OPCIÓN A

1.A.- Sea **f(x)** una función derivable en **(0 , 1)** y continua en **[0 , 1]**, tal que **f(1) = 0** y $\int_0^1 2xf'(x)\,dx = 1$. Utilizar la fórmula de integración por partes para calcular $\int_0^1 f(x)\,dx$

$$I = \int 2xf'(x)\,dx = 2xf(x) - \int f(x)\,2\,dx = 2xf(x) - 2\int f(x)\,dx \Rightarrow \int_0^1 2xf'(x)\,dx = [2xf(x)]_0^1 - 2\int_0^1 f'(x)\,dx$$

$$\begin{cases} 2x = u \Rightarrow 2\,dx = du \\ f'(x)\,dx = dv \Rightarrow v = \int f'(x)\,dx = f(x) \end{cases}$$

$$1 = [2xf(x)]_0^1 - 2\int_0^1 f'(x)\,dx \Rightarrow 2\int_0^1 f'(x)\,dx = [2xf(x)]_0^1 - 1 \Rightarrow \int_0^1 f'(x)\,dx = \frac{[2xf(x)]_0^1 - 1}{2} \Rightarrow$$

$$\int_0^1 f'(x)\,dx = \frac{1}{2} \cdot \{[2.1.f(1) - 2.0.f(0)] - 1\} \Rightarrow \int_0^1 f'(x)\,dx = \frac{1}{2} \cdot \{[2.0 - 0] - 1\} = -\frac{1}{2}$$

1

2.A.- Calcular un polinomio de tercer grado $p(x) = ax^3 + bx^2 + cx + d$, sabiendo que verifica:

I) tiene un máximo relativo en **x = 1**

II) tiene un punto de inflexión en el punto **(0 , 1)**

III) se verifica $\displaystyle\int_0^1 p(x)\,dx = \dfrac{5}{4}$

$$\begin{cases} f'(x) = 3ax^2 + 2bx + c \\ f''(x) = 6ax + 2b \end{cases} \Rightarrow \begin{cases} f(0) = 1 \Rightarrow a.0^3 + b.0^2 + c.0 + d = 1 \Rightarrow d = 1 \\ f'(1) = 0 \Rightarrow 3a.1^2 + 2b.1 + c = 0 \Rightarrow 3a + 2b + c = 0 \Rightarrow 3a + c = 0 \\ f''(1) = 0 \Rightarrow 6a.0 + 2b = 0 \Rightarrow 2b = 0 \Rightarrow b = 0 \end{cases}$$

$$\int_0^1 \left(ax^3 + cx + 1\right)dx = \frac{5}{4} \Rightarrow \frac{a}{4}\cdot\left[x^4\right]_0^1 + \frac{c}{2}\cdot\left[x^2\right]_0^1 + \left[x\right]_0^1 = \frac{5}{4} \Rightarrow \frac{a}{4}\cdot\left(1^4 - 0^4\right) + \frac{c}{2}\cdot\left(1^2 - 0^2\right) + \left(1 - 0\right) = \frac{5}{4} \Rightarrow$$

$$\frac{a}{4} + \frac{c}{2} + 1 = \frac{5}{4} \Rightarrow a + 2c + 4 = 5 \Rightarrow \begin{cases} 3a + c = 0 \\ a + 2c = 1 \end{cases} \Rightarrow \begin{cases} -6a - 2c = 0 \\ a + 2c = 1 \end{cases} \Rightarrow -5a = 1 \Rightarrow a = -\frac{1}{5} \Rightarrow$$

$$-\frac{1}{5} + 2c = 1 \Rightarrow 2c = 1 + \frac{1}{5} \Rightarrow 2c = \frac{6}{5} \Rightarrow c = \frac{3}{5} \Rightarrow p(x) = -\frac{1}{5}\cdot x^3 + \frac{3}{5}x + 1$$

2

3.A.- Dado el sistema de ecuaciones: $\begin{cases} (m-1)x + y + z = 3 \\ mx + (m-1)y + 3z = 2m-1 \\ x + 2y + (m-2)z = 4 \end{cases}$

a) Discutirlo según los distintos valores de **m**

b) Resolverla cuando sea compatible indeterminado

$a)$

$$|A| = \begin{vmatrix} m-1 & 1 & 1 \\ m & m-1 & 3 \\ 1 & 2 & m-2 \end{vmatrix} = (m-1)^2(m-2) + 3 + 2m - (m-1) - 6(m-1) - m(m-2) \Rightarrow$$

$$|A| = (m^2 - 2m + 1)(m-2) + 3 + 2m - 7m + 7 - m^2 + 2m = m^3 - 2m^2 - 2m^2 + 4m + m - 2 - m^2 - 3m + 10$$

$$|A| = m^3 - 5m^2 + 2m + 8 \Rightarrow m^3 - 5m^2 + 2m + 8 = (m+1)(m^2 - 6m + 8) \Rightarrow m^2 - 6m + 8 = 0 \Rightarrow$$

$$\begin{array}{c|ccc} & 1 & -5 & 2 & 8 \\ 1 & & -1 & 6 & -8 \\ \hline & 1 & -6 & 8 & \underline{|0} \end{array} \qquad \Delta = 36 - 32 = 4 > 0 \Rightarrow m = \frac{6 \pm \sqrt{4}}{2} \Rightarrow \begin{cases} m = 4 \\ m = 2 \end{cases} \Rightarrow$$

$$m^3 - 5m^2 + 2m + 8 = (m+1)(m-4)(m-2)$$

$$Si\ |A| = 0 \Rightarrow m^3 - 5m^2 + 2m + 8 = 0 \Rightarrow (m+1)(m-4)(m-2) = 0 \Rightarrow \begin{cases} m = -1 \\ m = 2 \\ m = 4 \end{cases} \Rightarrow$$

$$\forall m \in \Re - \{-1, 2, 4\} \Rightarrow |A| \neq 0 \Rightarrow rang(A) = 3 \Rightarrow Sistema\ Compatible\ Deter\min ado$$

$Pr\ imer\ método. - Roche - Frobenius$

$$|A/B| = \begin{vmatrix} C_1 & C_2 & B \end{vmatrix} = \begin{vmatrix} m-1 & 1 & 3 \\ m & m-1 & 2m-1 \\ 1 & 2 & 4 \end{vmatrix}$$

$$|A/B| = 4(m-1)^2 + (2m-1) + 6m - 3(m-1) - 2(2m-1)(m-1) - 4m$$

$$|A/B| = 4(m^2 - 2m + 1) + 2m - 1 + 2m - 3m + 3 - 4m^2 + 4m + 2m - 2$$

$$|A/B| = 4m^2 - 8m + 4 + 7m - 4m^2 = -m + 4 \Rightarrow Si\ |A/B| = 0 \Rightarrow -m + 4 = 0 \Rightarrow m = 4$$

$$\forall m \in \Re - \{4\} \Rightarrow |A/B| \neq 0 \Rightarrow rang(A/B) = 3 \Rightarrow m = 4 \Rightarrow |A/B| = \begin{vmatrix} 3 & 1 \\ 4 & 3 \end{vmatrix} = 5 \neq 0 \Rightarrow rang(A/B) = 2$$

Continúa el problema 3.A.-

$a)Continuación$

$\Pr imer\ método. - Roche - Frobenius(Continua)$

$$|A/B| = |C_1 \quad C_3 \quad B| = \begin{vmatrix} m-1 & 1 & 3 \\ m & 3 & 2m-1 \\ 1 & m-2 & 4 \end{vmatrix}$$

$$|A/B| = 12(m-1) + 2m - 1 + 3m(m-2) - 9 - 4m - (m-1)(m-2)(2m-1)$$

$$|A/B| = 12m - 12 - 2m - 10 + 3m^2 - 6m - (m^2 - 3m + 2)(2m-1) =$$

$$|A/B| = 3m^2 + 4m - 22 - 2m^3 + m^2 + 6m^2 - 3m - 4m + 2 = -2m^3 + 10m^2 - 3m - 20$$

$$\Rightarrow Si\ |A/B| = 0 \Rightarrow -2m^3 + 10m^2 - 3m - 20 = 0 \Rightarrow 2m^3 - 10m^2 + 3m + 20 = 0 \Rightarrow (2m^2 - 2m - 5)(m-4) = 0$$

$$\begin{array}{r|rrrr} & 2 & -10 & 3 & 20 \\ 4 & & 8 & -8 & -20 \\ \hline & 2 & -2 & -5 & \underline{0} \end{array} \qquad \Delta = 4 - 40 = 44 > 0 \Rightarrow m = \frac{2 \pm \sqrt{44}}{4} \Rightarrow \begin{cases} m = \dfrac{1 + \sqrt{11}}{2} \\ m = \dfrac{1 - \sqrt{11}}{2} \end{cases} \Rightarrow$$

$$\forall m \in \Re - \left\{ \frac{1 - \sqrt{11}}{2}\ ,\ 4\ ,\ \frac{1 + \sqrt{11}}{2} \right\} \Rightarrow |A/B| \neq 0 \Rightarrow rang(A/B) = 3 \Rightarrow$$

$$m = \begin{cases} 4 \\ \dfrac{1 - \sqrt{11}}{2} \\ \dfrac{1 + \sqrt{11}}{2} \end{cases} \Rightarrow |A/B| = \begin{vmatrix} 1 & 3 \\ 3 & 7 \end{vmatrix} = -2 \neq 0 \Rightarrow rang(A/B) = 2(como\ ejemplo)$$

$$|A/B| = |C_2 \quad C_3 \quad B| = \begin{vmatrix} 1 & 1 & 3 \\ m-1 & 3 & 2m-1 \\ 2 & m-2 & 4 \end{vmatrix}$$

$$|A/B| = 12 + 2(2m-1) + 3(m-1)(m-2) - 18 - 4(m-1) - (2m-1)(m-2)$$

$$|A/B| = 12 + 4m - 2 + 3(m^2 - 3m + 2) - 18 - 4m + 4 - (2m^2 - 4m - m + 2)$$

$$|A/B| = -4 + 3m^2 - 9m + 6 - 2m^2 + 4m + m - 2 = m^2 - 4m = m(m-4) \Rightarrow$$

$$|A/B| = 0 \Rightarrow \begin{cases} m = 0 \\ m = 4 \end{cases} \Rightarrow \forall m \in \Re - \{0\ ,\ 4\} \Rightarrow |A/B| \neq 0 \Rightarrow rang(A/B) = 3 \Rightarrow$$

$$m = \begin{cases} 0 \\ 4 \end{cases} \Rightarrow \begin{vmatrix} 1 & 3 \\ 3 & 7 \end{vmatrix} = -2 \neq 0 \Rightarrow rang(A/B) = 2$$

$Cuando\ m = 4 \Rightarrow rang(A) = rang(A/B) = 2 < Número\ de\ incognitas \Rightarrow Sist. Compat. In\det er\min ado$

$Cuando\ m = -1 \Rightarrow rang(A) = 2 \neq rang(A/B) = 3 \Rightarrow Sistema. Incompatible$

$Cuando\ m = 2 \Rightarrow rang(A) = 2 \neq rang(A/B) = 3 \Rightarrow Sistema. Incompatible$

4

Continúa el problema 3.A.-

Otra forma de realizar el apartado a) es utilizando la reducción por Gauss

$a)Continuación$

$Segundo\ método.-Gauss \Rightarrow$

$Con\ m = -1$

$$A = \begin{pmatrix} -2 & 1 & 1 & | & 3 \\ -1 & -2 & 3 & | & -3 \\ 1 & 2 & -3 & | & 4 \end{pmatrix} \equiv \begin{pmatrix} -2 & 1 & 1 & | & 3 \\ 2 & 4 & -6 & | & 6 \\ 2 & 4 & -6 & | & 8 \end{pmatrix} \equiv \begin{pmatrix} -2 & 1 & 1 & | & 3 \\ 0 & 5 & -5 & | & 9 \\ 0 & 5 & -5 & | & 11 \end{pmatrix} \equiv \begin{pmatrix} -2 & 1 & 1 & | & 3 \\ 0 & 5 & -5 & | & 9 \\ 0 & 0 & 0 & | & 2 \end{pmatrix} \Rightarrow$$

$Sistema\ Incompatible$

$Con\ m = 2$

$$A = \begin{pmatrix} 1 & 1 & 1 & | & 3 \\ 2 & 1 & 3 & | & 3 \\ 1 & 2 & 0 & | & 4 \end{pmatrix} \equiv \begin{pmatrix} 1 & 1 & 1 & | & 3 \\ 0 & -1 & 1 & | & -3 \\ 0 & 1 & -1 & | & 1 \end{pmatrix} \equiv \begin{pmatrix} 1 & 1 & 1 & | & 3 \\ 0 & -1 & 1 & | & -3 \\ 0 & 0 & 0 & | & -2 \end{pmatrix} \Rightarrow Sistema\ Incompatible$$

$Con\ m = 4$

$$A = \begin{pmatrix} 3 & 1 & 1 & | & 3 \\ 4 & 3 & 3 & | & 7 \\ 1 & 2 & 2 & | & 4 \end{pmatrix} \equiv \begin{pmatrix} 3 & 1 & 1 & | & 3 \\ 12 & 9 & 9 & | & 21 \\ 3 & 6 & 6 & | & 12 \end{pmatrix} \equiv \begin{pmatrix} 3 & 1 & 1 & | & 3 \\ 0 & 5 & 5 & | & 9 \\ 0 & 5 & 5 & | & 9 \end{pmatrix} \equiv \begin{pmatrix} 3 & 1 & 1 & | & 3 \\ 0 & 5 & 5 & | & 9 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \Rightarrow Sist.\ Compat.\ In\det er\min ado$$

$b)$

$Por\ Gauss$

$$5y + 5z = 9 \Rightarrow 5y = 9 - 5z \Rightarrow y = \frac{9-5z}{5} \Rightarrow 3x + \frac{9-5z}{5} + z = 3 \Rightarrow 15x + 9 - 5z + 5z = 15 \Rightarrow$$

$$15x = 6 \Rightarrow x = \frac{2}{5} \Rightarrow Solución\left(\frac{2}{5}, \frac{9}{5} - \lambda, \lambda\right)$$

$Por\ Rouche$

$$\begin{cases} 3x + y + z = 3 \\ 4x + 3y + 3z = 7 \end{cases} \Rightarrow \begin{cases} -9x - 3y - 3z = -9 \\ 4x + 3y + 3z = 7 \end{cases} \Rightarrow -5x = -2 \Rightarrow x = \frac{2}{5} \Rightarrow 3 \cdot \frac{2}{5} + y + z = 3 \Rightarrow$$

$$6 + 5y + 5z = 15 \Rightarrow 5y = 9 - 5z \Rightarrow y = \frac{9-5z}{5} = \frac{9}{5} - z \Rightarrow Solución\left(\frac{2}{5}, \frac{9}{5} - \lambda, \lambda\right)$$

Evidentemente hay que elegir uno de los dos procedimientos, el segundo valdrá para comprobar

4.A.- Dado el punto **P(1 , 3 , -1)**, se pide:

a) Escribir la ecuación que deben de verificar los puntos **X(x , y , z)** cuya distancia a **P** sea igual a **3**

b) Calcular los puntos de la recta $\begin{cases} x = 3\lambda \\ y = 1 + \lambda \\ z = 1 - 4\lambda \end{cases}$ cuya distancia a **P** es igual a **3**

$a)$

$$3 = \pm\sqrt{(x-1)^2 + (y-3)^2 + (z+1)^2} \Rightarrow 9 = (x-1)^2 + (y-3)^2 + (z+1)^2 \Rightarrow$$

$$9 = x^2 - 2x + 1 + y^2 - 6y + 9 + z^2 + 2z + 1 \Rightarrow x^2 + y^2 + z^2 - 2x - 6y + 2z + 2 = 0 \Rightarrow Una\ esfera$$

$b)$

$$3 = \pm\sqrt{(3\lambda-1)^2 + (1+\lambda-3)^2 + (1-4\lambda+1)^2} \Rightarrow 9 = (3\lambda-1)^2 + (\lambda-2)^2 + (2-4\lambda)^2 \Rightarrow$$

$$9 = 9\lambda^2 - 6\lambda + 1 + \lambda^2 - 4\lambda + 4 + 4 - 16\lambda + 16\lambda^2 \Rightarrow 26\lambda^2 - 26\lambda = 0 \Rightarrow \lambda^2 - \lambda = 0 \Rightarrow (\lambda-1)\lambda = 0 \Rightarrow$$

$$\begin{cases} \lambda = 0 \Rightarrow P_1 \begin{cases} x = 3.0 = 0 \\ y = 1 + 0 = 1 \Rightarrow P_1(0 , 1 , 0) \\ z = 1 - 4.0 = 0 \end{cases} \\ \\ \lambda - 1 = 0 \Rightarrow \lambda = 1 \Rightarrow P_2 \begin{cases} x = 3.1 = 3 \\ y = 1 + 1 = 2 \Rightarrow P_2(3 , 2 , -3) \\ z = 1 - 4.1 = -3 \end{cases} \end{cases}$$

OPCIÓN B

1.B.- a)Resolver el sistema: $\begin{cases} x + 2y + 3z = 1 \\ 2x + y - z = 2 \end{cases}$

b) Hallar dos constantes a y b de manera que al añadir al sistema anterior una tercera ecuación : **5x + y + az = b** el sistema resultante sea compatible indeterminado

$a)$

$$\begin{cases} x + 2y + 3z = 1 \\ 6x + 3y - 3z = 6 \end{cases} \Rightarrow 7x + 5y = 7 \Rightarrow 7x = 7 - 5y \Rightarrow x = \frac{7 - 5y}{7} = 1 - \frac{5}{7}y \Rightarrow 1 - \frac{5}{7}y + 2y + 3z = 1 \Rightarrow$$

$$3z = \frac{5}{7}y - 2y \Rightarrow 3z = -\frac{9}{7}y \Rightarrow z = -\frac{3}{7}y \Rightarrow Solución\left(1 - \frac{5}{7}\lambda \, , \, \lambda \, , \, -\frac{3}{7}\lambda\right)$$

$b)$

$$\begin{pmatrix} 1 & 2 & 3 & | & 1 \\ 2 & 1 & -1 & | & 2 \\ 5 & 1 & \alpha & | & \beta \end{pmatrix} \equiv \begin{pmatrix} 1 & 2 & 3 & | & 1 \\ 0 & -3 & -7 & | & 0 \\ 0 & -9 & \alpha - 15 & | & \beta - 5 \end{pmatrix} \equiv \begin{pmatrix} 1 & 2 & 3 & | & 1 \\ 0 & -3 & -7 & | & 0 \\ 0 & 0 & \alpha - 15 + 21 & | & \beta - 5 \end{pmatrix} \Rightarrow \begin{cases} \alpha + 6 \Rightarrow \alpha = -6 \\ \beta - 5 = 0 \Rightarrow \beta = 5 \end{cases}$$

2.B.-Hallar una matriz **X** tal que **A^{-1}XA = B**, siendo $A = \begin{pmatrix} 3 & 1 \\ -2 & -1 \end{pmatrix}$ y $B = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$

$$AA^{-1}XA = AB \Rightarrow IXA = AB \Rightarrow XAA^{-1} = ABA^{-1} \Rightarrow XI = ABA^{-1} \Rightarrow X = ABA^{-1}$$

$$\exists A^{-1} \Rightarrow |A| \neq 0 \Rightarrow |A| = \begin{vmatrix} 3 & 1 \\ -2 & -1 \end{vmatrix} = -3 + 2 = -1 \neq 0 \Rightarrow A^{-1} = \frac{1}{|A|} \cdot adj(A^t) \Rightarrow A^t = \begin{pmatrix} 3 & -2 \\ 1 & -1 \end{pmatrix} \Rightarrow$$

$$adj(A^t) = \begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{(-1)} \cdot \begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -2 & -3 \end{pmatrix} \Rightarrow$$

$$X = \begin{pmatrix} 3 & 1 \\ -2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} 9 & 11 \\ -6 & -7 \end{pmatrix}$$

Comprobación

$$A^{-1}XA = B \Rightarrow \begin{pmatrix} 1 & 1 \\ -2 & -3 \end{pmatrix} \cdot \begin{pmatrix} 9 & 11 \\ -6 & -7 \end{pmatrix} \cdot \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 1 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} = B \; (Comprobado)$$

3.B.- Calcular los siguientes límites:

$a)\ \lim\limits_{x\to\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)$

$b)\ \lim\limits_{x\to\infty} x\left[arc\,tg\left(e^x\right)-\frac{\pi}{2}\right]$

$a)\ \lim\limits_{x\to\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)=\infty-\infty=\lim\limits_{x\to\infty}\frac{\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)\left(\sqrt{x^2+x}+\sqrt{x^2-x}\right)}{\sqrt{x^2+x}+\sqrt{x^2-x}}=$

$=\lim\limits_{x\to\infty}\frac{x^2+x-\left(x^2-x\right)}{\sqrt{x^2+x}+\sqrt{x^2-x}}=\lim\limits_{x\to\infty}\frac{x^2+x-x^2+x}{\sqrt{x^2+x}+\sqrt{x^2-x}}=\lim\limits_{x\to\infty}\frac{2x}{\sqrt{x^2+x}+\sqrt{x^2-x}}=\frac{\infty}{\infty}=$

$=\lim\limits_{x\to\infty}\dfrac{\dfrac{2x}{x}}{\sqrt{\dfrac{x^2}{x^2}+\dfrac{x}{x^2}}+\sqrt{\dfrac{x^2}{x^2}-\dfrac{x}{x^2}}}=\lim\limits_{x\to\infty}\dfrac{2}{\sqrt{1+\dfrac{1}{x}}+\sqrt{1-\dfrac{1}{x}}}=\dfrac{2}{\sqrt{1+\dfrac{1}{\infty}}+\sqrt{1-\dfrac{1}{\infty}}}=\dfrac{2}{\sqrt{1+0}+\sqrt{1-0}}=\dfrac{2}{2}=1$

$b)\ \lim\limits_{x\to\infty} x\left[arc\,tg\left(e^x\right)-\frac{\pi}{2}\right]=\infty\cdot 0=\lim\limits_{x\to\infty}\dfrac{arc\,tg\left(e^x\right)-\dfrac{\pi}{2}}{\dfrac{1}{x}}=\dfrac{0}{0}=\xrightarrow{Utilizando\ L'Hopital}=\lim\limits_{x\to\infty}\dfrac{\dfrac{e^x}{1+e^{2x}}}{-\dfrac{1}{x^2}}=$

$=-\left(\lim\limits_{x\to\infty}\dfrac{x^2 e^x}{1+e^{2x}}\right)=\dfrac{\infty}{\infty}=\xrightarrow{Utilizando\ L'Hopital}=-\left(\lim\limits_{x\to\infty}\dfrac{2xe^x+x^2 e^x}{2e^{2x}}\right)=-\left(\lim\limits_{x\to\infty}\dfrac{xe^x\left(2+x\right)}{2e^{2x}}\right)=-\left(\lim\limits_{x\to\infty}\dfrac{x\left(2+x\right)}{2e^x}\right)=$

$=\dfrac{\infty}{\infty}=\xrightarrow{Utilizando\ L'Hopital}=-\left(\lim\limits_{x\to\infty}\dfrac{2+x+1}{2e^x}\right)=\dfrac{\infty}{\infty}=\xrightarrow{Utilizando\ L'Hopital}=-\left(\lim\limits_{x\to\infty}\dfrac{1}{2e^x}\right)=-\dfrac{1}{\infty}=0$

9

4.B.-Dadas las rectas: $r : \dfrac{x-1}{2} = \dfrac{y-1}{3} = \dfrac{z-1}{4}$ y $s : \dfrac{x+1}{1} = \dfrac{y-2}{-1} = \dfrac{z}{2}$

a) Hallar la ecuación de la recta t que corta a las dos y es perpendicular a ambas

b) Calcular la mínima distancia entre r y s

a) El vector director de la recta t, que arranca de puntos generales de r y s, es perpendicular a estas, por ello los productos escalares de los vectores directores (el de t con cada uno de los de r y s) son nulos. Hallado el punto de corte con r o s tendremos definida la recta t.

$$\begin{cases} r : \begin{cases} x = 1 + 2\lambda \\ y = 1 + 3\lambda \\ z = 1 + 4\lambda \end{cases} \\ s : \begin{cases} x = -1 + \mu \\ y = 2 - \mu \\ z = 2\mu \end{cases} \end{cases} \Rightarrow \begin{cases} \vec{v_t} = \left[1 + 2\lambda - (-1+\mu), 1 + 3\lambda - (2-\mu), 1 + 4\lambda - 2\mu \right] \\ \vec{v_r} = (2, 3, 4) \\ \vec{v_s} = (1, -1, 2) \end{cases} \Rightarrow$$

$$\begin{cases} \vec{v_t} = (2\lambda - \mu + 2, 3\lambda + \mu - 1, 4\lambda - 2\mu + 1) \\ \vec{v_r} = (2, 3, 4) \\ \vec{v_s} = (1, -1, 2) \end{cases} \Rightarrow \begin{cases} \vec{v_t} \perp \vec{v_r} \Rightarrow \vec{v_t} . \vec{v_r} = 0 \\ \vec{v_t} \perp \vec{v_s} \Rightarrow \vec{v_t} . \vec{v_s} = 0 \end{cases} \Rightarrow$$

$$\begin{cases} (2\lambda - \mu + 2, 3\lambda + \mu - 1, 4\lambda - 2\mu + 1) \cdot (2, 3, 4) = 0 \\ (2\lambda - \mu + 2, 3\lambda + \mu - 1, 4\lambda - 2\mu + 1) \cdot (1, -1, 2) = 0 \end{cases} \Rightarrow \begin{cases} 4\lambda - 2\mu + 4 + 9\lambda + 3\mu - 3 + 16\lambda - 8\mu + 4 = 0 \\ 2\lambda - \mu + 2 - 3\lambda - \mu + 1 + 8\lambda - 4\mu + 2 = 0 \end{cases} \Rightarrow$$

$$\begin{cases} 29\lambda - 7\mu + 5 = 0 \\ 7\lambda - 6\mu + 5 = 0 \end{cases} \Rightarrow \begin{pmatrix} 29 & -7 & | & -5 \\ 203 & -174 & | & -145 \end{pmatrix} = \begin{pmatrix} 29 & -7 & | & -5 \\ 0 & -125 & | & -110 \end{pmatrix} \Rightarrow -125\mu = -110 \Rightarrow \mu = \frac{110}{125} = \frac{22}{25}$$

$$7\lambda - 6 \cdot \frac{22}{25} = -5 \Rightarrow 7\lambda = -5 + \frac{132}{25} = \frac{-125 + 132}{25} = \frac{7}{25} \Rightarrow \lambda = \frac{1}{25}$$

$$R \begin{cases} x = 1 + 2 \cdot \dfrac{1}{25} = \dfrac{27}{25} \\ y = 1 + 3 \cdot \dfrac{1}{25} = \dfrac{28}{25} \\ z = 1 + 4 \cdot \dfrac{1}{25} = \dfrac{29}{25} \end{cases} \Rightarrow$$

$$\vec{v_t} = \left(2 \cdot \frac{1}{25} - \frac{22}{25} + 2, 3 \cdot \frac{1}{25} + \frac{22}{25} - 1, 4 \cdot \frac{1}{25} - 2 \cdot \frac{22}{25} + 1 \right) = \left(\frac{2 - 22 + 50}{25}, \frac{3 + 22 - 25}{25}, \frac{4 - 44 + 25}{25} \right)$$

$$R\left(\frac{27}{25}, \frac{28}{25}, \frac{29}{25} \right) \begin{cases} x = 1 + 2 \cdot \dfrac{1}{25} = \dfrac{27}{25} \\ y = 1 + 3 \cdot \dfrac{1}{25} = \dfrac{28}{25} \\ z = 1 + 4 \cdot \dfrac{1}{25} = \dfrac{29}{25} \end{cases} \Rightarrow t : \begin{cases} x = \dfrac{27}{25} + 2\beta \\ y = \dfrac{28}{25} \\ z = \dfrac{29}{25} + \beta \end{cases}$$

$$\vec{v_t} = \left(\frac{30}{25}, \frac{0}{25}, \frac{15}{25} \right) = \left(\frac{6}{5}, 0, \frac{3}{5} \right) \equiv (6, 0, 3) \equiv (2, 0, 1)$$

10

Continúa el problema 4.B.-

b) La mínima distancia entre las rectas r y s es la que existe entre los puntos de corte, R y S, de la perpendicular hallada con cada una de ellas, nos servirá `l` y `m` halladas para buscarlos

$$\left\{ \begin{array}{l} R\left(\dfrac{27}{25},\dfrac{28}{25},\dfrac{29}{25}\right) \\ \\ S\left\{ \begin{array}{l} x=-1+\dfrac{22}{25}=-\dfrac{3}{25} \\ y=2-\dfrac{22}{25}=\dfrac{3}{25} \\ z=2\cdot\dfrac{22}{25}=\dfrac{44}{25} \end{array} \right. \Rightarrow R\left(-\dfrac{3}{25},\dfrac{3}{25},\dfrac{44}{25}\right) \end{array} \right\} \Rightarrow$$

$$d_{rs}=d_{RS}=\sqrt{\left[\dfrac{27}{25}-\left(-\dfrac{3}{25}\right)\right]^2+\left(\dfrac{28}{25}-\dfrac{3}{25}\right)^2+\left(\dfrac{29}{25}-\dfrac{44}{25}\right)^2}=\sqrt{\left(\dfrac{30}{25}\right)^2+\left(\dfrac{25}{25}\right)^2+\left(-\dfrac{15}{25}\right)^2}$$

$$d_{rs}=\sqrt{\left(\dfrac{6}{5}\right)^2+1^2+\left(-\dfrac{3}{5}\right)^2}=\sqrt{\dfrac{36}{25}+1+\dfrac{9}{25}}=\sqrt{\dfrac{36+25+9}{25}}=\sqrt{\dfrac{70}{25}}=\dfrac{\sqrt{70}}{5}\,u$$

11