MATEMÁTICAS 1º BACHILLERATO

Ejercicios de exámenes de complejos

1. Calcular: $\sqrt{-2+2\sqrt{3}i} =$

2. Calcular: $a)\left(1-\frac{\sqrt{3}-i}{2}\right)^{12}$ $\qquad b)\left(\frac{1+\sqrt{3}i}{1-i}\right)^{\frac{1}{4}}$

3. Expresar en forma binómica el resultado de: $a)\left(\frac{3\sqrt{3}}{2}+\frac{3}{2}i\right)^{5}$ $\qquad b)(-1+i)^{10}$

4. Calcular: $\sqrt[5]{\frac{-8-8\sqrt{3}i}{\left(-2\sqrt{3}\right)^2}}+2i$

5. Resolver:

$$\left\{\begin{array}{l} z^4 + 27z = 0 \\ z^5 + 125\, z^2 = 0 \end{array}\right.$$

6. Resolver, expresando el resultado de forma binómica y trigonométrica: $\sqrt[3]{\frac{5+i}{2+3i}}$

7. Resolver: $a)\sqrt[3]{\frac{1-i}{1+i}}$ $\qquad b)\sqrt[3]{\frac{-1+i}{1+\sqrt{3}i}}$ $\qquad c)\sqrt[4]{81(\cos 120^{\circ}+i sen 120^{\circ})}$

8. Siendo $z=1-3i$ y $w=\sqrt{8}_{45^{\circ}}$, calcular: *z + w, z·w, 2z,* $\frac{z^3}{w}$

9. Calcula: $\frac{i^{35}-i^5}{2\cdot i^{10}}$

10. Opera: $\frac{(1-2i)(-2+i)}{3i\cdot(1-i)}$

11. Resuelve la siguiente ecuación y expresa el resultado en forma binómica: $\frac{3-zi+2i}{2}=z+i$

12. Siendo $z=4\sqrt{3}-4i$, se pide:

a) z^5

b) Las raíces cúbicas de z.

13. Hallar el valor de x para que el cociente $\frac{1+3xi}{3-4i}$:

a) Sea un número imaginario puro.

b) Sea un número real.

c) Tenga módulo 1.

14. Encontrar a y b para que $\frac{a-6i}{3+bi}=\sqrt{2}_{315^{\circ}}$

① $\sqrt{-2+2\sqrt{3}\,i} = \sqrt{4_{120^\circ}} = \sqrt{4}\left\{\begin{array}{l}\frac{120+k360}{2} \\ k=0,1\end{array}\right. =$

$= 2_{60^\circ}\ ,\ 2_{240^\circ}$

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② a) $\left(1-\frac{\sqrt{3}-i}{2}\right)^{12} = \left(\frac{2-\sqrt{3}+i}{2}\right)^{12} = \left(\frac{2-\sqrt{3}}{2}+\frac{1}{2}i\right)^{12} =$

$= \left(\sqrt{2-\sqrt{3}}\ \underline{|75^\circ}\right)^{12} = \left(\sqrt{2-\sqrt{3}}\right)^{12}\ \underline{|75\cdot 12} = (2-\sqrt{3})^6{}_{900^\circ} =$

$= (2-\sqrt{3})^6{}_{180^\circ}$

b) $\left(\frac{1+\sqrt{3}\,i}{1-i}\right)^{1/4} = \sqrt[4]{\frac{2_{60^\circ}}{\sqrt{2}_{315}}} = \sqrt[4]{\frac{2}{\sqrt{2}}}_{-255^\circ} = \sqrt[4]{\sqrt{2}_{105^\circ}} =$

$= \sqrt[8]{2}\left\{\begin{array}{l}\frac{105+k360}{4} \\ k=0,1,2,3\end{array}\right.$ $\quad \sqrt[8]{2}_{26'25^\circ}\ ,\ \sqrt[8]{2}_{116'25^\circ}$

$\sqrt[8]{2}_{206'25^\circ}\ ,\ \sqrt[8]{2}_{296'25^\circ}$

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③ a) $\left(\frac{3\sqrt{3}}{2}+\frac{3}{2}i\right)^5 = \left(3_{30^\circ}\right)^5 = 3^5_{150^\circ} =$

$243_{150^\circ} = 243(\cos 150^\circ + i\,\text{sen}\,150^\circ) =$

$= 243\left(\frac{-\sqrt{3}}{2}+\frac{1}{2}i\right) = \frac{-243\sqrt{3}}{2}+\frac{243}{2}i$

b) $(-1+i)^{10} = \left(\sqrt{2}_{135^\circ}\right)^{10} = (\sqrt{2})^{10}_{1350^\circ} =$

$= 2^5_{270} = 32_{270} = 32(\cos 270 + \text{sen}\,270\, i) = 0-32i$

$= -32i$

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④ $x = \sqrt[5]{\frac{-8-8\sqrt{3}i}{(-2\sqrt{3})^2}} + 2i$

Calculemos a parte la raíz

$y = \sqrt[5]{\frac{16_{240^\circ}}{12}} = \sqrt[5]{\frac{4}{3}_{240^\circ}} = \sqrt[5]{\frac{4}{3}} \left\{ \begin{array}{l} \frac{240+k360}{5} \\ k=0,1,2,3,4 \end{array} \right.$

$x_1 = y_1 + 2i = \sqrt[5]{\frac{4}{3}}_{48^\circ} + 2i = \sqrt[5]{\frac{4}{3}}(\cos 48 + i\,\text{sen}\,48) + 2i$

$x_2 = \sqrt[5]{\frac{4}{3}}_{120^\circ} + 2i = \ldots$

$x_3 = \sqrt[5]{\frac{4}{3}}_{192^\circ} + 2i = \ldots$

$x_4 = \sqrt[5]{\frac{4}{3}}_{264^\circ} + 2i = \ldots$

$x_5 = \sqrt[5]{\frac{4}{3}}_{336^\circ} + 2i = \ldots$

Calculadora y decimales

¡¡Quién se inventa estos problemas para un examen!!! ④

(5) $\begin{cases} z^4 + 27z = 0 \\ z^5 + 125z^2 = 0 \end{cases}$

• $z^4 + 27z = 0 \qquad z(z^3+27) = 0 \qquad \begin{array}{l} z = 0 \\ z^3 + 27 = 0 \end{array}$

$$z = \sqrt[3]{-27} = \sqrt[3]{27_{180^\circ}} = \sqrt[3]{27} \begin{cases} \dfrac{180 + k360}{3} \\ k = 0, 1, 2 \end{cases} =$$

$$= \begin{cases} 3_{60^\circ} \\ 3_{180^\circ} \\ 3_{300^\circ} \end{cases}$$

• $z^5 + 125z^2 = 0 \qquad z^2 = 0$

$z^2(z^3+125) = 0 \qquad z^3 = -125 \qquad z = \sqrt[3]{-125} = \sqrt[3]{125_{180^\circ}}$

$$= 5 \begin{cases} \dfrac{180 + k360}{3} \\ k = 0, 1, 2 \end{cases} = \begin{cases} 5_{60^\circ} \\ 5_{180^\circ} \\ 5_{300^\circ} \end{cases}$$

Solución del sistema: $z = 0$

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(6) • $\sqrt[3]{\dfrac{5+i}{2+3i}} = \sqrt[3]{\sqrt{26}} = \sqrt[3]{\sqrt{2}_{315^\circ}} = \sqrt[6]{2} \begin{cases} \dfrac{315^\circ + k360^\circ}{3} \\ k = 0, 1, 2 \end{cases} =$

$= \sqrt[6]{2}_{105^\circ}, \ \sqrt[6]{2}_{225^\circ}, \ \sqrt[6]{2}_{345^\circ}$

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(7) a) $\sqrt[3]{\frac{1-i}{1+i}} = \sqrt[3]{-i} = \sqrt[3]{1_{270^\circ}} = \sqrt[3]{1} \begin{cases} \frac{270+k360}{3} \\ k=0,1,2 \end{cases} =$

$= 1_{90^\circ}, 1_{210^\circ}, 1_{330^\circ}$

b) $\sqrt[3]{\frac{-1+i}{1+\sqrt{3}i}} = \sqrt[3]{\frac{\sqrt{2}_{135^\circ}}{2_{60^\circ}}} = \sqrt[3]{\frac{\sqrt{2}}{2}_{75^\circ}} = \sqrt[3]{\frac{\sqrt{2}}{2}} \begin{cases} \frac{75+k360}{3} \\ k=0,1,2 \end{cases} =$

$= \sqrt[3]{\frac{\sqrt{2}}{2}} \begin{cases} 25^\circ \\ 145^\circ \\ 265^\circ \end{cases}$

c) $\sqrt[4]{81(\cos 120^\circ + i \operatorname{sen} 120^\circ)} = \sqrt[4]{81_{120^\circ}} = \sqrt[4]{81} \begin{cases} \frac{120+k360}{4} \\ k=0,1,2,3 \end{cases} =$

$= 3_{30^\circ}, 3_{120^\circ}, 3_{210^\circ}, 3_{300^\circ}$

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(8) $z = 1-3i$, $w = \sqrt{8}_{45^\circ} = \sqrt{8}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 2+2i$

$z+w = (1-3i)+(2+2i) = 3-i$

$z \cdot w = 8-4i$

$2 \cdot z = 2-6i$

$\frac{z^3}{w} = \frac{(1-3i)^3}{2+2i} = \frac{-26+18i}{2+2i} = -2+11i$

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(9) $\frac{i^{35} - i^5}{2 \cdot i^{10}} = \frac{(-i)-i}{2(-1)} = \frac{-2i}{-2} = i$

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(10) $\frac{(1-2i)(-2+i)}{3i(1-i)} = \frac{5i}{3+3i} = \frac{5}{6} + \frac{5}{6}i$

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(11) $\frac{3-zi+2i}{2} = z+i \rightarrow 3-zi+2i = 2z+2i \rightarrow$

$2z+zi = 3+2i-2i \rightarrow (2+i)z = 3 \rightarrow z = \frac{3}{2+i} \rightarrow$

$z = \frac{3(2-i)}{(2+i)(2-i)} = \frac{6-3i}{5} = \frac{6}{5} - \frac{3}{5}i$

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(12) $z = 4\sqrt{3} - 4i = 8_{330^\circ}$

a) $z^5 = (8_{330^\circ})^5 = 8^5 \underline{_{330 \cdot 5}} = 32768 \underline{_{1650^\circ}} =$

$= 32768_{210^\circ}$

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b) $\sqrt[3]{4\sqrt{3}-4i} = \sqrt[3]{8_{330^\circ}} = \sqrt[3]{8} \begin{cases} \frac{330^\circ + k360}{2} \\ k=0,1,2 \end{cases} =$

$= 2_{165^\circ}, \; 2_{285^\circ}, \; 2_{405^\circ}$

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(13) $\frac{1+3x i}{3-4i} \cdot \frac{(3+4i)}{(3+4i)} = \frac{3+9xi+4i+12xi^2}{9+16} = \frac{(3-12x)}{25} + \frac{(9x+4)i}{25}$

a) $\frac{3-12x}{25} = 0 \rightarrow x = \frac{1}{4}$; b) $\frac{9x+4}{25} = 0 \quad x = \frac{-4}{9}$

c) $\sqrt{\left(\frac{3-12x}{25}\right)^2 + \left(\frac{9x+4}{25}\right)^2} = 1 \rightarrow x = \pm\sqrt{\frac{8}{3}}$

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(14) Igual que el 10 de la otra hoja.