**1.** Expresa, utilizando logaritmos, las siguientes igualdades:

**1.** $4^3 = 64$ **2.** $7^{-2} = \frac{1}{49}$ **3.** $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$ **4.** $5^0 = 1$ **5.** $2^{x+1} = 12$

**2.** Expresa, usando potencias, las siguientes igualdades:

**1.** $\log_3 81 = 4$ **2.** $\log_5 0'04 = -2$ **3.** $x = \log_2 5$ **4.** $x = \log_3 2 + 1$ **5.** $y = \log_2 x + \log_2 3$

**3.** Halla, sin usar calculadora, los siguientes logaritmos:

**1.** $\log_2 32$ **2.** $\log_3 \frac{1}{27}$ **3.** $\log_5 0'04$ **4.** $\log_{\frac{2}{3}} 2'25$ **5.** $\log \sqrt{0'1}$

**4.** Calcula el valor de x para que la igualdad sea cierta:

**1.** $\log_x 64 = 6$ **2.** $\log_x 9 = -2$ **3.** $\log_x 4 = 4$ **4.** $\log_2 x = 4$ **5.** $\log_x 4 = 0'5$

**5.** Calcula el logaritmo de M y desarrolla por logaritmos:

**1.** $M = 2x^2y$ **2.** $M = \frac{2x^2}{3y}$ **3.** $M = \sqrt[3]{\frac{3x}{y^2}}$ **4.** $M = \frac{2\sqrt{x+1}}{(y+1)^2}$ **5.** $M = 2\left(\frac{2x^2y}{3z}\right)^3$

**6.** Halla el valor de M, siendo:

**1.** $\log M = 2\cdot\log x - 3\cdot\log y$ **2.** $\log M = \log x - \frac{\log y}{2} - 1$ **3.** $\log M = \log x - \frac{\log y}{2} - \frac{2\cdot\log z}{3}$

**7.** Comprueba, sin calculadora, si son ciertas las siguientes igualdades:

**1.** $\log_a b \cdot \log_b a = 1$ **2.** $\log_2 3 \cdot \log_3 4 = 2$ **3.** $\log_4 x - \log_8 x = \log_2 \sqrt[6]{x}$

**8.** Reduce al máximo las siguientes expresiones:

**1.** $2^{3\cdot\log_4 x}$ **2.** $\log_2 \sqrt{x} + \log_4 x$ **3.** $\log_{\sqrt{x}} 4 - \log_x 4 - \log_{x^2} 4$

**9.** Halla, razonadamente, los valores de x que cumplen la igualdad:

**1.** $4^{\log_2 x} = 3$ **2.** $2^{\log_4 x} = 3$ **3.** $4^{\log_2 x} = 2^{\log_2 x - 2}$ **4.** $4^{\log_2 x} = 2^{2-\log_4 x}$

**10.** Sabiendo que $\log 2 = 0'301$ y $\log 3 = 0'477$, halla, sin usar calculadora:

**1.** $\log 72$ **2.** $\log 56'25$ **3.** $\log \frac{0'125}{\sqrt{6}}$ **4.** $\log \frac{20\sqrt{6}}{\sqrt[3]{30}}$

**11.** Sabiendo que $\log x = 2'3$ y $\log y = 3'2$, calcula:

**1.** $\log x^3\sqrt{y}$ **2.** $\log_x y - \log_y x$ **3.** $\log_x 10 + \log_y 100$ **4.** $\log_{x^2 y} \sqrt{10}$

**12.** Calcula, aplicando logaritmos:

**1.** $\frac{0'5\sqrt{3}}{1'2}$ **2.** $\sqrt{\frac{1'22}{0'3^3}}$ **3.** $\frac{2100}{20\sqrt[3]{2'21}}$ **4.** $\left(2'1\sqrt[3]{2'2\sqrt{2'3}}\right)^3$

**13.** Despeja x en la siguiente igualdad, usando logaritmos decimales si es necesario:

**1.** $M = (y+1)^{x+1}$ **2.** $M = \frac{2y^{2x-1}}{3} + 1$ **3.** $M = \sqrt{\left(y^2-1\right)^{x-1}}$ **4.** $\log x - \log y = \log (x-y)$

**14.** Halla el valor de x (aproximando a las centésimas), para los valores que se indican:

**1.** $y = 2^{x+1}$ ; $y = 0'5$ **2.** $c = 2(a+2)^{2x}$ ; $c = 5, a = 0'3$ **3.** $y = \dfrac{2\cdot\sqrt[x-2]{k}}{k-1}$ ; $y = -2'3, k = 0'5$

**15.** Resuelve las siguientes ecuaciones exponenciales:

**1.** $2^{2x-3} = 8$ **2.** $3^{\frac{4-x}{3}} = 9$ **3.** $3^{x-1} - 9^{1-x} = 0$ **4.** $\dfrac{4^{2x+1}}{8^{2-x}} = 1$

**5.** $9^{x^2+x} = 3^{x+1}$ **6.** $5^{x^2-x-2} - 1 = 0$ **7.** $\sqrt{2^{2x-3}} = 4^{x-1}$ **8.** $\sqrt{3^{x+2}} = \sqrt[3]{9^{2x-1}}$

**9.** $\sqrt{2^x\sqrt{2^{x+2}\sqrt{2^{x-2}}}} = 4$ **10.** $\sqrt[3]{3^{x+1}\sqrt{3^x\sqrt[3]{3^{x+2}}}} = 3$ **11.** $\sqrt[3x-2]{2} = \sqrt[x+1]{4}$ **12.** $\sqrt[x+1]{4^{x+2}} = \sqrt[2x-1]{8^{x+2}}$

**13.** $\sqrt{2}^{\sqrt{x+1}} = \sqrt[3]{8^{x-2}}$ **14.** $\sqrt[3]{2}^{\sqrt{x+3}} = \sqrt[x+2]{2^{3-x}}$ **15.** $2^{x+1} = 5$ **16.** $2^{2x-1} = 3^{1-x}$

**17.** $2^{x^2-2} = 3^{x-1}$ **18.** $\sqrt{2^{x+2}} = \sqrt[x-1]{3^x}$ **19.** $2^{x+2} - 2^{x+1} - 2^x = 2^3$ **20.** $3^x - 3^{x-1} - 3^{x-2} = 5$

**21.** $\dfrac{11}{3} - \dfrac{1}{3^{1-x}} + \dfrac{1}{3^{2-x}} = 3^x$ **22.** $\dfrac{1}{2^{1-x}} + \dfrac{1}{2^{2-x}} + \dfrac{5}{8} = 2^{x+1}$ **23.** $\dfrac{1}{5^{1-x}} - 4\cdot5^{x-2} = 5$ **24.** $5\cdot2^{x-1} - \dfrac{3}{2^{1-x}} - \dfrac{3}{2^{2-x}} = 2$

**25.** $2^{2x-1} + 2^{2x-3} - 2^{x+1} = 2$ **26.** $3 + 3^{2x+1} - 3^{x+2} = 3^x$ **27.** $5^{1+x} + 5^{1-x} = 26$ **28.** $2^{2-x} - 2^{1-x} = 4 + 2^{4+x}$

**29.** $\dfrac{1}{2^{x-2}} - \dfrac{1}{2^{x-3}} = 2^x - 5$ **30.** $\dfrac{1}{5^{x-1}} = 5^x - 4$ **31.** $2^{x+4} - 2^{x+3} - 2^{x+2} = 4^{x+2}$ **32.** $3^{2x+3} - 3^{x+2} = 1 - 9^{x+1}$

**33.** $2\cdot3^{2x-1} + 3^{x+1} - 9^x = 1 - 3^{x-2}$ **34.** $\dfrac{3}{2^{1-x}} - 4^{x-1} = 1 - 5\cdot2^{x-3}$ **35.** $3^{2x+1} - 3^{x+1} - 2\cdot9^x = 1 - \dfrac{1}{3^{x-1}}$ **36.** $8^{x-1} - \dfrac{5}{4^{1-x}} + \dfrac{21}{2^{3-x}} = 1 + 4^{x-2}$

**37.** $5^{2x+1} - 4\cdot25^x + 25^{1-x} = 26$ **38.** $2\cdot4^{x+1} + \dfrac{1}{2^{3-2x}} - \dfrac{1}{2^{1-4x}} = 2$ **39.** $x^2 2^x - x2^{x+1} - 2^{x+3} = 0$ **40.** $\dfrac{x^2}{2^{1-x}} - 3x2^x + 2^{x+2} = 0$

**41.** $2^{x+1} - 2^x - 2^{x-1} = 3$ **42.** $3^x - \dfrac{2}{3^{1-x}} - \dfrac{1}{3^{2-x}} = 4$ **43.** $2\cdot3^{x-2} + 3^x - 9^{x-1} = 2$ **44.** $3\cdot2^x + \dfrac{1}{3\cdot2^{1-x}} - 2^{2x-1} = 1$

**16.** Resuelve los siguientes sistemas exponenciales:

**1.** $\begin{cases} 2^{2x-y} = 8 \\ 3^{2x+y} = 3 \end{cases}$ **2.** $\begin{cases} 4^{2x-y} = 8 \\ 9^{x-2y} = 1 \end{cases}$ **3.** $\begin{cases} 8^{x+y+1} - 4^{y-x} = 0 \\ 9^{x+1} - 3^{x+y-1} = 0 \end{cases}$ **4.** $\begin{cases} 2^{x+y-1} = 4^{2-x} \\ 3^{x-y} = 9^{2-x} \end{cases}$

**5.** $\begin{cases} 3^{x+y+1} = 9^{x-y-3} \\ 4^{x+y} = \sqrt{2^{2x+3y}} \end{cases}$ **6.** $\begin{cases} 3^{x-y} = \sqrt[3]{3^{x+1}} \\ 2^{x-2} = \sqrt[3]{4^{1-y}} \end{cases}$ **7.** $\begin{cases} \sqrt[2x-y]{3} = \sqrt[x+2]{9} \\ \sqrt[x+4]{4} = \sqrt[4y+1]{8} \end{cases}$ **8.** $\begin{cases} \sqrt[x]{4^{x+1}} = \sqrt[y]{2^{2y-1}} \\ \sqrt[x-1]{3^{2x-1}} = \sqrt[y-1]{9^{y-2}} \end{cases}$

**9.** $\begin{cases} 4^{x-1} = \sqrt{2^{y+2}} \\ \sqrt[x]{3^y} = 9^{x-2} \end{cases}$ **10.** $\begin{cases} \sqrt[2x-y]{3} = \sqrt[x+2]{9} \\ \sqrt[2x+1]{2^{x+1}} = \sqrt[y+1]{2^y} \end{cases}$ **11.** $\begin{cases} 3^y - 2^x = 1 \\ 2^{x+2} - 3^{y+1} = 5 \end{cases}$ **12.** $\begin{cases} 2^{x+3} - 3^{y-1} = 1 \\ 2^{x+2} + 3^{y-2} = 3 \end{cases}$

**13.** $\begin{cases} 2^{x+3} + 2^{x+2} - 3^{y-1} = 0 \\ 3^y - 3^{y-2} - 2^{x+4} = 4 \end{cases}$ **14.** $\begin{cases} 2^{y+1} + 3\cdot2^y - 3^{x+3} = 1 \\ 3^{x+2} - 5\cdot3^{x+1} + 2^{y+1} = 2 \end{cases}$ **15.** $\begin{cases} \sqrt{2^x} - \sqrt{3^y} = 1 \\ \sqrt{2^{x-2}} - \sqrt{3^{y-2}} = 1 \end{cases}$ **16.** $\begin{cases} \sqrt{2^{x+1}} - \sqrt{3^{y-1}} = 1 \\ \sqrt{3^{y+1}} - \sqrt{2^{x+3}} = 1 \end{cases}$

**17.** $\begin{cases} 3^{y+1} - 2^{x+1} = 1 \\ 9^{y-1} - 4^{x-2} = 0 \end{cases}$ **18.** $\begin{cases} 3^{y-1} - 2^{x+2} = 1 \\ 4^{x+1} - 5\cdot3^{y-2} = 1 \end{cases}$ **19.** $\begin{cases} 2^y + 3^{x+1} - 9^x = 4 \\ 5\cdot3^x - 2^y - 9^x = 2 \end{cases}$ **20.** $\begin{cases} 2^{x+2} - 9^{y+1} = 7 \\ 2\cdot3^{y+2} - 4^x = 2 \end{cases}$

**21.** $\begin{cases} 2^x = 3^y \\ x + y = 2 \end{cases}$ **22.** $\begin{cases} 2^{x+1} = 5^{y-1} \\ x + y = 1 \end{cases}$ **23.** $\begin{cases} 2^x - 3^y = 1 \\ 2^{x+3} - 3^{y+2} = 6 \end{cases}$ **24.** $\begin{cases} 3^y + 3\cdot2^x = 3 \\ 2\cdot9^y - 9\cdot2^{2x+1} = 7 \end{cases}$

**17.** Resuelve las siguientes ecuaciones logarítmicas:

**1.** $2\cdot\log x - \log 4 = \log 9$ **2.** $\log 2x + \log 2 = 2\cdot\log x$ **3.** $2\cdot\log x - \log 8 = \log\dfrac{x}{2}$

**4.** $5\cdot\log x = 3\cdot\log x + 2\cdot\log 3$ **5.** $3\cdot\log\dfrac{x}{2} + 2\cdot\log\dfrac{x}{3} = 3\cdot\log x - \log 8$ **6.** $3\cdot\log x - 2\cdot\log\dfrac{x}{3} = 2\cdot\log 3 + 2\cdot\log 2x$

**7.** $\log\dfrac{x}{5} + 1 = 2\cdot\log x$ **8.** $2\cdot\log 4x + \log\dfrac{x}{2} = 3$ **9.** $\log_2\sqrt{x} - \log_2\sqrt{2} = \dfrac{1}{2}$

**10.** $\log \sqrt[3]{x} - \log 2 = \log \sqrt[3]{2} - \log \sqrt{2x}$ **11.** $\log (2x+12) - \log (3x-2) = \log 2$ **12.** $\log x - \log 2 = 2\cdot\log (x-3)$

**13.** $\log(x-1) + \log(x-2) - \log 3 = 2\cdot\log(x-3)$ **14.** $2\cdot\log(x-1) - \log(x+2) = \log(x+1) - \log 2$ **15.** $\log (5x+4) - \log 2 = \log \sqrt{x+4}$

**16.** $\log (2x+14) - \log 2 = \log \sqrt{x+5} + \log 3$ **17.** $\log \sqrt{3x-2} - \log \sqrt{2x-3} = 1 - \log 5$ **18.** $\log \sqrt{3x+4} + \log \sqrt{5x+1} = 1 + \log 3$

**19.** $\dfrac{\log\left(x^2-3\right)}{\log (x+3)} = 2$ **20.** $\dfrac{\log\left(4-x^2\right)}{\log (2+x)} = 2$ **21.** $\dfrac{\log 2 + \log (x+2)}{\log (x+1)} = 2$

**22.** $\dfrac{1 + \log_2 (x+6)}{\log_2 (x+2)} = 2$ **23.** $\dfrac{1 + 2\cdot\log x}{\log x} - \log x = 2$ **24.** $\log_2 \sqrt{x} - \log_2 \sqrt{2} = \dfrac{1}{\log_2 x}$

**25.** $\log_2 x - \log_4 x = 1$ **26.** $\log_2 x - \log_x 8 = 2$ **27.** $\log_{2x} 16 - \log_x 2 = 0$

**28.** $\log_x 4 + \log_{2x} 4 = 3$ **29.** $\dfrac{3\cdot\log x}{x} + \log \sqrt{x} = \log x^x$ **30.** $\log_2 x^6 - 8\cdot\log_{2^x} x = \log_2 x^x$

**18.** Resuelve los siguientes sistemas logarítmicos:

**1.** $\begin{cases} x + y = 8 \\ \log x - \log y = \log 3 \end{cases}$ **2.** $\begin{cases} x + y = 6 \\ \log 2x - \log y = 1 \end{cases}$ **3.** $\begin{cases} 4^{x-1} - 2^{y+1} = 0 \\ \log 5x - \log \dfrac{y}{2} = 1 \end{cases}$ **4.** $\begin{cases} 3^{1-x} - 9^{y-2} = 0 \\ \log_2 2x - \log_2 3y = 1 \end{cases}$

**5.** $\begin{cases} x + y = 7 \\ \log x + \log y = 1 \end{cases}$ **6.** $\begin{cases} x + 2y = 5 \\ \log 5x + \log y = 1 \end{cases}$ **7.** $\begin{cases} 9^{x-6} - 3^{3-2y} = 0 \\ \log 2x + \log 4y = 2 \end{cases}$ **8.** $\begin{cases} 8^{x-2} - 2^{1-3y} = 0 \\ \log_2 2x + \log_2 3y = 2 \end{cases}$

**9.** $\begin{cases} 2\left(x^2+y\right) = 5(3x-4) \\ \log_2 y = 2\left(\log_2 x - 1\right) \end{cases}$ **10.** $\begin{cases} 6x - x^2 - y = 3 \\ \log x + \log y = 1 \end{cases}$ **11.** $\begin{cases} y - \log_2 x = 1 \\ \log_2 x^3 - y = 1 \end{cases}$ **12.** $\begin{cases} \log_2 x + y = 3 \\ \log_2 x^3 + y^2 = 7 \end{cases}$

**13.** $\begin{cases} \log 2x + \log 5x = 2 \\ \log 4x - \log y = 1 \end{cases}$ **14.** $\begin{cases} \log_2 x + \log_2 y = 3 \\ \log_2 8x - \log_2 2y = 1 \end{cases}$ **15.** $\begin{cases} \log x^2 + \log y^3 = 2 \\ \log x^3 - \log y^2 = 3 \end{cases}$ **16.** $\begin{cases} \log_2 x + \log_2 y = 3 \\ \log_2 x^2 - \log_2 y^3 = 1 \end{cases}$

**17.** $\begin{cases} \log_2 x + \log_4 y = 2 \\ \log_4 x - \log_2 y = 1 \end{cases}$ **18.** $\begin{cases} \log_3 x^3 - \log_2 y^2 = 1 \\ \log_9 x + \log_4 y = 1 \end{cases}$ **19.** $\begin{cases} \log_x 4 + \log_y 2 = 2 \\ \log_2 x - \log_2 y = 1 \end{cases}$ **20.** $\begin{cases} \log_4 x + \log_9 y = 1 \\ \log_x 4 - \log_y 3 = 1 \end{cases}$

— Soluciones

**1.1.** $\log_4 64 = 3$ **1.2.** $\log_{\frac{1}{7}} \frac{1}{49} = -2$ **1.3.** $\log_{\frac{1}{2}} \frac{1}{16} = 4$ **1.4.** $\log_5 1 = 0$ **1.5.** $\log_2 12 = x+1$ **2.1.** $3^4 = 81$ **2.2.** $5^{-2} = 0'04$ **2.3.** $2^x = 5$ **2.4.** $3^{x-1} = 2$ **2.5.** $2^y = 3x$ **3.1.** 5 **3.2.** -3 **3.3.** -2 **3.4.** -2 **3.5.** $\frac{-1}{2}$ **4.1.** 2 **4.2.** $\frac{1}{3}$ **4.3.** $\sqrt{2}$ **4.4.** 16 **4.5.** 2 **5.1.** $\log M = \log 2+2\log x+\log y$ **5.2.** $\log M = \log 2+2\log x-\log 3-\log y$ **5.3.** $\log M = \frac{\log 3+\log x-2\log y}{3}$ **5.4.** $\log M = \log 2+\frac{\log(x+1)}{2}-2\log(y+1)$ **5.5.** $\log M = 4\log 2+6\log x+3\log y-3\log z$ **6.1.** $\frac{x^2}{y^3}$ **6.2.** $\frac{x}{10\sqrt{y}}$ **6.3.** $\frac{x\sqrt[3]{z^2}}{\sqrt{y}}$ **8.1.** $x\sqrt{x}$ **8.2.** $\log_2 x$ **8.3.** $\log_x 2$ **9.1.** $\sqrt{3}$ **9.2.** 9 **9.3.** 2 **9.4.** $\sqrt[5]{4}$ **10.1.** 1'857 **10.2.** 1'75 **10.3.** -1'292 **10.4.** 1'198 **11.1.** 8'5 **11.2.** 0'673 **11.3.** -0'19 **11.4.** 0'064 **12.1.** 0'722 **12.2.** 6'722 **12.3.** 80'611 **12.4.** 9'847 **13.1.** $\frac{\log M}{\log(y+1)}-1$ **13.2.** $\frac{\log\frac{3(M+1)}{2y}}{2\log y}$ **13.3.** $\frac{2\log M}{\log\left(y^2-1\right)}+1$ **13.4.** $\frac{y^2}{y-1}$ **14.1.** -2 **14.2.** 0'55 **14.3.** 3'25 **15.1.** 3 **15.2.** -2 **15.3.** 1 **15.4.** 4 **15.5.** $-1, \frac{1}{2}$ **15.6.** -1, 2 **15.7.** $\frac{1}{2}$ **15.8.** 2 **15.9.** 2 **15.10.** 1 **15.11.** 1 **15.12.** -2, 5 **15.13.** 3 **15.14.** 1 **15.15.** 1'32 **15.16.** 0'72 **15.17.** -0'23, 1'81 **15.18.** -0'70, 2'87 **15.19.** 3 **15.20.** 2 **15.21.** 1 **15.22.** -1 **15.23.** 3 **15.24.** 3 **15.25.** 2 **15.26.** -1, 1 **15.27.** -1,1 **15.28.** -2 **15.29.** 0, 2 **15.30.** 1 **15.31.** -2 **15.32.** -1 **15.33.** -1, 2 **15.34.** -1, 3 **15.35.** 0, 1 **15.36.** -1, 1, 3 **15.37.** 0, 1 **15.38.** -1, 2 **15.39.** -2, 4 **15.40.** 2, 4 **15.41.** 2'58 **15.42.** 2'63 **15.43.** 0'63 **15.44.** -1'58, 2'58 **16.1.** (1,-1) **16.2.** $\left(1,\frac{1}{2}\right)$ **16.3.** (-1,2) **16.4.** $\left(\frac{3}{2},\frac{1}{2}\right)$ **16.5.** (1,-2) **16.6.** (2,1) **16.7.** (2,2) **16.8.** (2,-1) **16.9.** (1,-2), (3,6) **16.10.** (1,2), $\left(-2,\frac{1}{2}\right)$ **16.11.** (3,2) **16.12.** (-1,2) **16.13.** (-2,2) **16.14.** (-1,1) **16.15.** (4,2) **16.16.** (3,3) **16.17.** (2,1) **16.18.** (1,3) **16.19.** (0,1), (1,2) **16.20.** (1,-1), (2,0) **16.21.** $\left(\log_6 9,\log_6 4\right)$ **16.22.** $\left(-\log 2,\log 20\right)$ **16.23.** $\left(\log_2 3,\log_3 2\right)$ **16.24.** $\left(-\log_2 3,\log_3 2\right)$ **17.1.** 6 **17.2.** 4 **17.3.** 4 **17.4.** 3 **17.5.** 3 **17.6.** $\frac{1}{4}$ **17.7.** 2 **17.8.** 5 **17.9.** 4 **17.10.** 2 **17.11.** 4 **17.12.** $\frac{9}{2}$ **17.13.** 5 **17.14.** 7 **17.15.** 0 **17.16.** -1, -4 **17.17.** 2 **17.18.** 7 **17.19.** $\frac{-5}{3}$ **17.20.** 0 **17.21.** $\sqrt{3}$ **17.22.** 2 **17.23.** $\frac{1}{10}$, 10 **17.24.** $\frac{1}{2}$, 4 **17.25.** 4 **17.26.** $\frac{1}{2}$, 8 **17.27.** $\sqrt[3]{2}$ **17.28.** $2, \frac{\sqrt[3]{2}}{2}$ **17.29.** 1, 2 **17.30.** 1, 2, 4 **18.1.** (6,2) **18.2.** (5,1) **18.3.** (3,3) **18.4.** (3,1) **18.5.** (5,2), (2,5) **18.6.** (1,2), $\left(4,\frac{1}{2}\right)$ **18.7.** $\left(\frac{5}{2},5\right), \left(5,\frac{5}{2}\right)$ **18.8.** $\left(\frac{1}{3},2\right), \left(2,\frac{1}{3}\right)$ **18.9.** (2,1), (4,4) **18.10.** (2,5),(5,2) **18.11.** (2,2) **18.12.** (2,2), (4,1) **18.13.** (5,2) **18.14.** (2,4) **18.15.** (10,1) **18.16.** (4,2) **18.17.** (4,1) **18.18.** (3,2) **18.19.** $\left(\sqrt{2},\frac{\sqrt{2}}{2}\right)$ **18.20.** (2,3), $\left(16,\frac{1}{9}\right)$