

Examen de Matemáticas 1º de Bachillerato
Enero 2013

Problema 1 Dados los números complejos $z_1 = 2 - i$ y $z_2 = -3 + 7i$. Se pide calcular:

- a) $z_1 + z_2$ y $z_1 - z_2$
- b) $z_1 \cdot z_2$
- c) $\frac{z_1}{z_2}$

Solución:

- a) $z_1 + z_2 = -1 + 6i$ y $z_1 - z_2 = 5 - 8i$
- b) $z_1 \cdot z_2 = 1 + 17i$
- c) $\frac{z_1}{z_2} = -\frac{13}{58} - \frac{11}{58}i$

Problema 2 Resolver la siguiente ecuación de segundo grado:

$$z^2 - 2z + 5 = 0$$

Solución:

$$z^2 - 2z + 5 = 0 \implies z = \begin{cases} 1 + 2i \\ 1 - 2i \end{cases}$$

Problema 3 Si $z = 4 - 5i$ calcular z^{10} .

Solución:

$$z = 4 - 5i = \sqrt{41}_{308^\circ 39' 35''} = \sqrt{41}(\cos 308^\circ 39' 35'' + i \sin 308^\circ 39' 35'')$$
$$z^{10} = (4 - 5i)^{10} = 41^5_{206^\circ 35' 53''} = 41^5(\cos 206^\circ 35' 53'' + i \sin 206^\circ 35' 53'')$$

Problema 4 Resolver la ecuación $z^3 - i = 5$.

Solución:

$$z^3 = 5 + i \implies z = \sqrt[3]{5 + i}$$
$$5 + i = \sqrt{26}_{11^\circ 18' 36''} = \sqrt{26}(\cos 11^\circ 18' 36'' + i \sin 11^\circ 18' 36'')$$
$$z = \sqrt[3]{5 + i} = \begin{cases} \sqrt[6]{26}_{3^\circ 46' 12''} = \sqrt[6]{26}(\cos 3^\circ 46' 12'' + i \sin 3^\circ 46' 12'') \\ \sqrt[6]{26}_{123^\circ 46' 12''} = \sqrt[6]{26}(\cos 123^\circ 46' 12'' + i \sin 123^\circ 46' 12'') \\ \sqrt[6]{26}_{243^\circ 46' 12''} = \sqrt[6]{26}(\cos 243^\circ 46' 12'' + i \sin 243^\circ 46' 12'') \end{cases}$$

Problema 5 Calcular las raíces de $\sqrt[3]{-3+i}$

Solución:

$$z = -3 + i = \sqrt{10}_{161^{\circ}33'54''} = \sqrt{10}(\cos 161^{\circ}33'54'' + i \sin 161^{\circ}33'54'')$$

$$\sqrt[3]{z} = \begin{cases} \sqrt[6]{10}_{53^{\circ}51'18''} = \sqrt[6]{10}(\cos 53^{\circ}51'18'' + i \sin 53^{\circ}51'18'') \\ \sqrt[6]{10}_{173^{\circ}51'18''} = \sqrt[6]{10}(\cos 173^{\circ}51'18'' + i \sin 173^{\circ}51'18'') \\ \sqrt[6]{10}_{293^{\circ}51'18''} = \sqrt[6]{10}(\cos 293^{\circ}51'18'' + i \sin 293^{\circ}51'18'') \end{cases}$$