begin{array}{l} A = -17 \\ B = 37 \end{array}$$

$$\begin{aligned} \int \frac{2x^2 - x + 1}{x^2 + 8x + 16} dx &= 2 \int dx - 17 \int \frac{1}{x + 4} dx + 37 \int \frac{1}{(x + 4)^2} dx = \\ &= 2x - 17 \ln|x + 4| - \frac{37}{x + 4} + C \end{aligned}$$

$$g) \int \frac{x^4 + 3x^3 - 2x^2 + 1}{x + 5} dx$$

$$x^4 + 3x^3 - 2x^2 + 1 = (x^3 - 2x^2 + 8x - 40) \cdot (x + 5) + 201$$

$$\begin{aligned} \int \frac{x^4 + 3x^3 - 2x^2 + 1}{x + 5} dx &= \int (x^3 - 2x^2 + 8x - 40) dx + \int \frac{201}{x + 5} dx = \\ &= \frac{x^4}{4} - 2 \frac{x^3}{3} + 8 \frac{x^2}{2} - 40x + 201 \ln|x + 5| + C \end{aligned}$$