Resuelve las siguientes integrales indefinidas utilizando el método de integración por partes:

$a)\ \int \ln x dx$

$b)\ \int x \ln x dx$

$c)\ \int x e^x dx$

$d)\ \int x^2 e^x dx$

$e)\ \int arcsen x dx$

$f)\ \int e^x sen x dx$

$g)\ \int x^2 sen x dx$

Solución

$$a)\ \int \ln x dx = \begin{pmatrix} u = \ln x & \Rightarrow & du = \frac{dx}{x} \\ dv = dx & \Rightarrow & v = x \end{pmatrix} = x \ln x - \int x \frac{dx}{x} = x \ln x - \int dx =$$

$$= x \ln x - x + C$$

$$b)\ \int x \ln x dx = \begin{pmatrix} u = \ln x & \Rightarrow & du = \frac{dx}{x} \\ dv = x dx & \Rightarrow & v = \frac{x^2}{2} \end{pmatrix} = \frac{x^2}{2} \ln x - \frac{1}{2} \int x^2 \frac{dx}{x} =$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \int x dx = \frac{x^2}{2} \ln x - \frac{1}{2} \frac{x^2}{2} + C = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$c)\ \int x e^x dx = \begin{pmatrix} u = x & \Rightarrow & du = dx \\ dv = e^x dx & \Rightarrow & v = e^x \end{pmatrix} = x e^x - \int e^x dx = x e^x - e^x + C$$

$$d)\ \int x^2 e^x dx = \begin{pmatrix} u = x^2 & \Rightarrow & du = 2x dx \\ dv = e^x dx & \Rightarrow & v = e^x \end{pmatrix} = x^2 e^x - 2 \int x e^x dx =$$

$$= \begin{pmatrix} u = x & \Rightarrow & du = dx \\ dv = e^x dx & \Rightarrow & v = e^x \end{pmatrix} = x^2 e^x - 2 \left[x e^x - \int e^x dx \right] =$$

$$= x^2 e^x - 2 x e^x + 2 \int e^x dx = x^2 e^x - 2 x e^x + 2 e^x + C$$

$$e)\int arcsenx\,dx=\begin{pmatrix} u=arcsenx & \Rightarrow & du=\dfrac{dx}{\sqrt{1-x^2}} \\ dv=dx & \Rightarrow & v=x \end{pmatrix}=xarcsenx-\int\frac{x}{\sqrt{1-x^2}}dx=$$

$$=xarcsenx-\int x\left(1-x^2\right)^{-1/2}dx=\begin{pmatrix} t=1-x^2 \\ dt=-2xdx \end{pmatrix}=$$

$$=xarcsenx+\frac{1}{2}\int t^{-1/2}dt=xarcsenx+\frac{1}{2}\frac{t^{1/2}}{1/2}+C=$$

$$=xarcsenx+\left(1-x^2\right)^{1/2}+C=xarcsenx+\sqrt{1-x^2}+C$$

$$f)\int e^x senxdx=\begin{pmatrix} u=e^x & \Rightarrow & du=e^x dx \\ dv=senxdx & \Rightarrow & v=-\cos x \end{pmatrix}=-e^x\cos x+\int e^x\cos xdx=$$

$$=\begin{pmatrix} u=e^x & \Rightarrow & du=e^x dx \\ dv=\cos xdx & \Rightarrow & v=senx \end{pmatrix}=$$

$$=-e^x\cos x+e^x senx-\int e^x senxdx$$

$$\Rightarrow 2\int e^x senxdx=-e^x\cos x+e^x senx$$

$$\Rightarrow \int e^x senxdx=\frac{e^x\left(senx-\cos x\right)}{2}+C$$

$$g)\int x^2 senxdx=\begin{pmatrix} u=x^2 & \Rightarrow & du=2xdx \\ dv=senxdx & \Rightarrow & v=-\cos x \end{pmatrix}=-x^2\cos x+2\int x\cos xdx=$$

$$=\begin{pmatrix} u=x & \Rightarrow & du=dx \\ dv=\cos xdx & \Rightarrow & v=senx \end{pmatrix}=$$

$$=-x^2\cos x+2xsenx-2\int senxdx=$$

$$=-x^2\cos x+2xsenx+2\cos x+C$$