Maths 4th ESO

EXAM 3_2 (Statistics - Algebra)

Name:_____

1. The histogram below shows the heights (in cm) distribution of 30 people. (1 point)

a) How many people have heights between 159.5 and 169.5 cm?

b) How many people have heights less than 159.5 cm?

c) How many people have heights more than 169.5 cm?

d) What percentage of people have heights between 149.5 and 179.5 cm?

2. The number of books read in a given months by each of the students in a group of 4ºESO are listed below: (2 points)

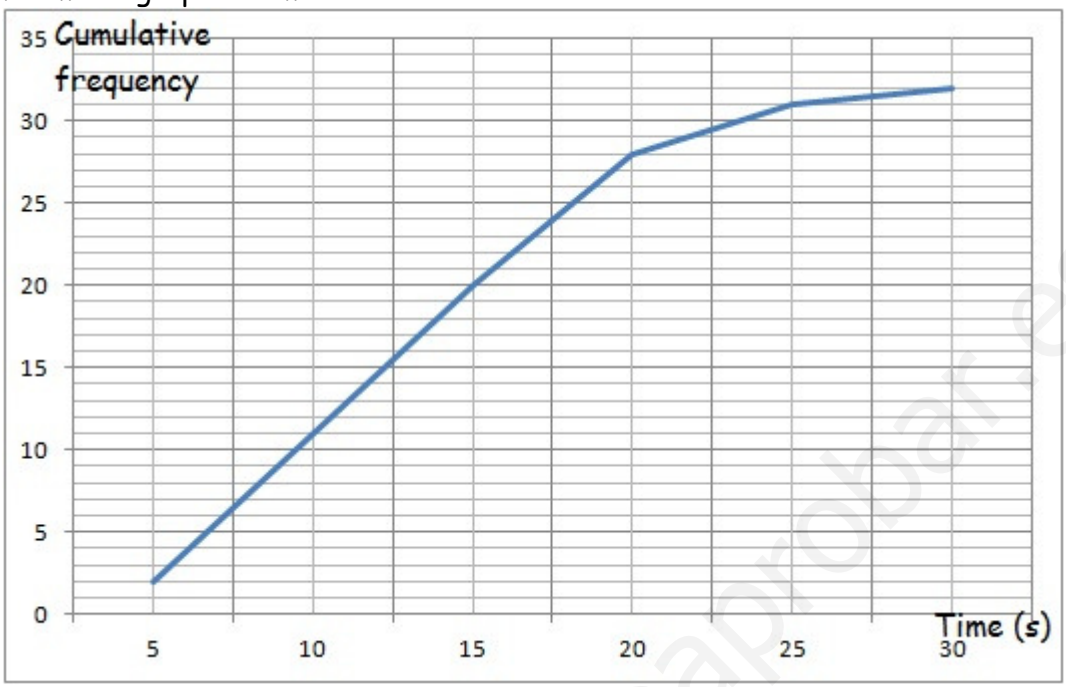
Number	Frequency
0	5
1	10
2	13
3	7
4	4
5	1

Calculate:

a) Range, Mode and Median
b) Mean and Standard deviation
c) Percentiles 15 and 90

3. The following graph shows the cumulative frequency curve for time taken for students to solve a puzzle. (2 points)

From the graph estimate:



a) The median time.
b) The time at the lower quartile and at the upper quartile.
c) After how many seconds were 70% of the puzzles made?
d) Using this data draw a box and whisker plot.

4. Solve the following equations: (3 points)

a) $6x^3 - 15x^2 + 12x - 3 = 0$

b) $1 - \sqrt{x-3} = x - 8$

c) $\frac{3x+1}{x-2} - \frac{x^2}{x^2-4} = \frac{x}{x+2} - 4$

5. Solve the simultaneous equations: (2 points)

a) $\left.\begin{array}{l}(x^2+1)y^2 = 5 \\ 4x - y = 0\end{array}\right\}$

b) $\left.\begin{array}{l}y^2 - 2y + 1 = x \\ \sqrt{x} + y = 5\end{array}\right\}$

SOLUTION

1. The histogram below shows the heights (in cm) distribution of 30 people.

a) How many people have heights between 159.5 and 169.5 cm? 7

b) How many people have heights less than 159.5 cm? 15

c) How many people have heights more than 169.5 cm? 8

d) What percentage of people have heights between 149.5 and 179.5 cm?

21 of 30, so the percentage is 70%

2. The number of books read in a given months by each of the students in a group of 4ºESO are listed below:

x_i	f_i	F_i	$x_i f_i$	$x_i^2 f_i$
0	5	5	0	0
1	10	15	10	10
2	13	28	26	52
3	7	35	21	63
4	4	39	16	64
5	1	40	5	25
			78	214

Calculate:

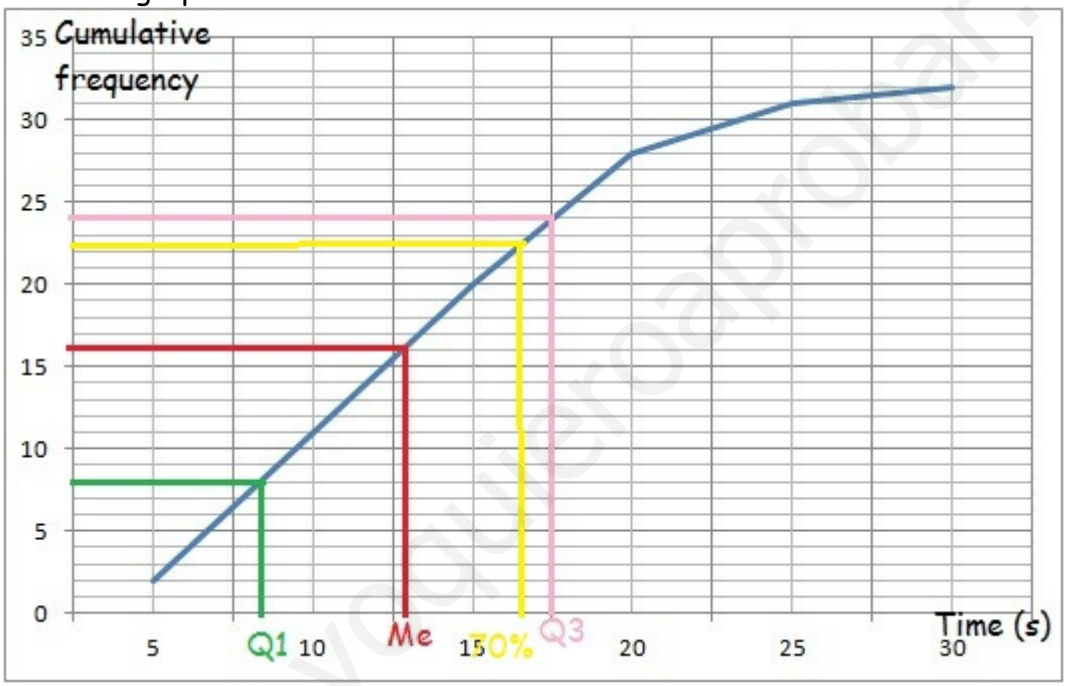
a) Range, Mode and Median: r=5-0=5 range 5 ; Mode 2 books
Median: 40/2=20, so the median is 2 books

b) Mean and Standard deviation

$$\bar{x} = \frac{78}{40} = 1.95 \text{ books} \; ; \; \sigma = \sqrt{\frac{214}{40} - 1.95^2} = 1.24 \text{ books}$$

c) Percentiles 15 and 90 $P_{15} \rightarrow \frac{15N}{100} = 6 \rightarrow 1$ book; $P_{90} \rightarrow \frac{90N}{100} = 36 \rightarrow 4$ books

3. The following graph shows the cumulative frequency curve for time taken for students to solve a puzzle.
From the graph estimate:

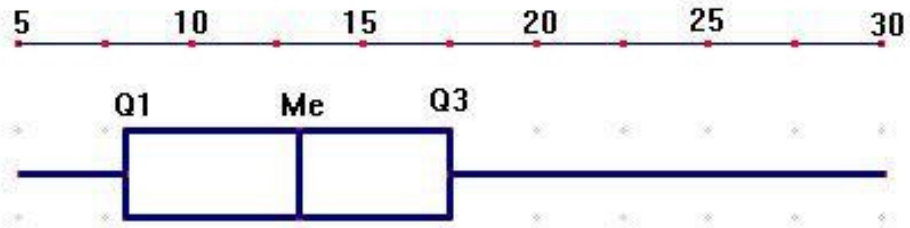


a) The median time. N=32, half is 16, so the median time is 13 seconds

b) The time at the lower quartile and at the upper quartile.
$Q_1 \rightarrow 32/4 = 8 \rightarrow Q_1 = 8.5\,s \; ; \; Q_3 \rightarrow 3 \cdot 32/4 = 24 \rightarrow Q_3 = 17.5\,s$

c) After how many seconds were 70% of the puzzles made?
$70\,\% \text{ of } 32 = 22.4 \rightarrow$ about 16.5 seconds

d) Using this data draw a box and whisker plot.



4. Solve the following equations:

a) $6x^3-15x^2+12x-3=0$, $P(1)=6-15+12-3=0$

	6	-15	+12	-3
1		+6	-9	+3
	6	-9	+3	0

$6x^2-9x+3=0\rightarrow 2x^2-3x+1=0$

$x=\dfrac{3\pm\sqrt{1}}{4}=\begin{cases}1\\1/2\end{cases}$

Solution: $x=1, x=\dfrac{1}{2}$

b) $1-\sqrt{x-3}=x-8\rightarrow -\sqrt{x-3}=x-8-1\rightarrow \left(-\sqrt{x-3}\right)^2=(x-9)^2$

$x-3=x^2-18x+81\rightarrow x^2-19x+84=0\rightarrow x=\dfrac{19\pm\sqrt{25}}{2}=\begin{cases}12\\7\end{cases}$

Checking: $x=12\rightarrow 1-\sqrt{12-3}=12-8\rightarrow 1-3=4$ NO

$x=7\rightarrow 1-\sqrt{7-3}=7-8\rightarrow 1-2=-1$ YES

Solution x = 7

c) $\dfrac{3x+1}{x-2}-\dfrac{x^2}{x^2-4}=\dfrac{x}{x+2}-4\rightarrow \text{LCD}=(x+2)(x-2)$

$\dfrac{(3x+1)(x+2)}{x^2-4}-\dfrac{x^2}{x^2-4}=\dfrac{x(x-2)}{x^2-4}-\dfrac{4\left(x^2-4\right)}{x^2-4}\rightarrow 3x^2+7x+2-x^2=x^2-2x-4x^2+16$

$3x^2+7x+2-x^2-x^2+2x+4x^2-16=0\rightarrow 5x^2+9x-14=0\rightarrow x=\dfrac{-9\pm\sqrt{361}}{10}=\begin{cases}1\\-\dfrac{14}{5}\end{cases}$

5. Solve the simultaneous equations:

a) $\left.\begin{matrix}\left(x^2+1\right)y^2=5\\4x-y=0\end{matrix}\right\}\rightarrow y=4x\Rightarrow\left(x^2+1\right)(4x)^2=5\rightarrow 16x^4+16x^2-5=0$

$z=x^2\rightarrow 16z^2+16z-5=0\rightarrow z=\dfrac{-16\pm\sqrt{576}}{32}=\dfrac{-16\pm 24}{32}=\begin{cases}\dfrac{1}{4}\\-\dfrac{5}{2}\end{cases}$

$z=x^2\rightarrow\begin{cases}x=\sqrt{\dfrac{1}{4}}=\pm\dfrac{1}{2}\\x=\sqrt{-\dfrac{5}{2}}\ \text{NO}\end{cases}\rightarrow\begin{cases}x=\dfrac{1}{2}\rightarrow y=4\cdot\dfrac{1}{2}=2\\x=-\dfrac{1}{2}\rightarrow y=-4\cdot\dfrac{1}{2}=-2\end{cases}\rightarrow\text{solution}\begin{cases}\left(\dfrac{1}{2}.2\right)\\\left(-\dfrac{1}{2},-2\right)\end{cases}$

b)

$\left.\begin{matrix}y^2-2y+1=x\\\sqrt{x}+y=5\end{matrix}\right\}\sqrt{y^2-2y+1}+y=5\rightarrow\sqrt{(y-1)^2}+y=5\rightarrow y-1+y=5\rightarrow 2y=6\rightarrow y=3$

$x=y^2-2y+1=9-6+1=4\rightarrow$ solution $(4,3)$