

- CUESTIÓN B.1 Calcular la inversa de la matriz $A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & -1 & 2 \\ 3 & 2 & -3 \end{pmatrix}$

$$(A | I) \xrightarrow{\text{transformaciones elementales}} (I | A^{-1})$$

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & -1 & 2 \\ 3 & 2 & -3 \end{pmatrix}$$

Utilizamos el método de Gauss:

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & -1 & 2 & 0 & 1 & 0 \\ 3 & 2 & -3 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{F_2 - 2F_1 \rightarrow F_2 \\ F_3 - 3F_1 \rightarrow F_3}} \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -7 & 0 & -2 & 1 & 0 \\ 0 & -7 & -6 & -3 & 0 & 1 \end{array} \right) \rightarrow$$

$$\xrightarrow{F_3 - F_2 \rightarrow F_3} \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -7 & 0 & -2 & 1 & 0 \\ 0 & 0 & -6 & -1 & -1 & 1 \end{array} \right) \rightarrow$$

$$\xrightarrow{\substack{7F_1 \rightarrow F_1 \\ 3F_2 \rightarrow F_2}} \left(\begin{array}{ccc|ccc} 7 & 21 & 7 & 7 & 0 & 0 \\ 0 & -21 & 0 & -6 & 3 & 0 \\ 0 & 0 & -6 & -1 & -1 & 1 \end{array} \right) \rightarrow$$

$$\xrightarrow{F_1 + F_2 \rightarrow F_1} \left(\begin{array}{ccc|ccc} 7 & 0 & 7 & 1 & 3 & 0 \\ 0 & -21 & 0 & -6 & 3 & 0 \\ 0 & 0 & -6 & -1 & -1 & 1 \end{array} \right) \rightarrow$$

$$\xrightarrow{\substack{6F_1 \rightarrow F_1 \\ 7F_2 \rightarrow F_2}} \left(\begin{array}{ccc|ccc} 42 & 0 & 42 & 6 & 18 & 0 \\ 0 & -21 & 0 & -6 & 3 & 0 \\ 0 & 0 & -42 & -7 & -7 & 7 \end{array} \right) \rightarrow$$

$$\xrightarrow{F_1 + F_3 \rightarrow F_1} \left(\begin{array}{ccc|ccc} 42 & 0 & 0 & -1 & 11 & 7 \\ 0 & -21 & 0 & -6 & 3 & 0 \\ 0 & 0 & -42 & -7 & -7 & 7 \end{array} \right) \rightarrow$$

$$\xrightarrow{\substack{\frac{1}{42} F_1 \\ -\frac{1}{21} F_2 \\ -\frac{1}{42} F_3}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{42} & \frac{11}{42} & \frac{7}{42} \\ 0 & 1 & 0 & \frac{6}{21} & -\frac{3}{21} & 0 \\ 0 & 0 & 1 & \frac{7}{42} & \frac{7}{42} & -\frac{7}{42} \end{array} \right)$$

OPERACIONES ADIC

$$\begin{array}{r} -2(1 \ 3 \ 1 \ 1 \ 0 \ 0) \rightarrow -2 \ -6 \ -2 \ -2 \ 0 \ 0 \\ + \quad 2 \ -1 \ 2 \ 0 \ 1 \ 0 \\ \hline 0 \ -7 \ 0 \ -2 \ 1 \ 0 \end{array}$$

$$\begin{array}{r} -3(1 \ 3 \ 1 \ 1 \ 0 \ 0) \rightarrow -3 \ -9 \ -3 \ -3 \ 0 \ 0 \\ + \quad 3 \ 2 \ -3 \ 0 \ 0 \ 1 \\ \hline 0 \ -7 \ -6 \ -3 \ 0 \ 1 \end{array}$$

$$\begin{array}{r} -(0 \ -7 \ 0 \ -2 \ 1 \ 0) \rightarrow 0 \ 7 \ 0 \ 2 \ -1 \ 0 \\ + \quad 0 \ 7 \ -6 \ -3 \ 0 \ 1 \\ \hline 0 \ 0 \ -6 \ -1 \ -1 \ 1 \end{array}$$

$$\begin{array}{r} 7 \ 21 \ 7 \ 7 \ 0 \ 0 \\ + \quad 0 \ -21 \ 0 \ -6 \ 3 \ 0 \\ \hline 7 \ 0 \ 7 \ 1 \ 3 \ 0 \end{array}$$

$$\begin{array}{r} 42 \ 0 \ 42 \ 6 \ 18 \ 0 \\ 0 \ 0 \ -42 \ -7 \ -7 \ 7 \\ \hline 42 \ 0 \ 0 \ -1 \ 11 \ 7 \end{array}$$

después $A^{-1} = \begin{pmatrix} -\frac{1}{42} & \frac{11}{42} & \frac{1}{6} \\ \frac{2}{7} & -\frac{1}{7} & 0 \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \end{pmatrix}$

Ahora la resolvemos utilizando determinantes con: $A^{-1} = \frac{1}{|A|} [\text{Adj}(A)]^t$

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & -1 & 2 \\ 3 & 2 & -3 \end{pmatrix} \quad \text{Para calcular } A^{-1}, \text{ utilizamos } A^{-1} = \frac{1}{|A|} [\text{Adj}(A)]^t$$

Calculamos el determinante asociado a la matriz A:

$$|A| = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 2 \\ 3 & 2 & -3 \end{vmatrix} = 1(-1)(-3) + 3 \cdot 2 \cdot 3 + 2 \cdot 2 \cdot 1 - 1(-1)3 - 3 \cdot 2(-3) - 2 \cdot 2 \cdot 1 = 3 + 18 + 4 + 3 + 18 - 4 = 42$$

Calculamos ahora la matriz adjunta:

$$A_{11} = \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = 3 - 4 = -1$$

$$A_{12} = -\begin{vmatrix} 2 & 2 \\ 3 & -3 \end{vmatrix} = -(-6 - 6) = 12$$

$$A_{13} = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 4 - (-3) = 7$$

$$A_{21} = -\begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix} = -(-9 - 2) = 11$$

$$A_{22} = \begin{vmatrix} 1 & 1 \\ 3 & -3 \end{vmatrix} = -3 - 3 = -6$$

$$A_{23} = -\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} = -(2 - 9) = 7$$

$$A_{31} = \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix} = 6 - (-1) = 7$$

$$A_{32} = -\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = -(2 - 2) = 0$$

$$A_{33} = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1 - 4 = -5$$

$$\text{Luego } \text{Adj}(A) = \begin{pmatrix} -1 & 12 & 7 \\ 11 & -6 & 7 \\ 7 & 0 & -5 \end{pmatrix} \quad \text{De ahí: } [\text{Adj}(A)]^t = \begin{pmatrix} -1 & 11 & 7 \\ 12 & -6 & 0 \\ 7 & 7 & -5 \end{pmatrix}$$

$$A^{-1} = \frac{1}{42} \begin{pmatrix} -1 & 11 & 7 \\ 12 & -6 & 0 \\ 7 & 7 & -5 \end{pmatrix} = \begin{pmatrix} -\frac{1}{42} & \frac{11}{42} & \frac{7}{42} \\ \frac{12}{42} & -\frac{6}{42} & 0 \\ \frac{7}{42} & \frac{7}{42} & -\frac{5}{42} \end{pmatrix} = \begin{pmatrix} -\frac{1}{42} & \frac{11}{42} & \frac{1}{6} \\ \frac{2}{7} & -\frac{1}{7} & 0 \\ \frac{1}{6} & \frac{1}{6} & -\frac{5}{42} \end{pmatrix}$$

LA TERCERA RESPUESTA ES LA MEJOR

ALINQUE LAS OTRAS DOS SON TAMBIÉN CORRECTAS