

Resuelve las siguientes ecuaciones y sistemas:

$$1) \quad \operatorname{sen} x = \frac{1}{2}$$

$$2) \quad \cos x = -1$$

$$3) \quad \operatorname{tg} x = \frac{\sqrt{3}}{3}$$

$$4) \quad 2\cos^2 x = \cos 2x$$

$$5) \quad \cos 2x = \cos x - 1$$

$$6) \quad 1 + \operatorname{sen} 2x = (\operatorname{sen} x + \cos x)$$

$$7) \quad \cos 2x = 1 + 4 \operatorname{sen} x$$

$$8) \quad \operatorname{tg}^2 x - 3 \operatorname{tg} x + 2 = 0$$

$$9) \quad \operatorname{sen} 2x - \operatorname{sen} x = 0$$

$$10) \quad 2\cos x + \cos x - 1 = 0$$

$$11) \quad \sec x + \operatorname{tg} x = 0$$

$$12) \quad \operatorname{sen} x + \cos x = 1$$

$$13) \quad 2 \cos x = 3 \operatorname{tg} x$$

$$14) \quad \operatorname{sen} x + \sqrt{3} \cos x = 2$$

$$15) \quad 4 \operatorname{sen} \frac{x}{2} + 2 \cos x = 3$$

SOLUCIONES

$$1) \quad \sin x = \frac{1}{2} \quad \begin{cases} x_1 = 30^\circ + 360^\circ \cdot k \\ x_2 = 150^\circ + 360^\circ \cdot k \end{cases}; k \in \mathbb{Z}$$

$$2) \quad \cos x = -1 \Rightarrow x = 180^\circ + 360^\circ \cdot k; k \in \mathbb{Z}$$

$$3) \quad \operatorname{tg} x = \frac{\sqrt{3}}{3} \Rightarrow \begin{cases} x_1 = 30^\circ + 360^\circ \cdot k \\ x_2 = 210^\circ + 360^\circ \cdot k \end{cases} \Rightarrow x = 30^\circ + 180^\circ \cdot k; k \in \mathbb{Z}$$

$$4) \quad 2\cos^2 x - 3\operatorname{tg} x + 2 = 0 \\ 2\cos^2 x = \cos 2x \Rightarrow 2\cos^2 x = \cos^2 x - \sin^2 x \quad \cos^2 x + \sin^2 x = 0 !!! \text{ Sinsolución.}$$

$$5) \quad \cos 2x = \cos x - 1 \Rightarrow \cos^2 x - \sin^2 x = \cos x - 1 \Rightarrow \cos^2 x - (1 - \cos^2 x) = \cos x - 1 \Rightarrow 2\cos^2 x - \cos x = 0$$

$$2\cos^2 x - \cos x = 0 \underset{t=\cos x}{\Rightarrow} 2t - t = 0 \Rightarrow t \cdot (2t - 1) = 0$$

$$\begin{array}{l} t = 0 \rightarrow \cos x = 0 \rightarrow x = 90^\circ \text{ y } 270^\circ \\ t = \frac{1}{2} \rightarrow \cos x = - \quad x = 60^\circ \text{ y } 300^\circ \end{array}$$

$$\text{Soluciones (con vueltas): } \begin{cases} x_1 = 90^\circ + 180^\circ \cdot k \\ x_2 = 60^\circ + 360^\circ \cdot k; k \in \mathbb{Z} \\ x_3 = 300^\circ + 360^\circ \cdot k \end{cases}$$

$$6) \quad 1 + \sin 2x = (\sin x + \cos x)$$

$$1 + 2\sin x \cos x = \sin x + \cos^2 x + 2\sin x \cos x \Rightarrow 1 = \sin x + \cos^2 x$$

Es una identidad, no es una ecuación.

$$7) \quad \cos 2x = 1 + 4 \sin x$$

$$\cos^2 x - \sin^2 x = 1 + 4 \sin x \Rightarrow 1 - \sin^2 x - \sin^2 x = 1 + 4 \sin x \Rightarrow 2\sin^2 x + 4 \sin x = 0 \Rightarrow$$

$$\Rightarrow 2\sin x \cdot (\sin x + 2) = 0$$

$$\begin{array}{l} \sin x = 0 \rightarrow x = 0^\circ \text{ y } 180^\circ \\ \sin x = -2 \end{array}$$

Solución: $x = 0^\circ + 180^\circ \cdot k; k \in \mathbb{Z}$

8)

$$\operatorname{tg}^2 x - 3\operatorname{tg} x + 2 = 0 \underset{t=\operatorname{tg} x}{\Rightarrow} t^2 - 3t + 2 = 0 \Rightarrow t = \frac{3 \pm \sqrt{(-3)^2 - 4 \cdot 2}}{2} = \frac{3 \pm 1}{2}$$

$$\begin{array}{l} 2 \Rightarrow \operatorname{tg} x = 2 \Rightarrow x = 63^\circ 26' 6'' \\ 1 \Rightarrow \operatorname{tg} x = 1 \Rightarrow x = 45^\circ \end{array}$$

La tangente es positiva en los cuadrantes I y III.

$$\text{Soluciones (con vueltas): } \begin{cases} x_1 = 45^\circ + 180^\circ \cdot k \\ x_2 = 63^\circ 26' 6'' + 180^\circ \cdot k \end{cases}; k \in \mathbb{Z}$$

$$9) \quad \sin 2x - \sin x = 0$$

$$2 \sin x \cdot \cos x - \sin x = 0 \Rightarrow \sin x \cdot (2 \cos x - 1) = 0$$

$$\begin{array}{l} \sin x = 0 \rightarrow x = 0^\circ \text{ y } 180^\circ \\ 2 \cos x - 1 = 0 \rightarrow \cos x = \frac{1}{2} \rightarrow x = 60^\circ \text{ y } 300^\circ \end{array}$$

$$\sin x + \cos x = 1$$

$$\text{Soluciones: } \begin{cases} x_1 = 0^\circ + 180^\circ \cdot k \\ x_2 = 60^\circ + 360^\circ \cdot k ; k \in \mathbb{Z} \\ x_3 = 300^\circ + 360^\circ \cdot k \end{cases}$$

$$10) \quad 2\cos^2 x + \cos x - 1 = 0$$

$$2\cos^2 x + \cos x - 1 = 0 \underset{t=\cos x}{\Rightarrow} 2t^2 + t - 1 = 0 \Rightarrow t = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} = \frac{-1 \pm \sqrt{1+8}}{4} =$$

$$\begin{array}{l} \frac{1}{2} \Rightarrow \cos x = - \quad x = 60^\circ \text{ y } 300^\circ \\ \frac{-1+3}{4} \Rightarrow -1 \Rightarrow \cos x = -1 \Rightarrow x = 135^\circ \text{ y } 225^\circ \end{array}$$

$$\text{Soluciones: } \begin{cases} x_1 = 60^\circ + 360^\circ \cdot k \\ x_2 = 300^\circ + 360^\circ \cdot k ; k \in \mathbb{Z} \\ x_3 = 135^\circ + 360^\circ \cdot k \\ x_4 = 225^\circ + 360^\circ \cdot k \end{cases}$$

$$11) \quad \sec x + \tan x = 0$$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x} = 0 \Rightarrow \frac{1 + \sin x}{\cos x} = 0 \Rightarrow \sin x = -1 \rightarrow x = 270^\circ + 360^\circ \cdot k; k \in \mathbb{Z}$$

$$12)$$

$$(\cos x + \sin x)^2 = 1^2 \rightarrow \underbrace{\cos^2 x + \sin^2 x}_1 + 2 \sin x \cos x = 1 \rightarrow 2 \sin x \cos x = 0$$

$$\begin{array}{l} \sin x = 0 \rightarrow x = 0^\circ + 360^\circ \cdot k \\ \cos x = 0 \rightarrow x = 90^\circ + 360^\circ \cdot k \end{array}$$

$$\begin{array}{l} \sin x = 0 \rightarrow x = 180^\circ + 360^\circ \cdot k \\ \cos x = 0 \rightarrow x = 270^\circ + 360^\circ \cdot k \end{array} \Rightarrow x = 0^\circ + 90^\circ \cdot k, k \in \mathbb{Z}$$

$$13) 2 \cos x = 3 \operatorname{tg} x$$

$$2 \cos x = \frac{3 \operatorname{sen} x}{\cos x} \Rightarrow 2 \cos^2 x = 3 \operatorname{sen} x \Rightarrow 2(1 - \operatorname{sen}^2 x) = 3 \operatorname{sen} x \Rightarrow 2 \operatorname{sen}^2 x + 3 \operatorname{sen} x - 2 = 0$$

$$2t^2 + 3t - 2 = 0 \Rightarrow t = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot (-2)}}{2 \cdot 2} = \frac{-3 \pm \sqrt{9 + 16}}{4} = \frac{-3 \pm 5}{4}$$

$\operatorname{sen} x = \frac{1}{2} \rightarrow x = 30^\circ \text{ y } 150^\circ$
 ~~$\operatorname{sen} x = -2$~~

$$\text{Soluciones: } \begin{cases} x_1 = 30^\circ + 360^\circ \cdot k \\ x_2 = 150^\circ + 360^\circ \cdot k \end{cases}; \quad k \in \mathbb{Z}$$

$$14) \operatorname{sen} x + \sqrt{3} \cos x = 2$$

$$\operatorname{sen} x + \sqrt{3} \cos x = 2 \quad \text{divido entre 2} \quad \frac{1}{2} \operatorname{sen} x + \frac{\sqrt{3}}{2} \cos x = 1 \Rightarrow \operatorname{sen}(x + 60^\circ) = 1$$

$$\operatorname{sen}(x + 60^\circ) = 1 \Rightarrow x + 60^\circ = 90^\circ + 360^\circ \cdot k \Rightarrow x = 30^\circ + 360^\circ \cdot k$$

$$\text{Soluciones: } x = 30^\circ + 360^\circ \cdot k; \quad k \in \mathbb{Z}$$

15)

$$4 \operatorname{sen} \frac{x}{2} + 2 \left(\cos 2 \frac{x}{2} \right) = 3 \Rightarrow 4 \operatorname{sen} \frac{x}{2} + 2 \left(\cos^2 \frac{x}{2} - \operatorname{sen}^2 \frac{x}{2} \right) = 3 \Rightarrow 4 \operatorname{sen} \frac{x}{2} + 2 \left(1 - \operatorname{sen}^2 \frac{x}{2} - \operatorname{sen}^2 \frac{x}{2} \right) = 3$$

$$4 \operatorname{sen} \frac{x}{2} + 2 - 4 \operatorname{sen}^2 \frac{x}{2} = 3 \Rightarrow 4 \operatorname{sen}^2 \frac{x}{2} - 4 \operatorname{sen} \frac{x}{2} + 1 = 0 \Rightarrow 4t^2 - 4t + 1 = 0 \Rightarrow$$

$$\Rightarrow t = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 4 \cdot 1}}{2 \cdot 4} = \frac{4 \pm \sqrt{16 - 16}}{8} = \frac{4}{8} = \frac{1}{2}$$

$$\text{Entonces: } \operatorname{sen} \frac{x}{2} = \frac{1}{2} \Rightarrow \begin{cases} \frac{x_1}{2} = 30^\circ + 360^\circ \cdot k \\ \frac{x_2}{2} = 150^\circ + 360^\circ \cdot k \end{cases} \Rightarrow \begin{cases} x_1 = 60^\circ + 720^\circ \cdot k \\ x_2 = 300^\circ + 720^\circ \cdot k \end{cases}; \quad k \in \mathbb{Z}$$