Ejercicio 1.

Calcula el valor de la expresión:

$$\frac{\log_a\left(\sqrt[3]{a^2\cdot\sqrt{a}}\right)^5-\log_{\sqrt{b}}\left(\frac{b}{\sqrt{b^{-3}}}\right)^{-2}}{\log_b\frac{1}{\sqrt{b^3}}\cdot\log_{\frac{1}{a}}\left(\sqrt{a}\cdot\sqrt[3]{\frac{1}{a}}\right)}=\frac{\log_a\left(\sqrt[3]{\sqrt{a^4\cdot a}}\right)^5-\log_{\sqrt{b}}\left(b^{1+\frac{3}{2}}\right)^{-2}}{\log_b b^{-\frac{3}{2}}\cdot\log_{\frac{1}{a}}\left(a^{\frac{1}{2}}\cdot a^{-\frac{1}{3}}\right)}=\frac{5\cdot\log_a a^{\frac{5}{6}}+2\cdot\log_{\sqrt{b}} b^{\frac{5}{2}}}{-\frac{3}{2}\cdot\log_{\frac{1}{a}} a^{\frac{1}{6}}}=$$

$$=\frac{5\cdot\frac{5}{6}+2\cdot\log_{\sqrt{b}}\left(\sqrt{b}\right)^5}{-\frac{3}{2}\cdot\log_{\frac{1}{a}}\left(\frac{1}{a}\right)^{-\frac{1}{6}}}=\frac{\frac{25}{6}+2\cdot5}{-\frac{3}{2}\cdot\left(-\frac{1}{6}\right)}=\frac{\frac{85}{6}}{\frac{1}{4}}=\frac{170}{3}$$

Ejercicio 2.

Si $\sqrt{2}=1+\cfrac{1}{2+\cfrac{1}{2+x}}$, ¿cuál es el valor de x? Comprueba las soluciones.

$$\sqrt{2}=1+\cfrac{1}{2+\cfrac{1}{2+x}}\Rightarrow\sqrt{2}=1+\cfrac{1}{\cfrac{4+2x+1}{2+x}}\Rightarrow\sqrt{2}=1+\frac{2+x}{5+2x}\Rightarrow\sqrt{2}=\frac{7+3x}{5+2x}\Rightarrow2=\frac{(7+3x)^2}{(5+2x)^2}\Rightarrow$$

$$\Rightarrow2=\frac{49+42x+9x^2}{25+20x+4x^2}\Rightarrow50+40x+8x^2=49+42x+9x^2\Rightarrow x^2+2x-1=0\Rightarrow\begin{cases}x=-1+\sqrt{2}\\ \cancel{x=-1-\sqrt{2}}\end{cases}$$

$$si\quad x=-1+\sqrt{2}\Rightarrow1+\cfrac{1}{2+\cfrac{1}{2+\left(-1+\sqrt{2}\right)}}=1+\cfrac{1}{2+\cfrac{1}{1+\sqrt{2}}}=1+\cfrac{1}{\cfrac{3+2\sqrt{2}}{1+\sqrt{2}}}=1+\frac{1+\sqrt{2}}{3+2\sqrt{2}}=$$

$$=\frac{4+3\sqrt{2}}{3+2\sqrt{2}}=\frac{\left(4+3\sqrt{2}\right)\left(3-2\sqrt{2}\right)}{\left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right)}=\frac{12+9\sqrt{2}-8\sqrt{2}-12}{9-8}=\sqrt{2}$$

$$si\quad x=-1-\sqrt{2}\Rightarrow1+\cfrac{1}{2+\cfrac{1}{2+\left(-1-\sqrt{2}\right)}}=1+\cfrac{1}{2+\cfrac{1}{1-\sqrt{2}}}=1+\cfrac{1}{\cfrac{3-2\sqrt{2}}{1-\sqrt{2}}}=1+\frac{1-\sqrt{2}}{3-2\sqrt{2}}=$$

$$=\frac{4-3\sqrt{2}}{3-2\sqrt{2}}=\frac{\left(4-3\sqrt{2}\right)\left(3+2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)}=\frac{12-9\sqrt{2}+8\sqrt{2}-12}{9-8}=-\sqrt{2}\neq\sqrt{2}$$

Ejercicio 3.

a) Escribe en forma de intervalo el conjunto de números que verifica la desigualdad: $\left|\dfrac{3+2x}{4}\right| \geq 2^{-2}$

$$\left|\frac{3+2x}{4}\right| \geq 2^{-2} \Rightarrow \left|\frac{3+2x}{4}\right| \geq \frac{1}{4} \Rightarrow \begin{cases} \dfrac{3+2x}{4} \geq \dfrac{1}{4} \Rightarrow 3+2x \geq 1 \Rightarrow x \geq -1 \ , \ x \in [-1,+\infty) \\ \\ \dfrac{3+2x}{4} \leq -\dfrac{1}{4} \Rightarrow 3+2x \leq -1 \Rightarrow x \leq -2 \ , \ x \in (-\infty,-2] \end{cases}$$

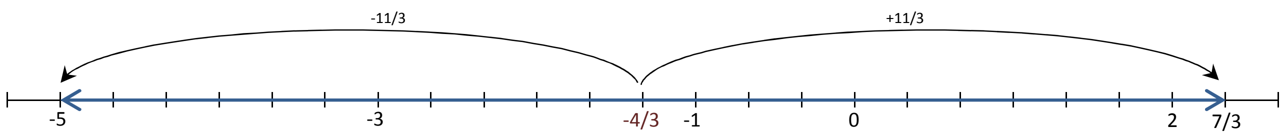
Los números $x \in (-\infty,-2] \cup [-1,+\infty)$ *verifican la condición* $\left|\dfrac{3+2x}{4}\right| \geq 2^{-2}$

b) Escribe, usando valor absoluto, la condición que deben cumplir los puntos del intervalo $\left(-5 \ , \ \dfrac{7}{3}\right)$

$$\left(-5 \ , \frac{7}{3}\right) \Rightarrow \textit{distancia entre los extremos}: \ \frac{7}{3}-(-5)=\frac{22}{3} \ ; \ \textit{la mitad} \ \frac{11}{3} = \textit{radio del entorno}$$

$$\textit{El centro del entorno será el punto medio entre } -5 \textit{ y } \frac{7}{3}: \quad \frac{-5+\frac{7}{3}}{2} = \frac{-4}{3}$$

$$d\left(x \ , -\frac{4}{3}\right) < \frac{11}{3} \Rightarrow \left|x-\left(-\frac{4}{3}\right)\right| < \frac{11}{3} \Rightarrow \left|x+\frac{4}{3}\right| < \frac{11}{3}$$



Ejercicio 4.

Calcula, razonadamente, los siguientes límites de sucesiones:

$$a) \ \lim_{n\to\infty}\left(\frac{1+2+2^2+2^3+\cdots+2^n}{1+3+3^2+3^3+\cdots+3^n}\right) = \lim_{n\to\infty}\left(\frac{\dfrac{2^{n+1}-1}{2-1}}{\dfrac{3^{n+1}-1}{3-1}}\right) = \lim_{n\to\infty}\left(\frac{2^{n+1}-1}{\dfrac{3^{n+1}-1}{2}}\right) = \lim_{n\to\infty}\left(\frac{2\cdot\left(2^{n+1}-1\right)}{3^{n+1}-1}\right) =$$

$$= \left(\frac{\infty}{\infty} \ , \textit{dividimos por } 3^{n+1}\right) = \lim_{n\to\infty}\left(\frac{\dfrac{2\cdot\left(2^{n+1}-1\right)}{3^{n+1}}}{\dfrac{3^{n+1}-1}{3^{n+1}}}\right) = \lim_{n\to\infty}\left(\frac{2\cdot\left(\left(\dfrac{2}{3}\right)^{n+1}-\dfrac{1}{3^{n+1}}\right)}{1-\dfrac{1}{3^{n+1}}}\right) = \frac{2\cdot(0-0)}{1-0} = 0$$

$b)\ \lim_{n\to\infty}\left(\frac{5n+2}{5n}\right)^{2n} = \left(indeterminación\ \ 1^{\infty}\right) = \lim_{n\to\infty}\left(\frac{5n}{5n}+\frac{2}{5n}\right)^{2n} = \lim_{n\to\infty}\left(1+\frac{2}{5n}\right)^{2n} = \lim_{n\to\infty}\left(1+\frac{1}{\frac{5n}{2}}\right)^{2n} =$

$= \lim_{n\to\infty}\left(1+\frac{1}{\frac{5n}{2}}\right)^{\frac{5n}{2}\cdot\frac{2}{5n}\cdot 2n} = e^{\lim_{n\to\infty}\frac{4n}{5n}} = e^{\lim_{n\to\infty}\frac{4}{5}} = e^{\frac{4}{5}}$

Ejercicio 5.

Resuelve la inecuación $2x\cdot\left(3x^2+3x+1\right)\le\left(2x+1\right)^2-3x^2$

$6x^3+6x^2+2x\le 4x^2+4x+1-3x^2 \ \Rightarrow\ 6x^3+5x^2-2x-1\le 0 \ \Rightarrow\ (x+1)\left(6x^2-x-1\right)\le 0$

$6x^2-x-1=0 \ \Rightarrow\ x=\frac{1}{2}\ ,\ x=-\frac{1}{3} \ \Rightarrow\ 6x^2-x-1=6\left(x-\frac{1}{2}\right)\left(x+\frac{1}{3}\right)$

Entonces la inecuación queda $6(x+1)\left(x-\frac{1}{2}\right)\left(x+\frac{1}{3}\right)\le 0$

Ahora analizamos el signo del producto:

	$(-\infty\ ,-1)$	-1	$\left(-1\ ,\ -\frac{1}{3}\right)$	$-\frac{1}{3}$	$\left(-\frac{1}{3}\ ,\ \frac{1}{2}\right)$	$\frac{1}{2}$	$\left(\frac{1}{2}\ ,+\infty\right)$
$x+1$	$-$	0	$+$	$+$	$+$	$+$	$+$
$x+\frac{1}{3}$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x-\frac{1}{2}$	$-$	$-$	$-$	$-$	$-$	0	$+$
$(x+1)\left(x+\frac{1}{3}\right)\left(x-\frac{1}{2}\right)$	$-$	0	$+$	0	$-$	0	$+$

Solución: $x\in(-\infty,-1]\cup\left[-\frac{1}{3},\frac{1}{2}\right]$

Ejercicio 6.

Resuelve las siguientes ecuaciones:

$a)\ 4^{x+1}+3=7\cdot 2^{x}$

$$\left(2^{2}\right)^{x+1}-7\cdot 2^{x}+3=0 \ \Rightarrow\ 2^{2x+2}-7\cdot 2^{x}+3=0 \ \Rightarrow 4\cdot 2^{2x}-7\cdot 2^{x}+3=0 \ \Rightarrow \left(cambio\ \ 2^{x}=z\right) \ \Rightarrow$$

$$\Rightarrow\ 4\cdot z^{2}-7\cdot z+3=0 \ \Rightarrow \begin{cases} x=1 \ \Rightarrow\ 2^{x}=1 \ \Rightarrow \boxed{x=0} \\ x=\dfrac{3}{4} \ \Rightarrow\ 2^{x}=\dfrac{3}{4} \ \Rightarrow\ \log\ 2^{x}=\log\dfrac{3}{4} \ \Rightarrow x\log 2=\log 3-\log 4 \ \Rightarrow \end{cases}$$

$$\Rightarrow \boxed{x=\frac{\log 3-\log 4}{\log 2}}$$

$b)\ \log(2x+5)-\dfrac{1}{2}\log(6x+1)-\log 3=0$

$$\log(2x+5)=\frac{1}{2}\log(6x+1)+\log 3 \ \Rightarrow\ \log(2x+5)=\log(6x+1)^{1/2}+\log 3 \ \Rightarrow$$

$$\Rightarrow\ \log(2x+5)=\log\left(\sqrt{6x+1}\right)\cdot 3 \ \Rightarrow\ 2x+5=3\cdot\sqrt{6x+1} \ \Rightarrow\ (2x+5)^{2}=9(6x+1) \ \Rightarrow$$

$$\Rightarrow\ 4x^{2}+20x+25=54x+9 \ \Rightarrow\ 4x^{2}-34x+16=0 \ \Rightarrow\ 2x^{2}-17x+8=0 \ \Rightarrow$$

$$\Rightarrow \boxed{\begin{cases} x=8 \\ x=\dfrac{1}{2} \end{cases}} \quad \textit{ambas soluciones son válidas.}$$